Canceling the U(1) Anomaly in the S Matrix of $\mathcal{N}=4$ Supergravity

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$\mathcal{N}=4$ supergravity is understood to contain a U(1) anomaly which manifests itself via the nonvanishing of loop-level scattering amplitudes that violate a tree-level charge conservation rule. In this Letter we provide detailed evidence that at one loop such anomalous amplitudes can be set to zero by the addition of a finite local counterterm. We show that the same counterterm also cancels evanescent contributions that play an important role in the analysis of ultraviolet divergences in dimensionally regularized gravity. These cancellations call for a reanalysis of the four-loop ultraviolet divergences previously found in this theory without the addition of such counterterms.

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Introduction.—Whenever a classical symmetry of a quantum field theory is broken by quantum corrections it is said to exhibit an anomaly. This may be either an off-shell symmetry (i.e., of the Lagrangian) or an on-shell symmetry (i.e., of the equations of motion or the S matrix). Anomalies can have important physical consequences. For example, they may render field theories nonrenormalizable and nonunitary or they may yield nonzero loop-level S-matrix elements whose tree-level counterparts vanish identically due to the classical symmetry. In general, quantum theories are ambiguous up to the addition of local counterterms and sometimes these can be chosen to remove an anomaly, perhaps at the cost of breaking another symmetry.

In the context of extended four-dimensional supergravity theories an interesting example of a symmetry susceptible to anomalies is given by their duality symmetries [1–3]. These symmetries involve electric or magnetic duality transformations of Abelian vector fields which leave invariant the equations of motion and, as such, are symmetries of the on-shell type. In addition, they act nonlinearly on scalars parametrizing a G/H sigma model. The presence of anomalies in such sigma models when coupled to fermions was first addressed in Ref. [4] and revisited in the setting of extended supergravities in Ref. [5]. While the duality groups of $\mathcal{N}>4$ supergravities are expected to be preserved at the quantum level, it is argued in Ref. [6] that the classical $SU(1,1)$ duality group of $\mathcal{N}=4$ supergravity is anomalous. This anomaly can be pushed into a $U(1)$ subgroup that "sources" certain classes of amplitudes that vanish at tree level [7].

The duality symmetries have received renewed interest due to their implications on the ultraviolet properties of these theories [7–11]. The precise connection between the anomalies in these symmetries and ultraviolet divergences remains to be fully unraveled [10–12], but there are hints that in $\mathcal{N}=4$ supergravity the two are tied [7,13,14]. For instance, the amplitudes sourced by the anomaly, when inserted into unitarity cuts at higher loops, lead to potential ultraviolet divergent contributions [7], which cancel at three loops [12], but not at four loops [13]. Another connection is the recent observation that the anomalous amplitudes are intertwined with evanescent contributions [15], similar to those which played an important role in the interpretation of the two-loop ultraviolet divergence of pure Einstein gravity in dimensional regularization [16,17]. Evanescent operators—such as the Gauss-Bonnet R^2 operator—are those whose tree-level matrix elements vanish identically in four but not general dimensions. Because of this property, they can contribute to divergences in dimensional regularization while otherwise having no physical consequences in the amplitudes, since their effects can be removed by the addition of local counterterms [18]. Similar curvature-squared evanescent contributions also appear at one loop in $\mathcal{N}=4$ supergravity [15], although with an ultraviolet-finite coefficient. From the perspective of the double-copy construction of $\mathcal{N}=4$ supergravity [19,20], in terms of $\mathcal{N}=4$ super-Yang-Mills and pure Yang-Mills theory, both these and the anomalous contributions originate in the same rational pieces of the corresponding Yang-Mills amplitudes.

Given the relation between the anomalous terms and the evanescent contributions, and the understanding that the effects of the latter can be absorbed into finite local counterterms, we are immediately led to the following questions: Can we absorb the effects of the anomaly in the one-loop scattering amplitudes into a local counterterm? More generally, can the anomaly be canceled in such a way? Motivated by these questions we explicitly computed a variety of anomalous amplitudes in $\mathcal{N}=4$ supergravity, including infinite classes, and show that they can indeed be set to zero by the addition of a finite local counterterm given by the supersymmetrization of a curvature-squared operator multiplied by scalars. The restoration of the anomalous $U(1)$ symmetry via the addition of a local counterterm has already been discussed in Ref. [10] in the context of the ungauge-fixed effective action. Here we address a related, but somewhat different, issue of finding a specific counterterm that removes anomalous scattering amplitudes.

