# Anomalous Breaking of Scale Invariance in a Two-Dimensional Fermi Gas

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The frequency of the breathing mode of a two-dimensional Fermi gas with zero-range interactions in a harmonic confinement is fixed by the scale invariance of the Hamiltonian. Scale invariance is broken in the quantized theory by introducing the two-dimensional scattering length as a regulator. This is an example of a quantum anomaly in the field of ultracold atoms and leads to a shift of the frequency of the collective breathing mode of the cloud. In this work, we study this anomalous frequency shift for a two-component Fermi gas in the strongly interacting regime. We measure significant upwards shifts away from the scale-invariant result that show a strong interaction dependence. This observation implies that scale invariance is broken anomalously in the strongly interacting two-dimensional Fermi gas.

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Symmetries are an indispensable ingredient to any attempt of formulating a fundamental theory of nature. Yet, it is not always true that one can make accurate predictions about the behavior of some complex system based on the symmetries of its Hamiltonian alone. The fundamental reason behind this is the concept of symmetry breaking [1]. Symmetry violations often have drastic effects on the state of the system, for example, when some metal breaks rotational invariance and becomes ferromagnetic. There are three different mechanisms through which a given system may violate some of the symmetries of its Hamiltonian: explicit, spontaneous, and anomalous symmetry breaking [2].

Quantum anomalies are violations of an exact symmetry of a classical action in the corresponding quantized theory [3]. They may occur when a cutoff has to be introduced to regularize divergent terms. This regulator may explicitly break some symmetry of the theory. If this symmetry is not restored even after the cutoff is removed at the end of the renormalization procedure, the symmetry is broken anomalously.

Quantum anomalies are ubiquitous in quantum field theories and provide important constraints on physical gauge theories like the standard model [4,5] or on string theories [6,7]. Whereas the formalisms of explicit and spontaneous symmetry breaking are frequently applied across many fields in physics [8–10], anomalous symmetry breaking is typically associated only with high-energy physics. One exception was found in molecular physics [11,12], and here we report an observation of a quantum anomaly in the field of cold atom physics.

A particular class of anomalies, called conformal anomalies, break the scale invariance of a theory, that is, invariance of the Hamiltonian under $\boldsymbol{r} \to \lambda \boldsymbol{r}$. The most prominent examples are found in field theories like QED or QCD where the renormalized coupling constants depend on the energy scale and thus break scale invariance explicitly. In ordinary quantum mechanics the $1/r^2$ and the $\delta^2$ potential in 2D are well-known examples of conformal anomalies [13,14].

Notably, the $\delta^2$ potential is used to model contact interactions in cold atom gases in two dimensions as $V_{\mathrm{int}} \propto \sum g_0 \delta^2(r_i - r_j)$. Including the kinetic term $E_{\mathrm{kin}} \propto \sum p_i^2$, the total Hamiltonian scales as $H \to H/\lambda^2$ and it is thus scale invariant. A direct quantization of the $\delta^2$ potential gives rise to inconsistent results like a bound state with diverging energy. A renormalization procedure is required to obtain a well-defined and quantized theory. To this end, the bare coupling constant $g_0$ in the Hamiltonian is replaced by a renormalized coupling $g$ and a new length scale, the 2D scattering length $a_{2\mathrm{D}}$, has to be introduced [15,16]. This additional length scale anomalously breaks the scaling symmetry of the bare Hamiltonian.

For an atom cloud trapped in an external 2D harmonic potential, the scale invariance of the unregularized $\delta^2$ potential translates directly into a SO(2,1) symmetry of the full Hamiltonian [17,18]. As a consequence of this symmetry, the breathing or monopole mode of the cloud follows undamped oscillations at twice the trap frequency $\omega_B = 2\omega_R$ irrespective of the interaction strength. While it was already noted in Ref. [17] that the required regularization of the $\delta^2$ potential leads to small deviations from this result, Olshanii *et al.* [19] pointed out that this is in fact an example of a quantum anomaly that can directly be accessed via accurate frequency measurements of the breathing mode in Fermi or Bose gas experiments. The anomaly originates from the SO(2,1) symmetry of the classical action that is broken in the quantized theory. A substantial theoretical effort has been made to quantify the anomalous corrections of the breathing mode frequency $\omega_B$ both at zero [19–22] and finite [23–25] temperature.

For two-component Fermi gases anomalous shifts up to 10% are expected that show a strong dependence on both interaction strength and temperature. At zero temperature a rather large shift to frequencies above $2\omega_R$ is predicted [21,22] while perturbative solutions at finite temperatures show significantly reduced shifts on the order of 1%–2% that even go below the scale-invariant value of $2\omega_R$ in the strongly interacting region [23,24].

