Quantum Spin Dynamics in a Normal Bose Gas with Spin-Orbit Coupling

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In this Letter, we investigate spin dynamics of a two-component Bose gas with spin-orbit coupling realized in cold atom experiments. We derive coupled hydrodynamic equations for number and spin densities as well as their associated currents. Specializing to the quasi-one-dimensional situation, we obtain analytic solutions of the spin helix structure and its dynamics in both adiabatic and diabatic regimes. In the adiabatic regime, the transverse spin decays parabolically in the short-time limit and exponentially in the long-time limit, depending on initial polarization. In contrast, in the diabatic regime, transverse spin density and current oscillate in a way similar to the charge-current oscillation in an undamped *LC* circuit. The effects of Rabi coupling on the short-time spin dynamics is also discussed. Finally, using realistic experimental parameters for ^{87}Rb, we show that the timescales for spin dynamics is of the order of milliseconds to a few seconds and can be observed experimentally.

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Introduction.—It has long been recognized that collective spin dynamics of quantum mechanical origin can exist in a dilute gas at temperature $T \gtrsim T_d$, where T_d is the degeneracy temperature. It arises due to indistinguishability of identical atoms in binary scattering and is known as the identical spin rotation effect (ISRE) [1–3]. This effect is operative for both bosons [4–6] and fermions [7,8] and has led to the observations of spin waves and anomalous spin segregation for weakly interacting bosons [9] and fermions [10]. Similar effects also occurs in a degenerate Fermi liquid like ^{3}He where it leads to anomalous spin diffusion known as the Leggett-Rice effect [11,12]. Recently, the Leggett-Rice effect has also been observed in unitary Fermi gas in both two [13] and three dimensions [14,15].

The ISRE effects explored so far are limited to systems with spin-SU(2) symmetry where the total spin is a good quantum number and its dynamics decouples from that of the density [4–8]. In this Letter, we investigate the spin dynamics of a normal Bose gas with spin-orbit coupling (SOC) that was recently realized in cold atom experiments [16–29]. The coupling between spin and orbit degrees of freedom breaks the SU(2) symmetry and leads to more intricate dynamics that has no analog in the usual dilute gases discussed above. In particular, we show how the long-wavelength and low-frequency hydrodynamic equations are modified in the presence of SOC, and how it leads to the appearance of a persistent spin helical (PSH) structure. The decay of the spin helical structure is discussed in both the adiabatic and diabatic limits. The general equations we obtain should serve as the starting point for investigating spin dynamics in a spin-orbit coupled Bose gas such as spin waves and their attenuations. In Ref. [30], a simple model for spin dynamics in a Bose gas with SOC based on auxiliary trajectories is given and spin dynamics for Fermi gas with spin-orbital coupling is discussed in Refs. [30–37].

General setup.—For definiteness, let us consider a gas of bosonic atoms ^{87}Rb of mass m with two hyperfine-Zeeman sublevels $|F, m_F\rangle \equiv |1,0\rangle \equiv |\uparrow\rangle$ and $|1,-1\rangle \equiv |\downarrow\rangle$ that are coupled by a pair of Raman lasers with momentum transfer $\mathbf{q} = q\hat{\mathbf{x}}$ along the $\hat{\mathbf{x}}$ direction. We set the two-photon detuning to be zero for simplicity in the following discussion. The harmonic trapping potential $V(\mathbf{r})$, independent of spin, is assumed to be strong in the $\hat{\mathbf{y}}$ and $\hat{\mathbf{z}}$ directions but weak in the $\hat{\mathbf{x}}$ direction and the system can be considered quasi-one-dimensional. The s-wave interaction is almost SU(2) invariant for ^{87}Rb and is given by a single coupling constant g. The Hamiltonian can be written as $\hat{\mathcal{H}} = \int d^3\mathbf{r} \sum_{\mu,\nu=\uparrow,\downarrow} \psi^\dagger_\mu(\mathbf{r}) H_{\mu\nu} \psi_\nu(\mathbf{r}) + \frac{1}{2} g \int d^3\mathbf{r} \colon \hat{n}(\mathbf{r})\hat{n}(\mathbf{r})\colon$ where $H_{\mu\nu}$ is given by

$$H_{\mu\nu} = \left(\frac{-\hbar^2\nabla^2}{2m} + V(\mathbf{r})\right)\delta_{\mu\nu} - \frac{i\hbar q}{m}\sigma^z_{\mu\nu}\partial_x + \frac{\hbar\Omega_R}{2}\sigma^x_{\mu\nu}. \quad (1)$$