Review.—The physical complex scalar of $\mathcal{N}=4$ supergravity parametrizes the coset $SU(1,1)/U(1)$; the choice of parametrization determines the anomalous $U(1)$. In a classically equivalent formulation with a global $SU(1,1)$ symmetry and an auxiliary $U(1)$ gauge symmetry [1] it is the latter symmetry that is anomalous. The former framework is recovered upon gauge fixing [5,21]. In the so-called $SU(4)$ gauge fixing the $SU(1,1)$ transformations are (a) shifting the physical scalar τ, (b) rescaling τ while rescaling oppositely the vector fields, and (c) nonlinearly transforming τ while chirally rotating the fermions and dualizing the vector fields. It is the third generator that is anomalous.

On shell, the spectrum of $\mathcal{N}=4$ supergravity consists of two different supermultiplets which can be represented using an on-shell superspace [22] as

$$\begin{aligned}\Phi^+ &= h^{++} + \eta^A\psi_A^+ + \frac{1}{2!}\eta^A\eta^B A_{AB}^+ \\ &\quad + \frac{1}{3!}\eta^A\eta^B\eta^C\varepsilon_{ABCD}\chi^{+D} + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D\varepsilon_{ABCD}\bar{t}, \\ \Phi^- &= t + \eta^A\chi_A^- + \frac{1}{2!}\eta^A\eta^B A_{AB}^- + \frac{1}{3!}\eta^A\eta^B\eta^C\varepsilon_{ABCD}\psi^{-D} \\ &\quad + \frac{1}{4!}\eta^A\eta^B\eta^C\eta^D\varepsilon_{ABCD}h^{--}. \end{aligned} \tag{1}$$

The indices A, B, C, D are $SU(4)$ R-symmetry indices. The Φ^+ multiplet contains the positive-helicity graviton h^{++}, the four positive-helicity gravitinos ψ_A^+, and so forth down to the complex scalar $\bar{t}$. The second supermultiplet is the CPT conjugate to first one and contains the negative-helicity graviton h^{--} and the conjugate scalar t. The relation between t and τ is

$$\tau = i + it + \mathcal{O}(t^2). \tag{2}$$

Superamplitudes in this theory are classified according to a maximally-helicity-violating (MHV) degree, $k=0,\ldots,n-4$ and the numbers n_+ and n_- of particles in the Φ^+ and Φ^- multiplets [7]. We shall denote the n-point $\mathrm{N}^k\mathrm{MHV}^{(n_+,n_-)}$ amplitudes as

$$M_{n,k}^{(n_+,n_-)} \equiv M_{n,k}(\Phi_1^+,\ldots,\Phi_{n_+}^+,\Phi_{n_++1}^-,\ldots,\Phi_n^-), \tag{3}$$

where $n=n_++n_-$. Only the $\mathrm{N}^k\mathrm{MHV}^{(n-k-2,k+2)}$ tree-level scattering superamplitudes of this theory are nonvanishing. This is a consequence of the $U(1)$ symmetry, which assigns charges $(0,\pm1/2,\pm1,\pm3/2,-2,2)$ to the states $(h^{\pm\pm},\psi^\pm,A^\pm,\chi^\pm,t,\bar{t})$ in Eq. (1). The $U(1)$ charges of spinors and vectors identify it as the on-shell form of the third (anomalous) generator. The main consequence of the anomaly in the S matrix is that the selection rule does not hold at loop level and all the amplitudes become nonvanishing [7].

From the double-copy perspective, the two multiplets in Eq. (1) correspond to the tensor products of an $\mathcal{N}=4$ super-Yang-Mills multiplet and a positive or negative-helicity gluon state of a pure Yang-Mills theory. In this way, the MHV degree k corresponds to the one of the supersymmetric side of the double copy and the n_+ and n_- labels refer to the two helicities of the gluon on the pure Yang-Mills side. The $U(1)$ charge of a given state is given by the difference of the helicities on the two sides of the double copy: $q_{U(1)} = h(\mathrm{YM}) - h(\mathrm{SYM})$. The anomalous tree amplitudes with $n_-=0$, 1, $n-1$ or n vanish trivially because the corresponding pure Yang-Mills tree amplitudes vanish in the double copy. The other cases are less trivial and rely on identities between gauge-theory amplitudes, such as those described in Ref. [23].