Experimentally, scale invariance has been studied with 2D Bose gases in Refs. [26–28], showing no significant symmetry violation in the weakly interacting regime. The strongly interacting regime in a 2D Fermi gas was studied in Refs. [29,30], where on the level of the experimental precision no significant deviation from the scale-invariant result was observed. This was attributed to the relatively high temperatures of their system and statistical errors on the same order as the expected shifts at these temperatures [21].

In this work, we study the anomalous frequency shift of the breathing mode in a 2D Fermi gas with high accuracy. We perform our experiments with a two-component mixture of $^6$Li atoms and approximately $10^4$ particles per spin state. The mixture is loaded into a highly anisotropic harmonic trap. The trap frequencies are $\omega_z = 2\pi \times 7.14$ kHz and $\omega_R \approx 2\pi \times 22.5$ Hz. The radial confinement is created by an approximately equal superposition of an optical dipole trap and a magnetic confinement proportional to $\sqrt{B}$, where $B$ is the magnetic offset field. A detailed discussion of frequency, anharmonicity, and anisotropy measurements of the trap can be found in the Supplemental Material [31].

The aspect ratio of approximately 300:1 between axial and radial trap axes allows us to reach the kinematically 2D regime for low enough temperature $T$ and chemical potential $\mu$ [32]. We tune the scattering length $a_{2D}$ by means of a broad magnetic Feshbach resonance [33]. In 2D the phase diagram of the BEC-BCS crossover is characterized by the two dimensionless parameters $\ln(k_F a_{2D})$ and $T/T_F$ with Fermi wave vector $k_F$ and temperature $T_F$. The temperature of the cloud $T$ is extracted from the *in situ* density distribution with the method established in Ref. [34]. $T/T_F$ varies from 0.1 in the BEC limit to 0.18 in the BCS regime. The biggest effect of the quantum anomaly is expected to appear in the strongly interacting region around $\ln(k_F a_{2D}) \approx 0$ [21].

To excite the breathing mode, we reduce our optical confinement adiabatically such that the cloud expands in the trap. A sudden quench of the trap depth back to its original value initiates the breathing mode oscillation. By tuning the magnitude of the quench, we set the amplitude to around 8% of the cloud width. In addition to the breathing mode, the quench leads to a small collective dipole oscillation of the center of the cloud. We do not observe any excitations of higher order collective modes in our trap using this procedure. We study both excited collective modes simultaneously by taking *in situ* absorption images along the axial direction of the cloud at different times after the quench [see Figs. 1(a)–1(c)].

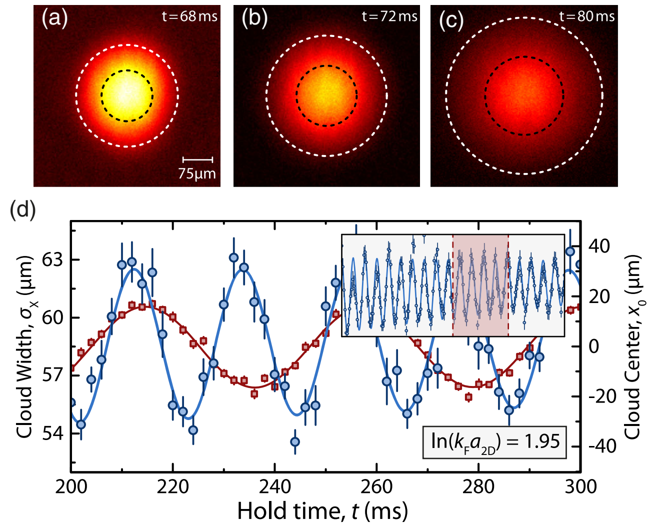


FIG. 1. Breathing mode in a harmonic confinement. (a)–(c) *In situ* images of the cloud along the strongly confined axial direction at different hold times $t = 68$–$80$ ms after quenching the trap depth at $t = 0$ ms. The inner (outer) dashed lines indicate the $1\sigma$ ($2\sigma$) width of a Gaussian fit to the cloud profile. The images are averaged over several experimental realizations and were taken at larger amplitudes to make the oscillation more apparent. (d) The cloud width $\sigma_x$ as a function of the hold time $t$ after the quench (blue circles). The inset shows the complete data set from $t = 0$ to 400 ms. The center-of-mass oscillations of the same cloud are shown in red as a comparison (right, $y$ axis). The solid lines are fits of a damped sine function to the measurements.