$\hat{\psi}_\mu(\mathbf{r})$ $[\hat{\psi}^\dagger_\mu(\mathbf{r})]$ is the annihilation (creation) operator for boson with spin μ at position $\mathbf{r}$. Ω_R is the two-photon Rabi coupling. The number and spin densities are then given by $\hat{n}(\mathbf{r}) = \sum_\mu \hat{\psi}^\dagger_\mu(\mathbf{r})\hat{\psi}_\mu(\mathbf{r})$ and $\hat{s}_i(\mathbf{r}) = \frac{1}{2}\sum_{\mu,\nu} \hat{\psi}^\dagger_\mu(\mathbf{r})\sigma^i_{\mu\nu}\hat{\psi}_\nu(\mathbf{r})$, respectively. σ^i are the Pauli matrices. In what follows, we use an arrow on top of an operator to indicate that it is a vector in spin space while boldface $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, $\hat{\mathbf{z}}$ describes the spatial direction. Properties of condensation described by $\hat{\mathcal{H}}$ have been discussed extensively in the literature, including its phase diagram and collective excitations [38–50] as well as spin dynamics [51].

Transport equations.—We first derive the continuity equations for number and spin densities and also identify the modifications to the associated number and spin currents due to spin-orbit coupling. We restrict ourselves to transport along the $\hat{\mathbf{x}}$ direction. Using Heisenberg's equation of motion $i\hbar\langle\partial_t\hat{A}\rangle = \langle[\hat{A},\hat{\mathcal{H}}]\rangle$ with $\hat{A}$ being the number $\hat{n}(\mathbf{r})$ and spin $\hat{\vec{s}}(\mathbf{r})$ densities, we find immediately

$$\partial_t n + \partial_x j_0 = 0, \tag{2}$$

where the number current along the $\hat{\mathbf{x}}$ direction $j_0 = \langle\hat{j}_0\rangle$ with $\hat{j}_0 = -i\hbar/(2m)\sum_\mu(\hat{\psi}^\dagger_\mu\partial_x\hat{\psi}_\mu - \partial_x\hat{\psi}^\dagger_\mu\hat{\psi}_\mu) + (2q/m)\hat{s}_z$. We note that due to SOC, the number current is coupled to the $\hat{z}$ component of the spin density. This redefinition is recently found to cause the violation of irrotationality of the velocity field in spin-orbit couple condensate and the reduction of the quantum of circulation [49].

In the presence of spin-orbit coupling, the total spin is no longer conserved and the definition of spin current operator $\hat{\vec{j}}$ is not entirely obvious. In our case, we identify the spin current by grouping all the gradient terms in the continuity equation for spin density

$$\partial_t\vec{s} + \partial_x\vec{j} = \Omega_R\hat{x}\times\vec{s} + (2q/\hbar)\hat{z}\times\vec{j}, \tag{3}$$

where the spin current operator is given by $\hat{\vec{j}} = -i\hbar/(4m)\sum_{\mu,\nu}\hat{\psi}^\dagger_\mu\vec{\sigma}_{\mu\nu}\partial_x\hat{\psi}_\nu + \text{H.c.} + \hat{n}q/(2m)\hat{z}$. We note two important modifications due to SOC. First, the spin current is now coupled to the total density of the system. Second, apart from the usual spin precessing term due to Rabi coupling, there is an additional precessing term, proportional to the strength of SOC, of spin current along the $\hat{z}$ direction in Eq. (3). We note that the modified definition of spin current operator $\hat{\vec{j}}$ can also be motivated from semiclassical considerations. Let the distribution function (a matrix in spin space) be given by $\hat{f}(\mathbf{r},\mathbf{p},t)$, then one can define the spin current as

$$\vec{j}(\mathbf{r},t) = \frac{1}{2}\mathrm{Tr}\int\frac{d^3\mathbf{p}}{(2\pi\hbar)^3}\hat{f}(\mathbf{r},\mathbf{p},t)\frac{1}{2}\left(\vec{\sigma}\frac{\partial H}{\partial p_x} + \frac{\partial H}{\partial p_x}\vec{\sigma}\right). \tag{4}$$

