# Spatial-Translation-Induced Discrete Time Crystals

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A discrete time crystal is a phase unique to nonequilibrium systems, where discrete time translation symmetry is spontaneously broken. Most conventional time crystals proposed so far rely on the spontaneous breaking of on-site symmetries and their corresponding on-site symmetry operations. In this Letter, we propose a new time crystal dubbed the "spatial-translation-induced discrete time crystal," which is realized by spatial translation and its symmetry breaking. Owing to the properties of spatial translation, in this new time crystal, various time crystal orders can only emerge by changing the filling but not changing the driving protocol. We demonstrate that the local transport of charges or spins shows a nontrivial oscillation, enabling detection and applications of time crystal orders, and also provide promising platforms including quantum circuits. Our proposal opens up a new avenue of realizing time crystal orders by spatial translation in various quantum simulators.

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*Introduction.*—A system whose Hamiltonian is periodic in time is called a Floquet system. The past decade has seen a tremendous growth of interest in such Floquet systems, which produce a variety of phases controlled by periodic driving [1–8]. It has also turned out that Floquet systems have novel phases that cannot exist in equilibrium, such as anomalous Floquet topological phases hosting chiral edge states despite the vanishing Chern numbers [9–13].

One of the most striking phases in Floquet systems is a discrete time crystal phase. It has indeed been proved that time crystals, where time translation symmetry is spontaneously broken, cannot exist in thermal equilibrium [14]. Thus, time crystals are inherent in nonequilibrium systems. In particular, time crystals realized in Floquet systems, called discrete time crystals (DTCs), are phases where discrete time translation symmetry is spontaneously broken and the resulting oscillation frequency of local observables is robust to perturbations. DTCs have attracted much interest because of their theoretical developments [15,16] and experimental realizations in various systems [17–21].

In most of conventional DTCs proposed so far, symmetry operation and phases brought by many-body localization (MBL) or spontaneous symmetry breaking (SSB) are utilized to realize DTC orders [15,16,22–27]. However, among them, only on-site symmetries by finite groups $\mathbb{Z}_n$ have been focused on, thus leading to a restriction that changing the driving protocol is required to realize different types of DTC orders.

In this Letter, focusing on spatial translation symmetry, which is a nonlocal but *infinite* group symmetry, we propose new DTCs realized by spatial translation and its symmetry breaking, and we thereby provide a feasible platform to realize various kinds of time crystals. First, note that spatial translation symmetry breaking can induce various orders. For example, a variety of charge density wave (CDW) orders can be realized by changing the filling when discrete spatial translation symmetry is spontaneously broken. Therefore, by utilizing these characteristics, in the new DTC, which is dubbed "spatial-translation-induced DTC"(STI-DTC), various DTC orders can be realized and controlled without changing the protocol in sharp contrast to the previously proposed DTCs. We further demonstrate that, in STI-DTCs, spatial translation induces local transport that shows nontrivial oscillation due to the time crystal orders. This property is characteristic of STI-DTCs, having merits for the detection and application of DTC orders. We also provide a general scheme to implement STI-DTCs with quantum circuits. With the above novel properties, STI-DTCs will open up a new way of realizing time crystals in various quantum simulators and their application to quantum information processing.

*Definition and example of DTCs.*—First of all, let us clarify the definition of DTCs. Originally, time crystals were introduced as systems where spontaneous time translation symmetry breaking (TTSB) occurs [28]. Since we consider only the cases when the Hamiltonian is periodic in time with period $T$, this means that local observables have a period different from the Hamiltonian's period $T$ in its (quasi)steady states. However, the definition characterized only by TTSB is inadequate because trivial examples such as Rabi oscillations are included. To preclude such examples and define DTC as a stable phase of matter, it should be defined as a phase where not only TTSB occurs but also the period of oscillation is robust to perturbations that do not change the driving period [16]. When the period of the local observables is $nT$, the phase is called $nT$-DTC.