Amplitudes.—One-loop scattering amplitudes in $\mathcal{N}=4$ supergravity have been studied in Refs. [7,20,24–27]. Following Ref. [20], we straightforwardly obtain all the anomalous four- and five-point gravity amplitudes from the gauge-theory ones [28] using the double-copy construction [19,29]. We present them here in the spinor helicity conventions of Ref. [30]. For an n-point amplitude we omit the overall factor of $(\kappa/2)^n/(4\pi)^2$, where κ is the gravitational coupling and the conserved supermomentum is denoted by $Q^A = \sum_{j=1}^n \lambda_j^\alpha \eta_j^A$. At four points the two independent anomalous superamplitudes are

$$\begin{aligned} M_{4,0}^{(0,4)} &= i\delta^{(8)}(Q), \\ M_{4,0}^{(1,3)} &= -i\frac{[1\,2]\langle 2\,3\rangle\langle 2\,4\rangle}{\langle 1\,2\rangle\langle 1\,3\rangle\langle 1\,4\rangle}\delta^{(8)}(Q). \end{aligned} \tag{4}$$

Similarly, the five independent anomalous five-point superamplitudes are given by

$$M_{5,0}^{(0,5)} = i2\delta^{(8)}(Q),$$
$$M_{5,0}^{(1,4)} = -i\sum_{r=2}^{3}\frac{[1\,r]\langle r\,4\rangle\langle r\,5\rangle}{\langle 1\,r\rangle\langle 1\,4\rangle\langle 1\,5\rangle}\delta^{(8)}(Q),$$
$$M_{5,0}^{(2,3)} = -i\varepsilon(1,2,3,4)\frac{\langle 3\,4\rangle^2\langle 4\,5\rangle^2\langle 5\,3\rangle^2}{\prod_{i<j}\langle i\,j\rangle}\delta^{(8)}(Q),$$
$$M_{5,0}^{(4,1)} = -i\frac{s_{12}s_{34}\delta^{(8)}(Q)}{\langle 1\,4\rangle\langle 4\,3\rangle\langle 3\,5\rangle\langle 5\,2\rangle\langle 2\,1\rangle}\left(\frac{2[1\,4]^2[1\,3][2\,4]}{\langle 2\,3\rangle[4\,5][1\,5]} - \frac{[1\,2]^2\langle 2\,5\rangle^2\langle 5\,1\rangle^2 + [3\,4]^2\langle 4\,5\rangle^2\langle 5\,3\rangle^2}{\langle 1\,2\rangle\langle 2\,3\rangle\langle 3\,4\rangle\langle 4\,5\rangle\langle 5\,1\rangle}\right) + (2\leftrightarrow 3),$$
$$M_{5,0}^{(5,0)} = i\sum_{i<j}\frac{(\hat{\gamma}_{ij})^2}{s_{ij}}\delta^{(8)}(Q), \tag{5}$$

where $s_{ij} = (k_i + k_j)^2$, $\varepsilon(i,j,k,l) = 4i\varepsilon_{\mu\nu\sigma\rho}k_i^\mu k_j^\nu k_k^\sigma k_l^\rho$ and [31]

$$\hat{\gamma}_{12} = \frac{[1\,2]^2[3\,4][4\,5][3\,5]}{\varepsilon(1,2,3,4)}. \tag{6}$$

The (1,4), (2,3), and (4,1) amplitudes are new and the rest match the results in Ref. [7]. The result with n_V vector multiplets running in the loop is the same with an additional overall multiplicative factor of $(n_V + 2)/2$. As one would expect for an anomaly in dimensional regularization, all of these amplitudes are nonvanishing because of ϵ/ϵ effects where $\epsilon = (4 - D)/2$. It is also worth noting that through the lens of the double-copy construction some of these amplitudes are obtained from nonsupersymmetric gauge-theory amplitudes which also have been suggested to be nonzero because of another type of duality anomaly [32].

