# Optimal Work Extraction and Thermodynamics of Quantum Measurements and Correlations

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We analyze the role of indirect quantum measurements in work extraction from quantum systems in nonequilibrium states. In particular, we focus on the work that can be obtained by exploiting the correlations shared between the system of interest and an additional ancilla, where measurement backaction introduces a nontrivial thermodynamic tradeoff. We present optimal state-dependent protocols for extracting work from both classical and quantum correlations, the latter being measured by discord. Our quantitative analysis establishes that, while the work content of classical correlations can be fully extracted by performing local operations on the system of interest, accessing work related to quantum discord requires a specific driving protocol that includes interaction between system and ancilla.

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One of the aims of quantum thermodynamics [1,2] is the precise identification of the role of genuine quantum resources, such as coherence [3], correlations [4], or squeezing [5], both in the performance of thermodynamic tasks by nanomachines [6–8], and, on a more fundamental ground, in the description of finite-time nonequilibrium thermodynamic processes [9–11]. In this context, interest has been raised toward the process of work extraction from quantum systems [12,13], and on its enhancement in feedback protocols [14]. Although entanglement generation is not essential for optimal work extraction from quantum batteries [15], it can, nevertheless, be exploited to increase the amount of extractable work [16–18]. Quantum discord can enter prominently in enhancing the performance of Maxwell demons [19], heat engines [20], and work extraction protocols [21] as well, and both entanglement and discord have been shown to play a role in the work gain obtained thanks to a feedback enhanced extraction protocol [22]. However, obtaining quantitative connections between the extracted work and both classical and quantum correlations has been shown very challenging, since these may depend on the allowed operations [18].

The prototypical scenario considers a cyclic unitary transformation to extract work from a quantum system $S$ [12]. This case has been extended by considering that $S$ shares correlations with an ancilla $A$ [22]. The unitary transformation setting has been generalized as well, by including thermalization processes [23,24]. The scope of this Letter is to obtain quantitative relations between the optimal work gain and classical and quantum correlations in this general framework of work extraction, where both access to a thermal reservoir is allowed [23,24] and feedback is provided by a measurement performed on $A$. We will show that a tight link exists between the optimal work gain obtainable in presence of feedback and the *classical* correlations. Turning to the role of quantum correlations, we will introduce a work contribution due to quantum discord, arguing that it cannot be extracted in a feedback enforced protocol, as it is unavoidably lost after a local measurement is performed. However, an improved protocol can be designed in which the work content of quantum discord can be possibly extracted *before* the measurement and the feedback enhanced protocol are performed. In doing this, we will elucidate the role and the energetic value of both classical and quantum correlations and, at the same time, discuss the energetic cost of the measurement that is necessary to provide feedback. Contrary to the classical case, quantum measurements introduce a tradeoff between the gain in extractable work due to the measurement-induced local entropy reduction, and its loss due to correlations erasure. Despite this, we will show that the total amount of work extractable with the generalized protocol can overcome the one obtained without measurement and feedback, provided optimal measurements are performed.

*Setup.*—We consider work extraction from a quantum system with Hamiltonian $H_S$ in an arbitrary nonequilibrium state $\rho_S$. In the most simple situation, only unitary operations are allowed, where the Hamiltonian changes according to some cyclic protocol, in which $H_S$ is the same

 