The symmetrization is necessary because of the noncommutivity of $\vec{\sigma}$ and $\partial H/\partial p_x$. Since $\partial H/\partial p_x = p_x/m + (q/m)\hat{\sigma}^z$. The first term p_x/m corresponds to the standard spin-current operator, while the second term $(q/m)\hat{\sigma}^z$ only modifies the $\hat{z}$ component of the spin current by an additional term $\hat{n}q/(2m)$.

Using the operator forms of the number and spin currents, it is now straightforward to obtain their equations of motion, which are much more complicated because of the involvement of the momentum flux tensors. However, in the normal state above the degeneracy temperature, the momentum flux tensors can be simplified using Boltzmann distribution (recall $T \gtrsim T_d$) and gradient expansion (for detailed derivation, see Supplemental Material [52]). As a result, we obtain

$$\partial_t j_0 + \frac{k_B T}{m}\partial_x n = \frac{2q}{m}\Omega_R s_y - \frac{g}{2m}\partial_x\left(\frac{3}{4}n^2 + \vec{s}^2\right) \tag{5}$$

$$\partial_t\vec{j} + \alpha\partial_x\vec{s} = \left(\Omega_R\hat{x} + \frac{g}{\hbar}\vec{s}\right)\times\vec{j} + \frac{2q\alpha}{\hbar}\hat{z}\times\vec{s} + \frac{qn\Omega_R}{2m}\hat{y} - \frac{3g}{4m}(\partial_x n)\vec{s} - \gamma\vec{j}, \tag{6}$$

where $\alpha = k_B T/m + ng/(4m)$. A phenomenological spin current relaxation term $-\gamma\vec{j}$ is added to Eq. (6). In the absence of the spin-orbit coupling ($\Omega_R = 0$ and $q = 0$), Eqs. (3), (6) reduce to the standard Leggett-Rice form for a degenerate Fermi liquid [11,12]. It is noteworthy that the spin gradient term $ng/(4m)\partial_x\vec{s}$ in Eq. (6) is usually omitted in comparison to the Leggett-Rice term $(g/\hbar)\vec{s}\times\vec{j}$ when the spatial variation of $\vec{s}$ is small. In the presence of SOC, however, it has to be retained because the natural scale of variation for $\vec{s}$ is set by the spin-orbit scale q, which can be quite large. In addition, due to the fast temporal variation of spin density, it is necessary to go beyond the adiabatic approximation $|\partial_t\vec{s}/\vec{s}| \lesssim \gamma$ usually assumed in literature and discuss the dynamics in the diabatic regime as well.

Equations (2), (3), (5), (6) form the basic equations for the spin dynamics of a SOC boson above the degeneracy temperature. In the following, we first discuss the limit when the effect of Rabi coupling Ω_R is small, or what is equivalent, for time $t \ll 1/\Omega_R$, and discuss the existence of the persistent spin helix (PSH) at wave vector $k = 2q$ (hereafter $\hbar = 1$) and its decay when k deviates from $2q$. The effects of the Rabi term on PSH will be discussed at the end of the Letter.