In Ref. [7], the n-point amplitude,

$$M_{n,0}^{(0,n)} = i(n-3)!\delta^{(8)}(Q), \tag{7}$$

is obtained via inverse-soft-scalar limits. We have extended this to the cases with $n_- \geq 3$ and n_+ arbitrary, which correspond to all infinite classes of MHV anomalous amplitudes except for those with $n_- = 0$, 1. All superamplitudes of this kind are nonlocal, and can be obtained from the local ones in Eq. (7) via the inverse-soft construction [33] which we implement using the soft-lifting functions of Ref. [26]. A convenient way of presenting them is as the minors, with rows and columns $M = \{m_1, \ldots, m_r\}$ removed [34], $S[M] = |\Phi|^{m_1\ldots m_r}_{m_1\ldots m_r}$, of Hodges' Φ matrix [35] with components

$$\phi_i^j = \frac{[i\,j]}{\langle i\,j\rangle} \quad \text{for } i \neq j, \qquad \phi_i^i = -\sum_{j\neq i}\frac{[i\,j]\langle j\,x\rangle\langle j\,y\rangle}{\langle i\,j\rangle\langle i\,x\rangle\langle i\,y\rangle}, \tag{8}$$

where x, y are arbitrary reference spinors. We find that the $n_- > 3$ anomalous amplitudes are simply given in terms of the soft-lifting functions,

$$M_{n,0}^{(n_+,n_-)} = i(n_- - 3)!S[M]\delta^{(8)}(Q), \tag{9}$$

where M is the set of Φ^- external states. The soft-lifting function was used in Ref. [26] to obtain the rational terms needed to complete the construction of the n-point nonanomalous ($n_- = 2$) amplitudes. We suspect that a similar inverse-soft formula exists for the $n_- < 2$ amplitudes, but finding it would require a detailed understanding of the kinematic deformations that implement the inverse-soft procedure, which is trivial for the $n_- \geq 3$ cases but not for the rest.

Equation (9) reproduces the corresponding results in Eqs. (4) and (5). In addition, as explained in Ref. [26], the soft-lifting functions encode all the correct soft and collinear limits involving the legs added in the inverse-soft procedure. Indeed, one can straightforwardly check that Eq. (9) has the correct soft limits

$$M_{n,0}^{(n_+,n_-)} \xrightarrow{k_i\to 0} \mathcal{S}_{p_i}M_{n-1,0}^{(n_+-1,n_-)}, \quad \text{for } \Phi_i^+,$$
$$M_{n,0}^{(n_+,n_-)} \xrightarrow{k_i\to 0} (n_- - 3)M_{n-1,0}^{(n_+,n_--1)}, \quad \text{for } \Phi_i^-, \tag{10}$$

where $\mathcal{S}_{p_i} = \phi_i^i$ is the usual graviton leading soft factor. Similarly, taking the supersymmetric collinear limits

$$(\lambda_a, \tilde{\lambda}_a, \eta_a) \to \sqrt{z}(\lambda_K, \tilde{\lambda}_K, \eta_K),$$
$$(\lambda_b, \tilde{\lambda}_b, \eta_b) \to \sqrt{1-z}(\lambda_K, \tilde{\lambda}_K, \eta_K), \tag{11}$$

we find that the amplitudes in Eq. (9) have a universal phase singularity, i.e.,

$$M_{n,0}(\ldots, \Phi_a^{h_a}, \Phi_b^{h_b}, \ldots) \xrightarrow{a\|b} \sum_{h_K=\pm}\mathrm{Sp}^{h_a h_b}_{-h_K}M_{n-1,0}(\ldots, \Phi_P^{h_K}, \ldots), \tag{12}$$

where the h_i denote the supermultiplet and the relevant splitting functions are

$$\mathrm{Sp}_-^{++} = -\frac{1}{z(1-z)}\frac{[a\,b]}{\langle a\,b\rangle}, \qquad \mathrm{Sp}_+^{-+} = -\frac{z}{(1-z)}\frac{[a\,b]}{\langle a\,b\rangle}. \tag{13}$$

As usual for gravity collinear limits with real momenta, we are only concerned with the terms that contain phase singularities. (See Ref. [36] for further details.) These checks fall short of a proof of Eq. (9), but as usual they give us confidence that this formula is correct.