 

DTCs are realized in several ways [15,23,29,30]. Among them, we focus on the one that relies on SSB and its corresponding symmetry operation. Assume that Floquet operator $U_f$, which is the time evolution operator of one period under the time-periodic Hamiltonian $H(t)$, is written in the form of

$$U_f \equiv \mathcal{T}\exp\left(-i\int_0^T H(t)dt\right) = X\exp(-iDT), \qquad (1)$$

where the effective Hamiltonian $D$ induces spontaneous $\mathbb{Z}_n$-symmetry breaking and $X$ is the corresponding symmetry operation. Then, TTSB occurs. Intuitively, this is because some Floquet eigenstates become cat states that are superpositions of $n$ symmetry broken ordered states. Since the quasienergies of such cat states are equidistantly separated by $2\pi/nT$, the oscillation of the cat states with the period $T$ becomes unstable, and instead, the superpositions thereof (i.e., macroscopically stable ordered states) show the nontrivial oscillation. Robustness of TTSB behavior is supported by several ways, such as prethermalization and MBL [23,31]. In both cases, the Floquet operator is unitarily equivalent to the form of Eq. (1), even if there is a small perturbation, and hence, TTSB behavior is robust.

*Spatial-translation-induced DTC.*—Here, we propose a new type of time crystals: spatial-translation-induced DTCs (STI-DTCs), where the symmetry operation $X$ is a spatial translation and $D$ shows spontaneous spatial translation symmetry breaking.

Let us consider a spinless fermion system in a one-dimensional ring. We consider a lattice system at half filling. If the spinless fermions have long-range repulsive interactions and the temperature of the initial state is low enough, spatial translation symmetry breaking occurs, and one of the two symmetry-broken states realizes [Fig. 1(a)]. In each state, one fermion localizes in every two sites and forms a CDW state. Suppose that spatial translation operation by one site can be realized by a certain periodic driving. Then, the dynamics of this system can be described as Fig. 1(a). Since positions of localized fermions change every period, the particle density at a certain site oscillates

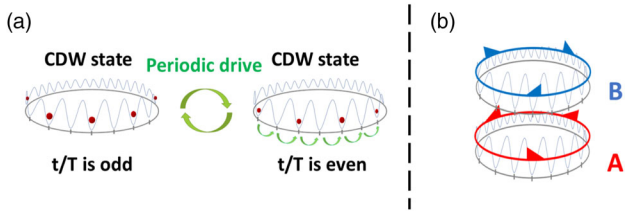


FIG. 1. (a) Intuitive picture of the STI-DTCs. If spatial translation is realized, the particle number oscillates with a period of $nT$. (b) The model for STI-DTCs. During one period, particles on the lower ring are translated anticlockwise, and those on the upper one are translated clockwise; thus in total, there is no pumping.

with a double period of the driving. The nontrivial $2T$-period oscillation is expected to be stabilized by the CDW order. Therefore, this system would be a $2T$-DTC if the assumption were correct [32].

Compared with conventional DTCs, it is notable that STI-DTCs can utilize $\mathbb{Z}_n$ orders brought by CDW for any integer $n(\geq 2)$ to realize DTC orders. In other words, $nT$-DTCs are expected to be realized at $1/n$ filling, since a particle localizes in every $n$ site in each of CDW states. Thus, STI-DTCs can realize various DTC orders with the same protocol only by changing the filling.

One important question is how the spatial translation operation can be realized by local Hamiltonians. If any long time can be taken for one period, spatial translation is possible in a one-dimensional ring by Thouless pumping [33], which is adiabatically performed. It has already been experimentally realized in cold atoms [34–36], and the combination with CDW has been theoretically suggested [37–40]. However, Thouless pumping, which requires infinite time even for one period, is not suitable for the realization of DTCs. To overcome this difficulty, we propose below a one-dimensional ladder ring as a candidate of STI-DTCs, which is nonadiabatically realizable [Fig. 1(b)].

*Model in 1D ladder.*—Here, we describe how to realize a STI-DTC in a one-dimensional ladder ring. In this model, as shown in Fig. 1(b), spatial translation $\mathbb{T}_A$ by one site in the sublattice $A$ and the opposite one $\mathbb{T}_B^{-1}$ in the sublattice $B$ are induced every period. Since the total amount of pumping is zero, the time-dependent Hamiltonian is nonadiabatically realizable by switching local Hamiltonians as follows [12] (Fig. 2(a)):

$$H(t) = \begin{cases} H_1 & (0 \leq t \leq \tau/2) \\ H_2 & (\tau/2 < t \leq \tau) \\ H_{\mathrm{SSB}} & (\tau < t \leq T), \end{cases} \qquad (2)$$

where each Hamiltonian is defined as

$$H_1 = -\frac{\pi}{\tau}\sum_i (c_{i,A}^\dagger c_{i,B} + \mathrm{H.c.}), \qquad (3)$$

$$H_2 = -\frac{\pi}{\tau}\sum_i (c_{i+1,A}^\dagger c_{i,B} + \mathrm{H.c.}), \qquad (4)$$

$$H_{\mathrm{SSB}} = \sum_{\alpha=A,B}\sum_{i,j}\frac{U_{ij}}{2} n_{i,\alpha}n_{j,\alpha}. \qquad (5)$$