Hamiltonian before and after the operation. In such a case, the maximum work that can be extracted from an initial (nonpassive) state $\rho_S$ is the so-called ergotropy $\mathcal{W}_S$ [12]. This framework can be naturally extended, with a performance enhancement, by including also nonunitary transformations. In particular, if, in addition to unitary cyclic driving, contact with a thermal reservoir is also allowed, the extracted work may increase due to both the system entropy varying during the protocol, and to the reservoir providing some extra energy. In this case, the maximum amount of extractable work is given by the difference in nonequilibrium free energy between the state $\rho_S$ and the thermal equilibrium state at the reservoir inverse temperature $\beta \equiv 1/k_B T$ [23,24],

$$W_{\text{ext}} \leq \mathcal{W}_S^\beta = \mathcal{F}_\beta(\rho_S) - \mathcal{F}_\beta(\rho_S^\beta) = k_B T D(\rho_S || \rho_S^\beta), \quad (1)$$

where $\rho_S^\beta = e^{-\beta H_S}/Z_S$ is the equilibrium state, while $D(\rho||\sigma) = \mathrm{Tr}[\rho(\log\rho - \log\sigma)]$ stands for the quantum relative entropy [25,26]. The nonequilibrium free energy for a system in state $\rho_S$, with Hamiltonian $H_S$, and with respect to a thermal bath at temperature $T$ is defined as,

$$\mathcal{F}_\beta(\rho_S) = \mathrm{Tr}[H_S \rho_S] - k_B T S(\rho_S), \quad (2)$$

where $S(\rho) = -\mathrm{Tr}[\rho \ln \rho]$ is the von Neumann entropy, and where, for thermal states $\mathcal{F}_\beta(\rho_S^\beta) = -k_B T \ln Z_S$ reduces to the Helmholtz free energy. Equality in Eq. (1) may be obtained by implementing an operationally reversible isothermal process [13,23,24]. This is made up of two steps: first, a sudden quench is performed, in which the Hamiltonian $H_S$ is changed into $H_{\rho_S} = -k_B T \ln \rho_S$; then, a quasistatic isothermal transformation follows, during which the Hamiltonian turns back to $H_S$, while the system is kept in contact with the heat bath. In this second step, the system always stays in equilibrium with the reservoir, ending up in the state $\rho_S^\beta$ [23,24]. Notice that here it is assumed that the thermal reservoir always induces decoherence and dissipation in the instantaneous energy eigenbasis [27]. Such an isothermal transformation can be constructed by means of an infinite sequence of quantum maps acting over infinitesimal time steps (the demostration is left to Ref. [28]). This optimal isothermal work extraction procedure always outperforms cyclic unitary protocols: independently of the temperature, one can show that the decrease in free energy is larger than the ergotropy, $\mathcal{W}_S^\beta \geq \mathcal{W}_S, \ \forall \beta$, where the equality is achieved only when the temperature verifies $S(\rho_S^\beta) = S(\rho_S)$ (the proof is given in Ref. [28]). Notice that the presence of the environment plays here a constructive role, allowing an extra source of energy and increasing our ability to extract work.

In the following, we will extend the optimal isothermal protocol to the case in which the system of interest $S$ is prepared in a joint state $\rho_{SA}$ with an uncoupled ancillary system $A$, with which it may share classical and/or quantum correlations. Specifically, the total amount of correlations between the two parts can be measured by the quantum mutual information $I(\rho_{SA}) = D(\rho_{SA}||\rho_S \otimes \rho_A) \geq 0$, where $\rho_S = \mathrm{Tr}_A[\rho_{SA}]$ and $\rho_A = \mathrm{Tr}_S[\rho_{SA}]$ are the marginal (reduced) states.

We will first show that the amount of work extractable from the system of interest increases when some information is provided after a measurement is performed on the ancilla. In this way, we provide both a generalization of the nonequilibrium isothermal work extraction setup to include quantum measurement induced feedback, and a generalization to the case of entropy-changing transformations of the result of Ref. [22] for the ergotropy.

To start with, let us define $\mathcal{W}_{S|\pi_A}^\beta$ as the maximum amount of work extractable from $S$ by exploiting the feedback obtained from a measurement performed on $A$. In particular, we consider a projective measurement, described by the set of projectors $\pi_A = \{\Pi_A^k\}$ for $k = 1, \ldots, d_A$. After optimizing the extracted work over all possible sets of projectors $\pi_A$ (that is, over all possible measurements on $A$), we define $\mathcal{W}_{S|A}^\beta = \max_{\pi_A} \mathcal{W}_{S|\pi_A}^\beta$ (see Fig. 1). Therefore, during this first part, we take as an operational assumption that work is only extracted from the system and not from the ancilla.