Cancellation of anomalous amplitudes.—With these results in hand we can address the question of whether the anomalous amplitudes can be canceled by a local counterterm. Reference [7] noted that the local amplitudes (7) can be interpreted as arising from the following local $U(1)$-breaking terms in the one-loop effective action,

$$\Gamma^{\text{local}}_{\text{U}(1)} = \frac{1}{2(4\pi)^2}\int d^4x((1-\log(1-\bar{t}))(R^+)^2 + (1-\log(1-t))(R^-)^2) + \text{SUSY}, \tag{14}$$

where R^+ and R^- are the self-dual and anti-self-dual parts of the Riemann tensor, $R^{\pm}_{\mu\nu\rho\sigma} = \pm(i/2)\varepsilon_{\mu\nu}{}^{\alpha\beta}R^{\pm}_{\alpha\beta\sigma\rho}$ with $\varepsilon_{0123} = +1$. Again, in the presence of n_V vector multiplets there is an extra factor of $(n_V+2)/2$ in the coefficient.

As usual, definitions of quantum theories are ambiguous up to the addition of local counterterms. Ambiguities can be fixed by demanding that classical symmetries and associated constraints on the scattering amplitudes are preserved. In this spirit, we choose to define $\mathcal{N}=4$ supergravity to include the finite local counterterm $S_{\text{ct}} = -\Gamma^{\text{local}}_{U(1)}$ which subtracts away the local part of the anomalous effective action (14). While this subtraction effectively changes the gauge of the auxiliary $U(1)$ symmetry and thus departs [7] from the original formulation of the off-shell theory [2], it also sets to zero all one-loop local anomalous amplitudes in Eq. (7), so it is the one we desire.

Deformations of the classical action by a local operator also contribute to nonlocal amplitudes. Since the counterterm sets to zero the anomalous local amplitudes, it is interesting to consider its effect on the nonlocal ones as well. The presentation of the $n_- > 3$ amplitudes in Eq. (9) makes it clear that the same counterterm in Eq. (14) also cancels this entire class of amplitudes, because it cancels the seed $M^{(0,n)}_{n,0}$ of the inverse soft construction. That is, for this class of superamplitudes, at one loop we have

$$M^{(n_+,n_-),\text{full}}_{n,0} = M^{(n_+,n_-)}_{n,0} + M^{(n_+,n_-),S_{\text{ct}}}_{n,0} = 0, \tag{15}$$

where $M^{(n_+,n_-),S_{\text{ct}}}_{n,0}$ are tree superamplitudes with a single vertex from the counterterm action in Eq. (14).

To independently confirm this cancellation, and to also check the fate of the two amplitudes in Eq. (5) which are not given by Eq. (9), we compute the contribution of the counterterm action to one-loop amplitudes using the double-copy construction applied to higher-dimension operators [37]. References [37,38] show that, when applied to two copies of pure Yang-Mills theory with a $\text{Tr}(F^\mu{}_\nu F^\nu{}_\rho F^\rho{}_\mu)/3$ deformation, the double-copy approach yields the amplitudes in gravity deformed by $\phi^m R^2$ operators, where ϕ is a scalar field. Using instead $\mathcal{N}=4$ super-Yang-Mills theory as the first factor leads therefore to the tree-level amplitudes of $\mathcal{N}=4$ supergravity deformed by an operator $\mathcal{O}$ which is a supersymmetric completion of the $\phi^m R^2$ operators. Using the F^3-deformed Yang-Mills amplitudes found in Refs. [37,39], the Kawai-Lewellen-Tye (KLT) [40] double-copy formulas give

$$M^{(0,n),\mathcal{O}}_{n,0} = i(n-2)!\delta^{(8)}(Q), \tag{16}$$

for $n = 3, 4, 5$. Thus, at least for this number of external lines, the double copy yields the matrix elements of the supersymmetrization of the operator

$$\mathcal{O} = \frac{1}{2(4\pi)^2}\left((R^+)^2\sum_n \bar{t}^n + (R^-)^2\sum_n t^n\right), \tag{17}$$