Here, $c_{i,\alpha}$ and $n_{i,\alpha}$, respectively, represent the annihilation and the number operator of spinless fermions at site $i$ in a sublattice $\alpha = A, B$. $U_{ij}$ represents the strength of the long-range repulsive interaction, and then $H_{\mathrm{SSB}}$ is a Hamiltonian that induces spontaneous spatial translational symmetry breaking at low temperature. As seen below, the time

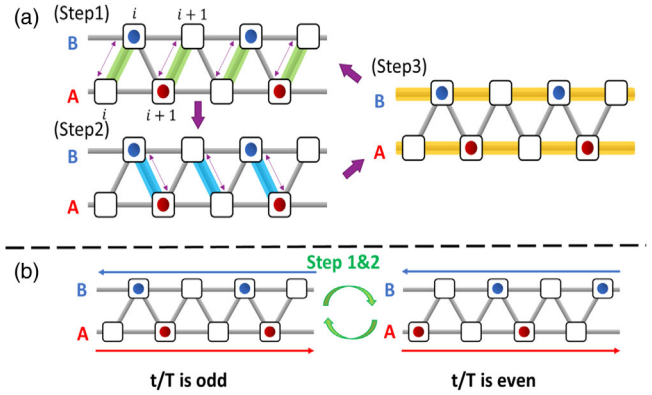


FIG. 2. (a) Hamiltonian described by Eq. (2) for one period. After Step 1 or Step 2, the particle numbers are exchanged between sites linked by green or blue lines. Yellow lines in Step 3 mean repulsive interaction. (b) Dynamics of the model. The period of particle number becomes $2T$ at half filling.

evolution, under Hamiltonians $H_1$ and $H_2$, generates the spatial translation $\mathbb{T}_A \otimes \mathbb{T}_B^{-1}$, and the time evolution under $H_{\mathrm{SSB}}$ stabilizes CDW states.

*Time translation symmetry breaking.*—Let us confirm that TTSB occurs in the system. How the spatial translation $\mathbb{T}_A \otimes \mathbb{T}_B^{-1}$ is generated by the Hamiltonian is explained intuitively here. The detailed calculation is provided in the Supplemental Material [41].

First, let us consider the time evolution under the Hamiltonian $H_1$ given in Eq. (3), which induces hopping between sites $(i, A)$ and $(i, B)$ for every $i$. Since the duration of imposing $H_1$ is fine-tuned, a particle at $(i, A)$ is completely transferred to $(i, B)$ and vice versa by the time evolution under $H_1$. Similarly, by the time evolution under $H_2$, exchanges of particles occur between sites $(i+1, A)$ and $(i, B)$ [Fig. 2(a)]. When we consider the dynamics under $H_1$ and $H_2$, a particle at $(i, A)$ moves to $(i, B)$, and after that, it reaches $(i+1, A)$. On the other hand, a particle at $(i, B)$ moves to $(i-1, B)$. Therefore, the spatial translation by one site in A and the opposite translation in B are realized by $H_1$ and $H_2$, as described by

$$e^{-iH_2\tau/2}e^{-iH_1\tau/2} = (\mathbb{T}_A \otimes \mathbb{T}_B^{-1})U_p, \tag{6}$$

where $U_p \equiv \exp\{-i\pi\sum_i n_{i,A}(n_{i,B} + n_{i+1,B})\}$ is the phase that stems from the commutation relation of fermion operators [41]. Here, a global phase other than $U_p$ is removed by a proper gauge transformation.