Persistent spin helical structure.—The full set of equations allow an exact solution corresponding to persistent spin helix with uniform density $n = n_0$, spin density $s_z = s_{z,0}$, and $\vec{s}^2 \equiv \vec{s}\cdot\vec{s}$ that are independent of time. If we write the transverse spin $\vec{s}_\perp = s_x\hat{x} + s_y\hat{y}$ in terms of $s^\pm = s_x \pm is_y$, and likewise for the spin currents $\vec{j}(x,t) = \vec{j}_\perp(x,t) + j_z(t)\hat{z}$. Then for the spin helical structure with definite wave number k, we can write $s^\pm(x,t) = e^{\pm ikx}\tilde{s}^\pm(t)$ and similarly $j^\pm(x,t) = e^{\pm ikx}\tilde{j}^\pm(t)$ and obtain the following set of equations

$$\partial_t\tilde{s}^+ = -i(k-2q)\tilde{j}^+, \tag{7}$$

$$\partial_t\tilde{j}^+ = (i\lambda s_{z,0} - \gamma)\tilde{j}^+ - i[\alpha(k-2q) + \lambda j_z]\tilde{s}^+, \tag{8}$$

$$\partial_t j_z = \lambda\mathrm{Im}[\tilde{s}^-\tilde{j}^+] - \gamma j_z, \tag{9}$$

where $\lambda = g/\hbar$ and Im denotes the imaginary part. When $k = 2q$, the transverse spin $\tilde{s}^+$ is independent of time and corresponds to a static spin helical structure in which spin density rotates about the $\hat{z}$ axis with wave vector $2q$ in the $\hat{\mathbf{x}}$ direction,

$$\vec{s}_{\rm psh} = s_{\perp,0}\cos(2qx)\hat{x} + s_{\perp,0}\sin(2qx)\hat{y} + s_{z,0}\hat{z}. \qquad (10)$$

In semiconductor heterostructure, it is understood that the persistent spin helix is due to an emergent SU(2) symmetry in the presence of spin-orbit coupling [53–55]. In the long time limit $t \gg 1/\gamma$, it is easy to see that both j_z and $\tilde{j}^+$ decay to zero, according to Eqs. (8), (9).

Vicinity of PSH.—In the following, we investigate the dynamics of spin helical structure when its wave vector deviates away from $2q$, described by the parameter $\varepsilon \equiv k/(2q) - 1$. Here it is important to distinguish two regimes. In the adiabatic regime where the spin currents can relax much faster than the spin densities and can thus follow adiabatically the time evolution of spin density $|\partial_t \vec{s}/\vec{s}| \lesssim \gamma$, we can set $\partial_t \tilde{j}^+ = \partial_t j_z = 0$ in Eqs. (8), (9) in the steady state. Writing $\tilde{s}^+(t) \equiv s_\perp(t)\exp[i\theta(t)]$, we obtain the following set of equations:

$$(\gamma^2 + \lambda^2 s_{z,0}^2)\ln\frac{s_\perp(t)}{s_{\perp,0}} + \frac{\lambda^2}{2}[s_\perp^2(t) - s_{\perp,0}^2] = -\alpha\gamma(k-2q)^2 t, \qquad (11)$$

$$\theta(t) = \frac{\lambda s_{z,0}}{\gamma}\ln\left[\frac{s_\perp(t)}{s_{\perp,0}}\right], \qquad (12)$$

$$j_z = -\frac{\lambda s_\perp^2 \alpha(k-2q)}{\gamma^2 + \lambda^2(s_\perp^2 + s_{z,0}^2)}, \qquad (13)$$

$$\tilde{j}^+ = \tilde{s}^+ \frac{\alpha(k-2q)(\lambda s_{z,0} - i\gamma)}{\gamma^2 + \lambda^2(s_\perp^2 + s_{z,0}^2)}, \qquad (14)$$

where $s_{\perp,0} = s_\perp(t=0)$. Substitution of $k = 2q$ recovers the previous solution of PSH. When $k \neq 2q$, the transverse spin magnitude decays according to Eq. (11). Depending on the relative magnitude of $s_\perp$ and $s_{z,0}$, one can distinguish two qualitatively different behaviors. (i) When $|s_{\perp,0}| \geq |s_\perp(t)| \gg |s_{z,0}|$, namely, when spins are polarized close to the xy plane, the first term on the left of Eq. (11) is negligible; hence the transverse spin magnitude decays parabolically in the short time limit $t \ll \tau_{\rm para}$,

$$s_\perp(t) \approx s_{\perp,0}\sqrt{1 - \frac{t}{\tau_{\rm para}}}, \qquad \tau_{\rm para} = \frac{\lambda^2 s_{\perp,0}^2}{2\alpha\gamma(k-2q)^2}, \qquad (15)$$

where the time constant $\tau_{\rm para}$ depends quadratically on the interaction parameter λ and inversely on the spin current relaxation rate γ. As expected, it diverges when $k = 2q$. (ii) In the long time limit $t \gg \tau_{\rm para}$ when $|s_\perp(t)| \ll |s_{z,0}|$, the decay becomes exponential in the adiabatic regime with a different time constant $\tau_{\rm exp}$