We extract the frequencies of the breathing mode $\omega_B$ and dipole mode $\omega_D$ by fitting a damped sine function to the oscillation of cloud width and center along both principal axes $x$ and $y$ of the trap. The principal axes of our confinement are determined and fixed by a principal component analysis of independent measurements using a noninteracting gas [31,35]. A typical data set along the $x$ axis is shown in Fig. 1(d). In total we obtain four frequency measurements per scattering length ($\omega_{B,x}$, $\omega_{B,y}$, $\omega_{D,x}$, and $\omega_{D,y}$).

We observe $\omega_B \equiv \omega_{B,x} = \omega_{B,y}$ for all interaction parameters that are accessible in our experiment. This is expected for the breathing mode in the hydrodynamic regime where the scattering rate is much larger than the oscillation frequency. The center-of-mass dipole modes, on the contrary, oscillate separately along both principal trap axes. From the measured difference of the two frequencies, $\omega_{D,x}$ and $\omega_{D,y}$, we estimate that the in-plane anisotropy of our trap is on the order of 2% [31].

In order to compare the measured breathing mode frequency $\omega_B$ to the scale-invariant result of $2\omega_R$, an accurate determination of the radial trap frequency is essential. To this end, we use the dipole frequencies that coincide with the trap frequency $\omega_R$, independent of interactions or temperature. We take the average of the two measured dipole frequencies as reference, $\omega_R \equiv 1/2(\omega_{D,x} + \omega_{D,y})$. This is justified by the observation that the hydrodynamic breathing mode in the classical limit in a slightly anisotropic trap oscillates at this average up to a correction on the order of less than 0.1% [31,36]. The insensitivity of the breathing mode frequency to small anisotropies is in agreement with calculations at zero temperature [21].

The measured breathing mode is very weakly damped with damping rates $\Gamma_B$ on the order of $\Gamma_B/\omega_R \approx 0.003$. The latter coincide with the background damping rate of a noninteracting cloud, confirming that, apart from technical limitations, the breathing mode is undamped. The only exception to this is the very strongly interacting region around $\ln(k_F a_{2D}) = 0$, where we observe significantly larger, yet still small, damping rates of up to $\Gamma_B/\omega_B \approx 0.01$. This is a first indication of a broken SO(2,1) symmetry in the strongly interacting degenerate gas.

The measured average breathing and dipole frequencies as a function of the magnetic offset field are shown in Fig. 2. In the strongly interacting region around the Feshbach resonance at $B_0 = 832$ G we find a significant shift of the breathing mode to frequencies above twice the dipole frequency (blue shaded area). In the weakly interacting BEC and BCS limits the shift disappears and the scale-invariant result $\omega_B = 2\omega_D \equiv 2\omega_R$ is restored. The data point at $B = 700$ G is shown grayed out due to the significant heating rates we observe this far in the BEC limit. Following Ref. [17], the observed frequency shift necessarily implies that scale invariance is broken in the strongly interacting region. As we will discuss in the following, the only conclusive explanation for the significant shift above $2\omega_R$ is the presence of the quantum anomaly. All other relevant effects which explicitly break

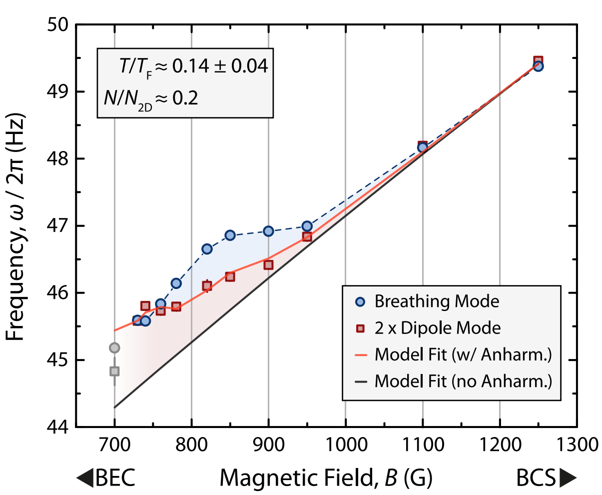


FIG. 2. Measured average breathing and dipole frequencies versus magnetic offset field $\boldsymbol{B}$. Statistical errors are on the order of the symbol size. The dipole frequencies were scaled by a factor of 2 to facilitate the comparison to the breathing mode. We fit a model $\omega_R(B,\sigma)$ for our trap frequencies to the dipole frequency measurements (solid orange line). The solid black line shows the same model for a fixed cloud size.

the SO(2,1) symmetry result in a reduced breathing mode frequency instead.