Then, from Eq. (6), the Floquet operator $U_f$ is described as follows:

$$U_f = (\mathbb{T}_A \otimes \mathbb{T}_B^{-1})\exp(-iH_{\mathrm{SSB}}(T-\tau))U_p. \tag{7}$$

Though its form is slightly different from Eq. (1) by the existence of $U_p$, TTSB can be induced since the spatial translation $\mathbb{T}_A \otimes \mathbb{T}_B^{-1}$ moves particles regardless of $U_p$. In fact, in the Heisenberg picture,

$$n_{i,A}(mT) = n_{i-m,A}, \qquad n_{i,B}(mT) = n_{i+m,B} \tag{8}$$

is satisfied for $m \in \mathbb{N}$ [41]. Thus, when a CDW state, where spatial translation symmetry is broken, is prepared as the initial state, then the particle density or the current at each site oscillates with a period different from the Hamiltonian [Fig. 2(b)]. At $1/n$ filling, since particles localize at every $n$ site, TTSB occurs, and $nT$ oscillation is observed.

*Robustness of TTSB.*—One of the nontrivial properties of DTCs is robustness, that is, the oscillation frequency of the observables is hardly influenced by some small perturbations. In the case of $nT$-DTCs, the peak structure of the Fourier component of the oscillation at $\omega T = 2\pi/n$ does not move nor split even if there is a small perturbation [16].

To confirm the robustness of DTCs, let us consider perturbations on symmetry operations [18,24,42]. In the case of STI-DTCs, the perturbation is assumed to be on the spatial translation operation, that is, on the duration of $H_2$. Then, the perturbed Hamiltonian $H(t)$ is described as follows:

$$H(t) = \begin{cases} H_1 & (0 \le t \le \tau/2) \\ H_2 & [\tau/2 < t \le (1+r)\tau/2] \\ H_{\mathrm{SSB}} & [(1+r)\tau/2 < t \le T] \end{cases}. \tag{9}$$

When $r = 1$, Eq. (9) is reduced to the unperturbed case described by Eq. (2). Thus, the value $|r-1|$ represents the strength of the perturbation. The independent parameters of the system are $r$ and $U_{\mathrm{int}}T_{\mathrm{SSB}}$, where $T_{\mathrm{SSB}} \equiv T - (1+r)\tau/2$ represents the duration of $H_{\mathrm{SSB}}$. Since the theorem about prethermalization in [23] is not necessarily applicable to the current system [43], robustness to the perturbation is examined by the exact diagonalization for finite systems. We assume a long-range repulsive interaction $U_{ij} = U_{\mathrm{int}}/r_{ij}^3$, where $r_{ij}$ and $U_{\mathrm{int}}$ represents the distance between sites $i$ and $j$ and the strength of the interaction, respectively. Such a long-range interaction is realized in trapped ions and diamond NV centers, which are platforms of DTCs [17,18].

Figure 3 represents the results when $U_{\mathrm{int}}T_{\mathrm{SSB}} = 1.0$, in the case of I, $2 \times 8$ sites at half filling, and II, $2 \times 9$ sites at $1/3$ filling. Here, the initial state is assumed to be an equilibrium state under $H_{\mathrm{SSB}}$ at low temperature, which spontaneously breaks the spatial translation symmetry. In this calculation, the initial temperature is zero, and the odd sites in both sublattices are occupied in the initial state [44]. The Supplemental Material provides the case with finite initial temperature, which shows a similar result [41]. In both cases, when $r = 1.05$, the oscillation of the particle number hardly decays [see (a)]; thus TTSB behavior is robust. On the other hand, when $r = 1.15$, the oscillation rapidly decays and then DTC order is lost [see (d)]. Robustness can be examined also from their Fourier spectrum described by (b) and (e). Each of the peaks at