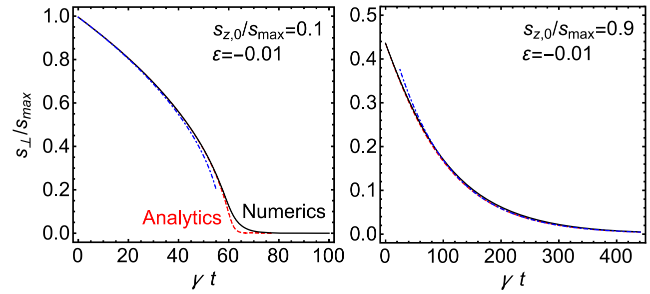

FIG. 1. Short- and long-time behaviors of the transverse spin in the adiabatic regime. Short-time decay for initial spin density polarized close to the xy plane. The decay is parabolic (left panel). Long-time decay for initial spin density polarized close to the z axis. The decay is exponential (right panel). Numerical simulations of the full set of Eqs. (2), (3), (5), (6) (black solid) agree very well with the analytical results (red dashed) given by Eq. (11). Blue dashed lines show the asymptotic results, Eqs. (15), (16). The deviation at the tail of the left panel between the simulation and analytic equation (11) indicates the failure of adiabatic approximation.

$$s_\perp(t) \approx s_{\perp,0}e^{-t/\tau_{\rm exp}}, \qquad \tau_{\rm exp} = \frac{\gamma^2 + \lambda^2 s_{z,0}^2}{\alpha\gamma(k-2q)^2}. \qquad (16)$$

Figure 1 shows the excellent agreement between above analytic formulas and numerical results. We note that the dynamical equation (11) is similar in form to the Leggett-Rice equation derived for a degenerate Fermi liquid [12], except for the explicit appearance of the spin-orbit coupling term on the right-hand side of Eq. (11). We emphasize that Eqs. (15), (16) apply so long as $1/\Omega_R \gg \tau_{\rm para}, \tau_{\rm exp}$ even in the presence of a small Rabi coupling.

Diabatic regime.—When the wave vector k of the spin helix deviates significantly away from the PSH wave vector $2q$, the adiabatic condition fails. In the diabatic regime when $|\partial_t \vec{s}/\vec{s}| \gg \gamma$, we can neglect $\lambda|j_z|$ in Eq. (8), and as a result $\tilde{s}^+(t)$, $\tilde{j}^+(t)$ form a closed dynamical system in Eqs. (7), (8). With the initial condition $(\tilde{s}^+, \tilde{j}^+, j_z)_{t=0} = (s_{\perp,0}, 0, j_{z,0})$, one obtains [52]

$$\tilde{s}^+(t) = s_{\perp,0}e^{-[(\gamma t)/2] + i[(\lambda s_{z,0})/2]t}\left[\cos\Gamma t + \frac{\gamma - i\lambda s_{z,0}}{2\Gamma}\sin\Gamma t\right], \qquad (17)$$

$$\tilde{j}^+(t) = i\sqrt{\alpha}s_{\perp,0}e^{-[(\gamma t)/2] + i[(\lambda s_{z,0})/2]t}\sin(\Gamma t)\,{\rm sgn}(2q-k), \qquad (18)$$

$$j_z(t) = j_{z,0}e^{-\gamma t} - e^{-\gamma t}\frac{\lambda s_{\perp,0}^2}{4(k-2q)} \times \left[1 + \gamma t - \cos(2\Gamma t) - \frac{\gamma}{2}\frac{\sin(2\Gamma t)}{\Gamma}\right], \qquad (19)$$