To enhance our confidence in using the dipole mode measurement as reference, we fit a model for our trap frequency $\omega_R(B,\sigma)$ to the measured dipole frequencies $\omega_D$. By its dependence on the offset field $B$ and the cloud width $\sigma$, our model incorporates the magnetic field dependence and the anharmonicity of our total confinement, respectively. Two free parameters of the model are determined from the fit.

Our model explains the measured dipole mode frequencies remarkably well (orange solid line). We note that the origin of the visible scatter of the dipole frequencies on top of their statistical errors is just given by the fluctuations in particle numbers of different data points. These fluctuations translate into small frequency shifts through an anharmonic correction term that is proportional to the cloud width $\sigma$ squared. The overall effect of the anharmonicity can be estimated by a comparison to the same model while keeping the cloud size $\sigma_0$ fixed. We choose $\sigma_0 = 65$ $\mu$m such that it matches with the measured cloud size in the BCS limit. The black solid line shows the resulting frequencies in the absence of anharmonic corrections. The effective trap frequency is shifted by the anharmonicity by around 2% in the BEC regime compared to the BCS regime (red shaded area). In the same range interactions reduce the cloud size from $\sigma = 65$ $\mu$m to $\sigma = 44$ $\mu$m in the BEC limit.

To exclude any further contributions of our trapping potential experimentally, we have performed measurements with two different spin mixtures. The difference in their Feshbach resonance positions leads to different values for $\ln(k_F a_{2D})$ at the same magnetic field $B$. We find no significant effect of the mixture on the measured anomalous shift (see Fig. 3), confirming that all magnetic field dependencies of the potential were treated properly.

As a final test, we compare the model to measurements in a noninteracting single spin component Fermi gas. Here, the anomalous frequency shift is absent and only systematic shifts from anisotropy or anharmonicity remain. Both breathing and dipole frequencies and their dependence on magnetic field and cloud width are very well explained by our model without any additional deviations and we observe no significant violation of scale invariance [31].

In Fig. 3 we show the relative frequency of the breathing mode $\omega_B/\omega_R$ as a function of the interaction parameter $\ln(k_F a_{2D})$. We observe an anomalous shift towards higher frequencies up to a maximum of 1.3% around $\ln(k_F a_{2D}) = 1$. The maximum position coincides with the region where we have found a many-body paired state in the normal phase of our system in a previous measurement [37] and is in agreement with zero temperature calculations [21,22] based on a quantum Monte Carlo simulation of the equation of state [38]. These predict an anomalous shift of up to 10% with a maximum at $\ln(k_F a_{2D}) \approx -0.5$ (Fig. 3 inset).

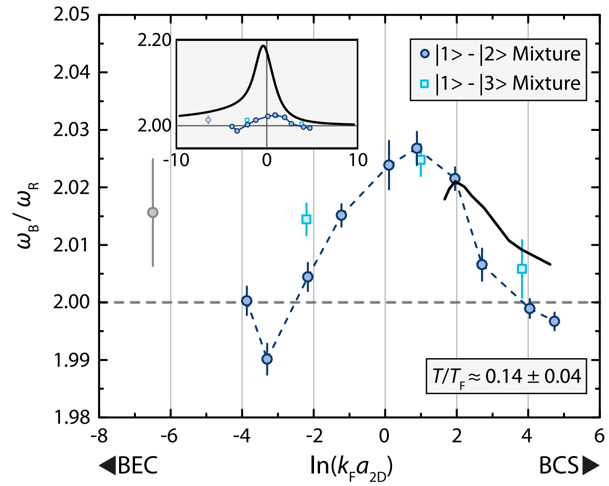


FIG. 3. Anomalous shift of the breathing mode frequency. At low temperatures the breathing mode shows a significant shift away from the scale-invariant result of $\omega_B = 2\omega_R$ towards higher frequencies, even after accounting for both the effects of anharmonicity and anisotropy of our trap. Our data agree with a beyond mean-field approximation from Ref. [24] at $T/T_F = 0.2$ (solid black line). The inset compares our data to a zero temperature calculation from Ref. [21]. Circles and squares represent measurements that were taken with different spin mixtures.