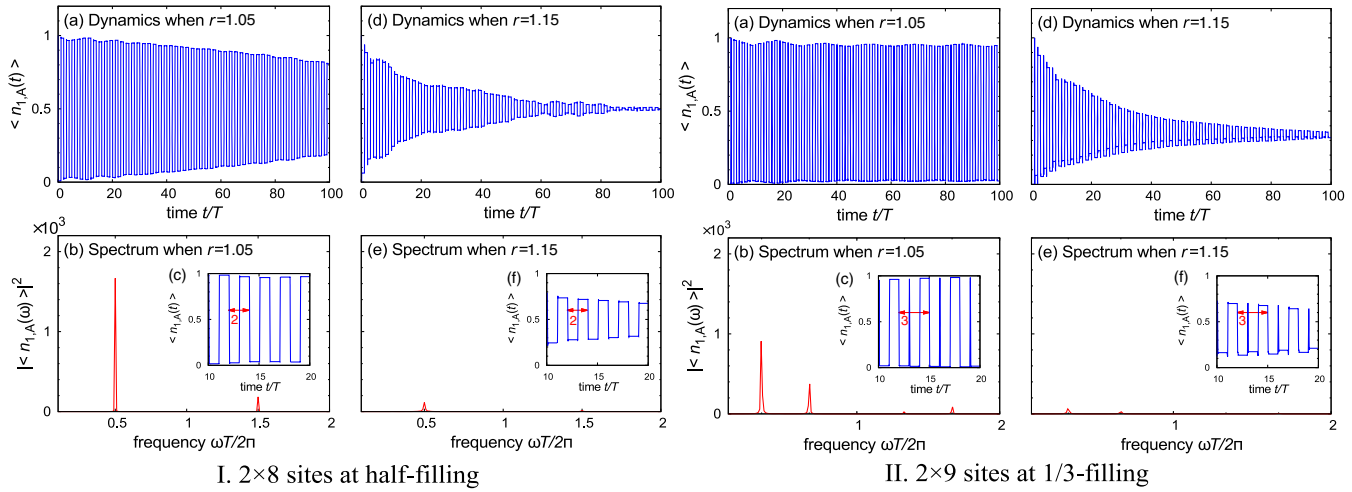


FIG. 3. Dynamics of the particle number and its Fourier spectrum in the case of (i) $2 \times 8$ sites at half filling and in the case of (ii) $2 \times 9$ sites at $1/3$ filling, (a,b) when $r = 1.05$ and (d,e) when $r = 1.15$, respectively. (c) and (f) are enlarged figures of (a) and (d), respectively. In each calculation, the interaction $U_{\mathrm{int}}T_{\mathrm{SSB}}$ is 1.0 and the duration $\tau/T$ is 0.1. The peaks at $\omega T/2\pi = 0$ in the spectrum are neglected since they are not related to TTSB behavior. In each situation, the dynamics shown in the figures are the full time evolutions but not the stroboscopic ones.

$\omega T/2\pi = 1/2$ and $\omega T/2\pi = 1/3$ corresponds to each $2T$ oscillation and $3T$ oscillation. It is notable that these peaks do not move from their original positions when $r = 1.05$. This property is unique to DTCs.

From these results, in both cases I and II of Fig. 3, there seem to be thresholds, where the DTC orders are lost, between $r = 1.05$ and $r = 1.15$. To examine the existence of the thresholds, the lifetime of STI-DTCs is also calculated for each $r$ and $U_{\mathrm{int}}T_{\mathrm{SSB}}$ (Fig. 4). Here, the lifetime is defined as the time when the amplitude of the oscillation becomes 90% of the initial value. Yellow regions indicate that the lifetime is more than $100T$; thus the DTC order is stable there. On the other hand, the black regions represent the cases where no robust oscillation is observed, and the boundary represents the thresholds where DTC orders are lost. Thus, Figs. 4(a) and 4(b) can be regarded as phase diagrams of $2T$-DTCs and $3T$-DTCs, respectively.

Note that, robustness is observed at half filling even without the interaction [Fig. 4(a)]. This phenomenon originates from the statistics of fermions. In the unperturbed case, where the Floquet operator is described by Eq. (7), the effective density-density interaction appears in $U_p$ because of the commutation relation of fermion operators. Even in perturbed cases, though the form is different from $U_p$, the effective density-density interaction appears and could stabilize CDW states at half filling.

*Experimental setup.*—STI-DTCs are expected to be realized in various systems since the essential ingredients for them are spatial translation and its symmetry breaking. For example, spin systems can also realize STI-DTCs by utilizing SWAP gates and antiferromagnetic order [41]. Thus it is expected that there are various platforms for realizing STI-DTCs.

For example, in trapped ions and Rydberg atoms, long-range interactions that can realize CDW order have been experimentally observed [18,45]. On the other hand, in cold atoms, selective hopping by $H_1$ and $H_2$ is theoretically proposed by moving optical lattices [12], and long-range interactions can be realized by dipole-dipole interactions and electric fields [42]. Thus, these are candidates for STI-DTCs.