The measurement affects both the ancilla and the system state. In particular, if the outcome $k$ occurs (with probability $p_k = \mathrm{Tr}[\Pi_A^k \rho_{SA}]$), then the state of the system is updated to

$$\rho_S \to \rho_{S|\Pi_A^k} = \mathrm{Tr}_A[(\mathbb{I}_S \otimes \Pi_A^k)\rho_{SA}(\mathbb{I}_S \otimes \Pi_A^k)]/p_k, \quad (3)$$

$\mathbb{I}_S$ being the identity operator for $S$. Notice that the measurement operators commute with the Hamiltonian of the system, $[H_S, \mathbb{I}_S \otimes \Pi_A^k] = 0 \ \forall \ k$; therefore the system energy remains constant during the measurement process.

For an initially uncorrelated $SA$ state, that is, for $\rho_{SA} = \rho_S \otimes \rho_A$—or, equivalently, $I(\rho_{SA}) = 0$—measurements on the ancilla do not induce any change in the system state.

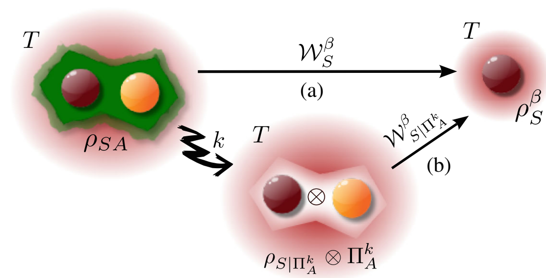


FIG. 1. Starting from $\rho_{SA}$, work can be extracted from $S$ either by a direct isothermal protocol [path (a)], or by first performing a measurement on $A$, and then applying an outcome dependent isothermal protocol [path (b)].

*Extracting work from classical correlations.*—In the optimal isothermal protocol discussed above, the maximum work that can be extracted on average from $S$ is increased at best by the amount of *classical* correlations initially present in the state $\rho_{SA}$.

To show this, we start by considering that, if the ancilla is subjected to the projective measurement introduced above, when outcome $k$ is obtained, $S$ suffers from the backaction corresponding to Eq. (3), and this amounts to a change in the system's free energy

$$\Delta\mathcal{F}^{\beta}_{S|\Pi_A^k} = \mathcal{F}_{\beta}(\rho_{S|\Pi_A^k}) - \mathcal{F}_{\beta}(\rho_S). \quad (4)$$

The outcome $k$ being known, we may adapt the optimal isothermal protocol introduced above, which now will depend on $k$: in the first step, a $k$-dependent sudden quench of the Hamiltonian is performed, with $H_S \to H_{\rho_{S|\Pi_A^k}} = -k_B T \ln \rho_{S|\Pi_A^k}$, which is then quasistatically brought back to $H_S$ while in contact with the thermal reservoir. This process requires precise knowledge of the state $\rho_{S|\Pi_A^k}$, which in turn implies knowing the initial state $\rho_{SA}$ and the set of projectors $\pi_A$.

With the same argument recalled above, one may conclude that the maximum amount of work extractable from the state $\rho_{S|\Pi_A^k}$ is given by $\mathcal{W}^{\beta}_{S|\Pi_A^k} \equiv \mathcal{F}_{\beta}(\rho_{S|\Pi_A^k}) - \mathcal{F}_{\beta}(\rho_S^{\beta})$. The average work extracted after many repetitions of this process is then

$$\mathcal{W}^{\beta}_{S|\pi_A} = \sum_k p_k \mathcal{W}^{\beta}_{S|\Pi_A^k} = \sum_k p_k \Delta\mathcal{F}^{\beta}_{S|\Pi_A^k} + \mathcal{W}^{\beta}_S, \quad (5)$$

where, in the last equality, we used Eqs. (4) and (1). A sketch of the protocol and of this result is given in Fig. 1.