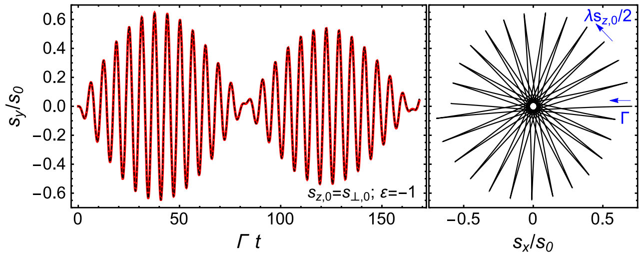

FIG. 2. Left panel shows time dependence of the y component of spin density for spin density polarized along the $\hat{x}$ direction at $t = 0$. Numerical result (black dashed line) agrees very well with the analytical result (red solid line), Eq. (17). Right panel is the trajectory of transverse spin component in the xy plane. Also indicated in the graph are the fast oscillations of the magnitude of transverse spin (Γ) and its slow rotations with rate $\lambda s_{z,0}/2$.

where $\Gamma = \sqrt{\alpha}|k - 2q|$. In obtaining the above simplified expressions we have assumed that the polarization of spin are close to the xy plane and as a result $\Gamma \gg \gamma, \lambda s_{z,0}$.

The dynamics of the transverse components consists of three parts: fast oscillation in magnitude with frequency Γ, slow precessing of the axis of oscillation with frequency $\lambda s_{z,0}/2$ and the damping of oscillation amplitude with the rate of $\gamma/2$, as shown in Fig. 2. If one neglects the small correction of the sine function in Eq. (17), then there is an exact $\pi/2$ phase difference between the oscillations of $\tilde{s}^+(t)$ and $\tilde{j}^+(t)$, similar to the undampened LC circuit, in contrast to the overdamped case, where $\tilde{j}^+$ follows adiabatically the dynamics of $\tilde{s}^+$.

The region of adiabaticity for various initial polarizations $s_{z,0}$ and $s_{\perp,0}$ are determined (approximately) by the condition $|\partial_t\vec{s}/\vec{s}| \sim \gamma$. It is shown in Fig. 3 that close to PSH, the adiabatic region prevails for most of the parameter regime except when the spin polarization is small. On the other hand, as one moves away from PSH, the region of nonadiabatic evolution grows much larger. Starting from an arbitrary initial conditions, the spin dynamics might traverse both adiabatic and diabatic regimes and becomes much richer. In particular, close to the boundaries, it is necessary to deal with the full set of hydrodynamic equations (2), (3), (5), (6) that we derived before.

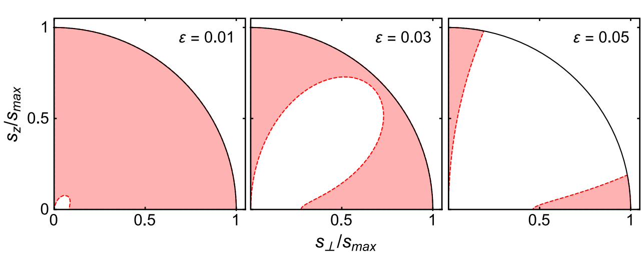

FIG. 3. The approximate demarcation of adiabatic from diabatic regions based on the condition $|\partial_t\vec{s}/\vec{s}| \sim \gamma$ (red shaded lines). The region of adiabaticity becomes smaller when the wave number of spin helix deviates away from $2q$.

Quenching of Rabi coupling on PSH.—In the above analysis, we have assumed that the Rabi coupling is weak and can be neglected. Inclusion of the Rabi term results in complex dynamics, e.g., an extra precession of spin density and spin current density in Eqs. (3), (6). In the equilibrium state, due to the breaking of the emergent SU(2) symmetry, PSH is no longer stable and decays with a rate that is determined by Ω_R.