One of the most promising platforms is a quantum circuit as a quantum simulator [46,47]. Since the dynamics of Floquet systems are described by $U_f$, STI-DTCs can be realized once the unitary gate of $U_f$ is prepared. In fact, $U_f$ described by Eq. (7) is realizable since the time evolution operators under $H_1$, $H_2$, and $H_{\mathrm{SSB}}$ can be composed of NOT gates, SWAP gates, and CPHASE gates [41]. Quantum circuits are, or will be, realized by various platforms, such as superconducting qubits and quantum dots [47–50]. Thus STI-DTCs on quantum circuits will be realized by various platforms.

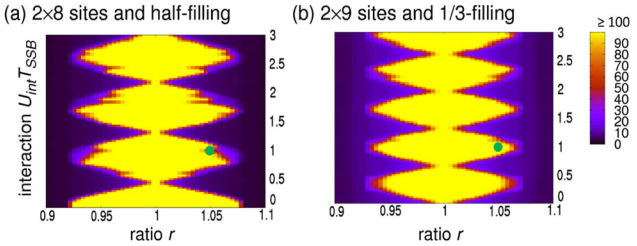


FIG. 4. Lifetime for each $r$ and $U_{\mathrm{int}}T_{\mathrm{SSB}}$ is described by the colors. In the yellow regions, lifetime is more than $100T$, where $T$ represents the driving period. The green points in (a) and (b) correspond to the cases described in Fig. 3 I(a) and Fig. 3 II(a), respectively.

*Conclusions.*—In this Letter, a new type of DTCs, spatial-translation-induced DTCs, have been proposed. In this time crystal, spatial translation operation changes the system from a CDW state to another CDW state; thus the particle number oscillates with a period different from its Hamiltonian. Unlike almost all the conventional DTCs, DTC orders in this system can be controlled by changing its filling. A one-dimensional ladder ring under a periodic drive has been proposed as a model of STI-DTCs, which is realizable without adiabaticity.

One question left open is to clarify the origin of robustness. In this Letter, robustness of the STI-DTCs has been confirmed by numerical calculations. Despite the inapplicability of the theorem about prethermalization in [23], the robustness of TTSB behavior has been observed, as seen in other examples [17,25,42,51]. Though it has been qualitatively demonstrated in this Letter that the CDW order supports the robustness, its quantitative evaluation by analytical calculation is desired.