The average change of the generalized free energy, entering the Eq. (5) above, has a clear information theoretic interpretation when it is expressed in terms of the entropy change: $\sum_k p_k \Delta\mathcal{F}^{\beta}_{S|\Pi_A^k} = k_B T[S(\rho_S) - \sum_k p_k S(\rho_{S|\Pi_A^k})] \equiv k_B T J(\rho_{SA})_{\pi_A}$. The quantity $J(\rho_{SA})_{\pi_A}$ gives the mutual information extracted by the local measurement performed on $A$ by using the set of projectors $\pi_A = \{\Pi_A^k\}$ [30,31]. The same quantity has been employed to discuss feedback controlled protocols in Ref. [32].

As a result, the average increase of the work extracted during the process reads

$$\Delta\mathcal{W}^{\beta}_{S|\pi_A} = \sum_k p_k \Delta\mathcal{F}^{\beta}_{S|\Pi_A^k} = k_B T J(\rho_{SA})_{\pi_A} \geq 0, \quad (6)$$

where $\Delta\mathcal{W}^{\beta}_{S|\pi_A} = \mathcal{W}^{\beta}_{S|\pi_A} - \mathcal{W}^{\beta}_S$ is the gain in extractable work, and where the inequality in Eq. (6) follows directly from concavity of von Neumann entropy and implies that an average enhancement in the extracted work is found for any measurement. No gain is obtained only if $\rho_{SA}$ is factorized, while, if $S$ and $A$ are correlated to some extent the extractable work can increase thanks to the feedback coming from the knowledge of the measurement outcome. Intuitively, this is due to the fact that a measurement can increase the free energy of $S$ [24].

When optimized over all possible measurements, the quantity $J(\rho_{SA})_{\pi_A}$ introduced above gives a measure of the classical correlations shared by $S$ and $A$ in the state $\rho_{SA}$, as defined in Refs. [30,31]. There, a measurement oriented framework is put forward and classical correlations are defined as $J(\rho_{SA}) = \max_{\pi_A} J(\rho_{SA})_{\pi_A}$.

Thus, if we maximize Eq. (6) over all sets of projectors $\pi_A$, we obtain that the maximum enhancement in work extraction $\Delta\mathcal{W}^{\beta}_{S|A} \equiv \max_{\pi_A} \Delta\mathcal{W}^{\beta}_{S|\pi_A}$ is precisely given by

$$\Delta\mathcal{W}^{\beta}_{S|A} = k_B T J(\rho_{SA}), \quad (7)$$

Eq. (7) is the first of our main results; it tells us that the gain in the work extracted from $S$ thanks to the feedback protocol in which $A$ is measured, is due to (and upper bounded by) the classical correlations shared by $S$ and $A$.

Even if quantum correlations do not contribute to Eq. (7), this does not imply that they do not play any role, as we will see in the remainder of this Letter.

In a classical context, where measurement backaction can be avoided in principle, the correlation function $J$ would coincide with the mutual information $I$, stemming from Bayes' rule. In the quantum regime, the difference between these two classically equivalent definitions of mutual information, called discord [30,31], gives a measure of the amount of nonclassical correlations in the state $\rho_{SA}$ [33],

$$\mathcal{D}(\rho_{SA}) \equiv I(\rho_{SA}) - J(\rho_{SA}) \geq 0. \quad (8)$$

Our result in Eq. (7) gets a clear physical interpretation if discord is used to understand it. From its definition, we can understand quantum discord as the amount of correlations present in a bipartite quantum state, which cannot be accessed by local measurements on one party. Therefore, intuition dictates that as long as this information is not available from measuring the ancilla $A$, it cannot be used in any way to improve our ability of extracting work from $S$. More precisely, the work extractable from the whole $SA$ system in the state $\rho_{SA}$ is given by the free-energy difference between this state and the thermal reference one,