Considering the situation that the Rabi coupling is turned on suddenly at $t = 0$ and remains fixed, all spin helices except the PSH with $k = 2q$ vanish long before $t = 0$. In the following the short-time effect of the Rabi coupling on the PSH will be studied. We can separate the densities and current densities into two parts, one from the PSH while the other from the leading correction due to Rabi coupling which vanishes for $t \le 0$,

$$n(x,t) = n_0 + \delta n(x,t), \tag{20}$$

$$\vec{s}(x,t) = \vec{s}_{\rm psh} + \delta\vec{s}(x,t), \tag{21}$$

$$j_0(x,t) = 0 + \delta j_0(x,t), \tag{22}$$

$$\vec{j}(x,t) = 0 + \delta\vec{j}(x,t). \tag{23}$$

Substituting the above expressions into the transport equations Eqs. (2), (3), (5), (6) and only keeping terms linear in small derivations and the Rabi coupling, we are led to an inhomogeneous diffusion equation of the form

$$\partial_t\delta\vec{V}(x,t) = \hat{\mathbf{H}}(x,\partial_x)\delta\vec{V}(x,t) + \vec{g}(x), \tag{24}$$

with $\delta\vec{V} = (\delta n, \delta j_0, \delta s_z, \delta j_z, \delta s_x, \delta j_x, \delta s_y, \delta j_y)^T$ and $\vec{g}(x) = \Omega_R(0, 2qs_{{\rm psh},y}/m, s_{{\rm psh},y}, 0, 0, 0, -s_{z,0}, qn_0/(2m))^T$. The explicit form of $\hat{\mathbf{H}}(x,\partial_x)$ is given in the Supplemental Material [52] and the solution is given by

$$\delta\vec{V}(x,t) = \hat{\mathbf{H}}^{-1}[\exp(\hat{\mathbf{H}}t) - \hat{\mathbf{I}}]\vec{g}(x). \tag{25}$$

To characterize the decay of the transverse component of spin density due to Rabi coupling, we define a quantity that measures the amplitude of the spin helical structure

$$R(t) = \frac{1}{|\vec{s}(x,0)|(\pi/q)}\int_0^{\pi/q}{\rm Re}[s^+(x,t)e^{-i2qx}]dx, \tag{26}$$

where $|\vec{s}(x,0)| = \sqrt{s_{\perp,0}^2 + s_{z,0}^2}$ is the initial spin magnitude. For $t > 0$, Rabi coupling destroys the helical structure and results in decay of $R(t)$. The short-time behavior is described by the leading terms of the series Eq. (25). When $\Omega_R t \ll 1$,

$$R(t) \approx \frac{s_{\perp,0}}{\sqrt{s_{\perp,0}^2 + s_{z,0}^2}}\left[1 - \frac{(\Omega_R t)^2}{4}\right] + \mathcal{O}(t^4). \tag{27}$$

Experimental considerations.—For ^{87}Rb used in the spin-orbit coupling experiment [16], the typical density is about $n_0 = 2 \times 10^{13}\ \mathrm{cm}^{-3}$. For our calculation, we assume $T = 700$ nK, well above the typical condensation temperature. The Raman laser defines a scale of wave number $k_L = (\sqrt{2}\pi/804.1\ \mathrm{nm})/10$, chosen to be 10 times smaller than that in Ref. [16] to make the spin helical structure more visible. In the numerical calculations presented, we chose $q = 0.5\hbar k_L$ and Rabi coupling $\hbar\Omega_R = 0.5\hbar^2 k_L^2/(2m)$, appropriate to experimental situations. We assume the system is initially polarized with $|\vec{s}_0| = s_{\max} = n_0/2$. The intrinsic spin current relaxation rate γ is chosen to be approximately 20 Hz, appropriate to ^{87}Rb [4,9]. Numerical simulations show that the timescale for spin dynamics in the adiabatic regime is of the order of seconds, while it is of the order of milliseconds in the diabatic regime, and can be observed experimentally. To initialize the system in a particular spin helical state, one can start with Rb atoms in the $|\downarrow\rangle$ state with no Raman lasers and apply a radio-frequency pulse to achieve a desired transverse polarization in the xy plane. Afterwards, a small magnetic field along the $\hat{z}$ direction with linear gradient ΔB can be applied to create the spin helical structure with desired wave vector k. To investigate the stability of the spin helical structure, one can now turn on the Raman fields that create the spin-orbit coupling with strength q, and measure the evolution of the transverse spin component.

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