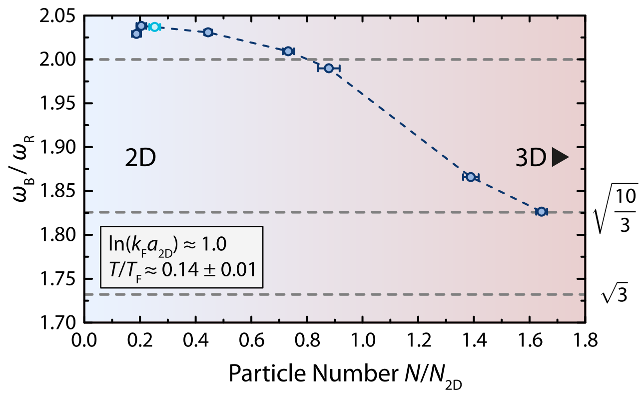


FIG. 4. Influence of the third dimension on the oscillation frequency. As we increase the particle number we observe a strong shift towards lower frequencies. On the $x$ axis we plot the measured particle number of one spin component $N$ divided by $N_{2D} = 48000$. $N_{2D}$ is the maximal number of noninteracting atoms per spin state in the axial ground state of our trap at this magnetic field. The dashed line is a straight connection between the data points.

The frequency shifts observed in the experiment are much smaller in magnitude. This issue is discussed extensively in literature and there are several proposed causes for the strongly reduced shift [20,24,39]. Thermal fluctuations are expected to reduce the anomalous shift at finite temperatures [21–24] and the beyond mean-field approximation from Ref. [24] shows anomalous frequency shifts of a similar order as our measurements at $T/T_F = 0.2$ (Fig. 3 solid black line). Consistently, when increasing the temperature of our sample by $\Delta T = 0.1T_F$, we observe a downwards frequency shift of the order of $-5\%$ [31].

In addition to the trap anharmonicity and anisotropy, we are aware of a third effect that breaks the SO(2,1) symmetry of our system explicitly: the presence of the third dimension. Figure 4 shows how the third dimension affects the breathing mode frequency at fixed temperature and interactions. As we increase $N$ the quasi-2D description of our system breaks down and the system becomes kinematically three-dimensional. As the system leaves the 2D limit, we observe a quick decrease of the measured frequencies below the scale-invariant value of $2\omega_R$. This is in agreement with theoretical calculations, which predict breathing mode frequencies of $\sqrt{10/3}\omega_R(\approx 1.83\omega_R)$ for a Bose gas and $\sqrt{3}\omega_R(\approx 1.73\omega_R)$ for a 3D Fermi gas confined to a "pancake" trap in the unitary limit [40]. Explicit breaking of scale invariance by the presence of the third dimension has been studied before both experimentally [41] and theoretically [41,42]. In a Bose gas a shift to lower frequencies has been observed when increasing the ratio of chemical potential to $\omega_z$.

Since both the expected and measured shifts of the breathing mode introduced by the third dimension are always negative, we conclude that measurements above $2\omega_R$ deep in the quasi-2D limit can only be attributed to the presence of the quantum anomaly. We do, however, identify the third dimension as one of the possible sources for a reduced frequency measurement at any finite particle number [39]. Whether the influence of finite temperature and third dimension alone explain our measurements or if additional effects, as suggested by Ref. [20], reduce the anomalous shift further is an interesting question to be investigated in the future.

The $\delta^2$ potential that we introduced as model for contact interactions is just an approximation of the actual scattering between cold atoms in nature. The exact scaling symmetry holds solely in this approximate theory. A more fundamental theory would contain a modified interaction term and the resulting Hamiltonian would break the SO(2,1) symmetry explicitly without requiring any renormalization procedure. In this equivalent picture, the same frequency shift of the breathing mode is then merely the consequence of the explicit violation of scale invariance by the Hamiltonian.

Any anomaly can be understood in this way. In the standard model, for instance, the appearance of quantum anomalies is related to the fact that the underlying field theories fail to accurately describe nature at small length scales. In contrast to our system, the fundamental physical description is still unknown in this case. This example highlights the significance of the concept of quantum anomalies in the formulation of effective theories that accurately describe physics at larger length scales.

To conclude, we observe a significant, interaction-dependent, frequency shift away from the scale-invariant frequency $\omega_B = 2\omega_R$. We have confirmed that other terms that explicitly break the symmetry of the Hamilton cannot explain the positive frequency shift of the breathing mode, and we attribute it to the presence of a quantum anomaly. We have identified both temperature and the third dimension as causes of the strongly reduced anomalous shift compared to zero temperature calculations.

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*Note added.*—Recently, we became aware of a study with comparable results that has been performed by Vale and co-workers simultaneously to our Letter [43].

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