$$\begin{aligned}\mathcal{W}^{\beta}_{SA}(\rho_{SA}) &\equiv \mathcal{F}_{\beta}(\rho_{SA}) - \mathcal{F}_{\beta}(\rho_S^{\beta} \otimes \rho_A^{\beta}) \\ &= \mathcal{W}^{\beta}_S(\rho_S) + \mathcal{W}^{\beta}_A(\rho_A) + k_B T I(\rho_{SA}), \quad (9)\end{aligned}$$

where $\mathcal{W}^{\beta}_A(\rho_A) = \mathcal{F}_{\beta}(\rho_A) - \mathcal{F}_{\beta}(\rho_A^{\beta}) \geq 0$ is the work locally extractable from the ancilla $A$ without using measurements. From Eq. (7), it then follows that the work extractable from $S$ through the optimal isothermal protocol supplemented by the feedback scheme, $\mathcal{W}^{\beta}_{S|A}$, plus the work extractable from $\rho_A$, can never exceed $\mathcal{W}^{\beta}_{SA}$:

$$\mathcal{W}^{\beta}_{S|A} + \mathcal{W}^{\beta}_{A}(\rho_A) = \mathcal{W}^{\beta}_{SA}(\rho_{SA}) - k_B T \mathcal{D}(\rho_{SA}). \quad (10)$$

Equation (10) has a clear interpretation. The intrinsic irreversibility of the measurement process destroys the quantum correlations present in state $\rho_{SA}$, as measured by quantum discord. As a consequence, the work extractable from system and ancilla decreases by an amount $k_B T \mathcal{D}(\rho_{SA})$, which corresponds to the work value of quantum correlations in the state $\rho_{SA}$. Result (7) is then an exact expression stressing the deep link between work and knowledge. This interpretation of the role of discord agrees with that provided in Ref. [19], when comparing local and global Maxwell demonlike configurations.

*Thermodynamic tradeoff of quantum measurement.*—In the above discussion, we summed up the two extractable works obtained (i) from $S$, with the optimal protocol including feedback, and, separately, (ii) from $A$. Although providing a nice interpretation for the work content of quantum discord, this does not properly take into account the measurement backaction on $A$, as $\mathcal{W}^{A}_{\beta}$ would be the work extractable from $A$ if no measurement had been performed. In fact, the projective measurement, in the first stage of the feedback scheme, modifies the whole $SA$ state. After the $k$th outcome, one has

$$\rho_{SA} \to \rho_{S|\Pi^k_A} \otimes \Pi^k_A, \quad (11)$$

Then, one may ask how the work extracted from $SA$ in the presence of the feedback gets modified and whether it can in fact surpass $\mathcal{W}^{\beta}_{SA}$ in Eq. (9). To answer this question, we consider the gain in work extraction obtained from the true postmeasurement state, with respect to $\mathcal{W}^{\beta}_{SA}$, i.e., $\Delta\mathcal{W}^{\beta}_{SA|\pi_A} = \sum_k p_k \mathcal{F}_\beta(\rho_{S|\Pi^k_A} \otimes \Pi^k_A) - \mathcal{F}_\beta(\rho_{SA})$.