which only differs by a numerical factor from the expected counterterm action (14): the number of scalars. The normalization of these matrix elements can be changed to obtain those of Eq. (14), thus confirming through an independent calculation that all three-, four-, and five-point anomalous one-loop amplitudes are canceled after deforming the classical action by the counterterm (14). Once lower-point amplitudes are canceled by a counterterm, consistency with soft and collinear graviton limits suggests that the cancellation continues to all multiplicities, even for $n_- = 0$, 1. These results strongly indicate that the counterterm action cancels both the local and the nonlocal one-loop anomalous amplitudes and thus restores the $U(1)$ symmetry. We expect that the addition of this counterterm moves the anomaly into other generators of $SU(1,1)$, which do not appear to impose any selection rule on the scattering amplitudes.

In addition, Ref. [15] explains that the nonanomalous four-point superamplitude has the form

$$M^{(2,2)}_{4,0} = M^{\text{tree}}_{R^2} + \cdots, \tag{18}$$

where $M^{\text{tree}}_{R^2}$ are evanescent matrix elements of the Gauss-Bonnet operator and its supersymmetric completion [41]. The same paper points out that the anomalous and evanescent contributions are intertwined by the double copy. The local term in Eq. (14) reflects this observation. Indeed, the t-independent term in Eq. (14) is the Gauss-Bonnet combination in the effective action,

$$\Gamma^{\text{local}}_{U(1)} = \frac{1}{2(4\pi)^2}\int d^4x R^* R^* + \cdots. \tag{19}$$

Counterterm (14) removes both the anomalous amplitudes and the evanescent contribution found at one loop in Ref. [15].

Reference [42] noted that the nonanomalous amplitudes can give rise to anomalous ones in the soft-scalar limit. This fact was interpreted as a consequence of the duality anomaly. Following the analysis above, we checked that the only effect of the counterterm on the four- and five-point amplitudes is restricted to the cancellation of evanescent pieces. In particular, one can check that the soft limits are unmodified. In general, higher-derivative corrections explicitly break the noncompact duality symmetry resulting in nonvanishing soft limits [9]. In light of this, the connection between soft scalar limits and the anomaly in the presence of counterterms requires further analysis.

Conclusions.—In summary, in this Letter we showed that both the local and nonlocal anomalous MHV one-loop amplitudes of $\mathcal{N}=4$ supergravity can be systematically canceled by adding a local counterterm to the classical action. These cancellations are nontrivial and strongly suggest that all anomalous amplitudes in the theory can be removed in the same way. Nevertheless, a number of issues remain. First, an off-shell supersymmetric completion of our counterterm might help shed light on the cancellations beyond scattering amplitudes. A related supersymmetric counterterm has been described in Ref. [43]; it would be worthwhile to directly compare the anomalous matrix elements of $\mathcal{N}=4$ supergravity with ones generated by this term, as well as the one described in Ref. [10]. It would also be interesting to understand the relation, if any, to other instances of cancellations of similar anomalies in other four-dimensional field and string-theory models [44–47]. Perhaps more importantly, we would like to investigate the anomalous amplitudes at higher loops. Reference [13] found that, at four loops, both anomalous and nonanomalous amplitudes in $\mathcal{N}=4$ supergravity carry a leading ultraviolet divergence. This is surprising because the integrands of the anomalous amplitudes must vanish in strictly four dimensions and therefore carry an extra factor the dimensional regularization parameter ϵ compared to the nonanomalous ones. One would then expect the divergences of the anomalous amplitudes to be suppressed, unless the actual source of the divergence is the anomaly. As mentioned above, the anomalous lower-loop amplitudes are expected to induce divergences at higher loops, even in the nonanomalous amplitudes [7]. Thus, the local counterterms removing the former should contribute nontrivially to the divergences of the latter. The complete four-loop divergence should thus be reanalyzed. To determine the counterterm effects on the four-loop divergence it is necessary to evaluate anomalous amplitudes at higher loops and understand whether further finite counterterms are necessary for their removal. In any case, there are clearly new lessons to be learned by investigating the higher-loop amplitudes of this theory.

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