To perform a proper energy balance in the presence of the measurement process, we should also consider its work cost. If $H_A$ is the Hamiltonian of the ancilla, and if $\rho_{A|\pi_A} = \sum_k p_k \Pi^k_A$ is its unconditional, post-measurement state, then the cost $\mathcal{C}(\pi_A) \equiv \mathrm{Tr}[H_A(\rho_{A|\pi_A} - \rho_A)]$ corresponds to the work needed to perform the measurement $\pi_A$ [34]. It vanishes as soon as measurements are performed in the energy eigenbasis, $[\Pi^k_A, H_A] = 0$, or when energy-less ancillas are considered ($H_A \propto \mathbb{I}_A$) [35]. More importantly, if the optimal set of projectors $\pi^{\mathrm{opt}}_A$ is taken, which maximizes the extracted classical information in Eq. (7), we have

$$\begin{aligned}\Delta\mathcal{W}^{\beta}_{SA|\pi^{\mathrm{opt}}_A} - \mathcal{C}(\pi^{\mathrm{opt}}_A) &= k_B T[S(\rho_{SA}) - \sum_k p_k S(\rho_{S|\Pi^k_A})] \\ &= k_B T[S(\rho_A) - \mathcal{D}(\rho_{SA})] \geq 0, \end{aligned} \quad (12)$$

where the final inequality in Eq. (12) follows from the fact that discord is always bounded from above by the entropy of the measured system [36].

Equation (12) is the second of our main results. It remarkably ensures that the amount of extractable work from system and ancilla does not decrease when using optimal quantum measurements and feedback in the work extraction process, even if the cost of the measurement is properly accounted for and subtracted. The interpretation of the two terms above becomes clear if one notices that the measurement induced free-energy change can be written $\Delta\mathcal{F}^{\beta}_A \equiv \sum_k p_k \mathcal{F}_\beta(\Pi^k_A) - \mathcal{F}_\beta(\rho_A) = \mathcal{C}(\pi_A) + k_B T S(\rho_A)$. Thus, even if the quantum measurement produces a decrease in the extractable work of the composite system by an amount $k_B T \mathcal{D}(\rho_{SA})$, corresponding to the loss of quantum discord, this is always (over-)compensated by an increase, $\Delta\mathcal{W}^{\beta}_A = \Delta\mathcal{F}^{\beta}_A$, of the work locally extractable from the ancillary system after the measurement. Indeed, this provides both a compensation for the measurement cost, as well as the extra work amount $k_B T S(\rho_A)$, exceeding the work value of discord. It is worth noticing here that if the optimal set of projectors $\pi^{\mathrm{opt}}_A$ were not used, then $\Delta\mathcal{W}^{\beta}_{SA|\pi^{\mathrm{opt}}_A} \geq \mathcal{C}(\pi^{\mathrm{opt}}_A)$ cannot be ensured anymore, and the tradeoff between the gain in extractable work due to the measurement, and its reduction due to correlation erasure may give a detrimental result, implying that the direct work extraction from $\rho_{SA}$ (without using measurements) is the best option.

*Extracting work from quantum correlations.*—Finally, we are interested in the possibility of extracting the work content of quantum correlations, which may also exceed classical correlations [37], without renouncing the benefits of the measurement. This may seem impossible at first sight, as including projective quantum measurements will eventually produce the loss of discord in state $\rho_{SA}$, as we already discussed. We propose a protocol for which this can be circumvented extracting the work content of quantum correlations *before* the projective measurement is performed on the ancilla. This means including a new initial step $\rho_{SA} \to \rho'_{SA}$ in the extraction protocol, performed before measurement and isothermal driving, sketched as step (c) in Fig. 2. Such a step unavoidably requires interaction between system and ancilla.

Leaving considerations motivating the construction of this reversible subprocess to the Supplemental Material [28], we require a final state of the step with zero quantum correlations, but intact classical ones,

$$\rho'_{SA} = \sum_k p_k \rho_{S|\Pi^k_A} \otimes \Pi^k_A. \quad (13)$$

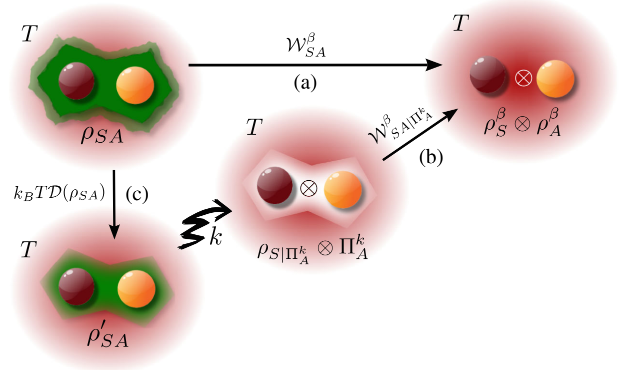


FIG. 2. The maximum amount of extractable work from $S$ and $A$ [path (a)] can be enhanced using quantum measurements [path (b)]. However, the work content of quantum discord is in general lost in this case. Optimal feedback enhanced work extraction can then be achieved by extracting it before the measurement is performed [step (c)].

where, once again, the projectors $\Pi^k_A$ are taken from the optimal set $\pi^{\rm opt}_A$. The step goes as follows: First we perform a sudden quench of the total Hamiltonian, so that $H_S + H_A \to H_{SA} \equiv -k_B T \ln \rho_{SA}$. Then, a quasistatic driving is applied to the compound system, transforming $H_{SA} \to H'_{SA} \equiv -k_B T \ln \rho'_{SA}$, which leads the compound system to end up in the state $\rho'_{SA}$, as it follows from the fact that $\rho'_{SA}$ is now the equilibrium state at temperature $T$ with respect to the Hamiltonian $H'_{SA}$. Finally, a second sudden change of the Hamiltonian is performed $H'_{SA} \to H_S + H_A$. The maximum work extractable in this reversible three-step process is, then,

$$\mathcal{W}^{\beta}_{SA}(\rho_{SA}) - \mathcal{W}^{\beta}_{SA}(\rho'_{SA}) = k_B T\mathcal{D}(\rho_{SA}). \tag{14}$$

The full extraction of work is finally completed by applying the feedback enhanced protocol to $SA$ (see Fig. 2). Summing up all of the contributions, the maximum work extractable from $\rho_{SA}$ is obtained by adding the work value of discord [Eq. (14)], the one extractable directly from $\rho'_{SA}$ [Eq. (9)], plus the entropic gain due to the measurement [Eq. (12) applied to $\rho'_{SA}$]; that is,

$$\begin{aligned}
&\mathcal{W}^{\beta}_{SA|\pi^{\rm opt}_A}(\rho_{SA}) - \mathcal{C}(\pi^{\rm opt}_A) \\
&= k_B T\mathcal{D}(\rho_{SA}) + \mathcal{W}^{\beta}_{SA}(\rho'_{SA}) + k_B T S(\rho_A) \\
&= \mathcal{W}^{\beta}_{S}(\rho_S) + \mathcal{W}^{\beta}_{A}(\rho_A) + k_B T I(\rho_{SA}) + k_B T S(\rho_A).
\end{aligned} \tag{15}$$

In particular, this implies that the process involving feedback from $A$ helps in increasing the extractable work even in comparison with the optimal isothermal protocol (without feedback) applied to the whole $SA$ system. This means that using a local quantum measurement may allow not only to extract the full work associated with the total amount of correlations present in $\rho_{AB}$, namely, $k_B T I(\rho_{AB})$, but also an enhancement proportional to the entropy of the ancilla. The latter, eventually, may be lost in restoring the initial state of $A$ [32]. Finally, the recognition of the maximum extractable work in Eq. (15) allows for the definition of a suitable information-to-work conversion efficiency in line with Ref. [38].

In conclusion, we derived quantitative relations linking the optimal work extractable from bipartite quantum systems and their classical and quantum correlations, assessing both the role of thermal environments and quantum measurements. Moreover, we proposed a protocol to extract the work associated to the presence of not only classical but also quantum correlations. Our results, beyond establishing a way to exploit quantum correlations thermodynamically, might be of interest in practical applications regarding the design of quantum batteries [39].

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