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[1] T. Oka and H. Aoki, Phys. Rev. B **79**, 081406 (2009).
[2] T. Kitagawa, T. Oka, A. Brataas, L. Fu, and E. Demler, Phys. Rev. B **84**, 235108 (2011).
[3] N. H. Lindner, G. Rafael, and V. Galitski, Nat. Phys. **7**, 490 (2011).
[4] A. G. Grushin, A. Gómez-León, and T. Neupert, Phys. Rev. Lett. **112**, 156801 (2014).
[5] K. Takasan, A. Daido, N. Kawakami, and Y. Yanase, Phys. Rev. B **95**, 134508 (2017).
[6] K. Takasan, M. Nakagawa, and N. Kawakami, Phys. Rev. B **96**, 115120 (2017).
[7] Y. Wang, H. Steinberg, P. Jallilo-Herrero, and N. Gedik, Science **342**, 453 (2013).
[8] G. Jotzu, M. Messer, R. Desbuquois, M. Lebrat, T. Uehlinger, D. Greif, and T. Esslinger, Nature (London) **515**, 237 (2014).
[9] T. Kitagawa, E. Berg, M. Rudner, and E. Demler, Phys. Rev. B **82**, 235114 (2010).
[10] M. S. Rudner, N. H. Lindner, E. Berg, and M. Levin, Phys. Rev. X **3**, 031005 (2013).
[11] P. Titum, E. Berg, M. S. Rudner, G. Refael, and N. H. Lindner, Phys. Rev. X **6**, 021013 (2016).
[12] H. C. Po, L. Fidkowski, T. Morimoto, A. C. Potter, and A. Vishwanath, Phys. Rev. X **6**, 041070 (2016).
[13] S. Mukherjee, A. Spracklen, M. Valiente, E. Andersson, P. Öhberg, N. Goldman, and R. R. Thomson, Nat. Commun. **8**, 13918 (2017).
[14] H. Watanabe and M. Oshikawa, Phys. Rev. Lett. **114**, 251603 (2015).
[15] D. V. Else, B. Bauer, and C. Nayak, Phys. Rev. Lett. **117**, 090402 (2016).
[16] N. Y. Yao, A. C. Potter, I.-D. Potirniche, and A. Vishwanath, Phys. Rev. Lett. **118**, 030401 (2017).
[17] S. Choi, J. Choi, R. Landig, G. Kucsko, H. Zhou, J. Isoya, F. Jelezko, S. Onoda, H. Sumiya, V. Khemani, C. V. Keyserlingk, N. Y. Yao, E. Demler, and M. D. Lukin, Nature (London) **543**, 221 (2017).
[18] J. Zhang, P. W. Hess, A. Kyprianidis, P. Becker, A. Lee, J. Smith, G. Pagano, I.-D. Potirniche, A. C. Potter, A. Vishwanath, N. Y. Yao, and C. Monroe, Nature (London) **543**, 217 (2017).
[19] J. Rovny, R. L. Blum, and S. E. Barrett, Phys. Rev. Lett. **120**, 180603 (2018).
[20] J. Rovny, R. L. Blum, and S. E. Barrett, Phys. Rev. B **97**, 184301 (2018).
[21] S. Pal, N. Nishad, T. S. Mahesh, and G. J. Sreejith, Phys. Rev. Lett. **120**, 180602 (2018).
[22] K. Sacha, Phys. Rev. A **91**, 033617 (2015).
[23] D. V. Else, B. Bauer, and C. Nayak, Phys. Rev. X **7**, 011026 (2017).
[24] Z. Gong, R. Hamazaki, and M. Ueda, Phys. Rev. Lett. **120**, 040404 (2018).
[25] W. W. Ho, S. Choi, M. D. Lukin, and D. A. Abanin, Phys. Rev. Lett. **119**, 010602 (2017).
[26] A. Russomanno, F. Iemini, M. Dalmonte, and R. Fazio, Phys. Rev. B **95**, 214307 (2017).
[27] T.-S. Zeng and D. N. Sheng, Phys. Rev. B **96**, 094202 (2017).
[28] F. Wilczek, Phys. Rev. Lett. **109**, 160401 (2012).
[29] T. Iadecola and T. H. Hsieh, Phys. Rev. Lett. **120**, 210603 (2018).
[30] V. Khemani, A. Lazarides, R. Moessner, and S. L. Sondhi, Phys. Rev. Lett. **116**, 250401 (2016).
[31] C. W. von Keyserlingk, V. Khemani, and S. L. Sondhi, Phys. Rev. B **94**, 085112 (2016).
[32] If only the stroboscopic dynamics is focused on, 2T-DTCs by spatial translation seem equivalent to the conventional DTC in spin systems with spin flip [23]. However, there is an essential difference in that STI-DTCs induce nontrivial currents originating from nonlocality of spatial translation.
[33] D. J. Thouless, Phys. Rev. B **27**, 6083 (1983).
[34] S. Nakajima, T. Tomita, S. Taie, T. Ichinose, H. Ozawa, I. Wang, M. Troyer, and Y. Takahashi, Nat. Phys. **12**, 296 (2016).
[35] M. Lohse, C. Schweizer, O. Zilberberg, M. Aidelsburger, and I. Bloch, Nat. Phys. **12**, 350 (2016).
[36] C. Schweizer, M. Lohse, R. Citro, and I. Bloch, Phys. Rev. Lett. **117**, 170405 (2016).
[37] M. Nakagawa and S. Furukawa, Phys. Rev. B **95**, 165116 (2017).
[38] T.-S. Zeng, C. Wang, and H. Zhai, Phys. Rev. Lett. **115**, 095302 (2015).
[39] T.-S. Zeng, W. Zhu, and D. N. Sheng, Phys. Rev. B **94**, 235139 (2016).

[40] L. Taddia, E. Cornfeld, D. Rossini, L. Mazza, E. Sela, and R. Fazio, Phys. Rev. Lett. **118**, 230402 (2017).
[41] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.121.093001 for the detailed derivations, the further numerical results, and the detailed descriptions of realization in quantum circuits.
[42] B. Huang, Y.-H. Wu, and W. V. Liu, Phys. Rev. Lett. **120**, 110603 (2018).
[43] In order to apply the theorem, $U_{\mathrm{int}}T \ll 1/L$ ($L$ is the number of the sites in each sublattice) is at least required. However, driving with the frequency proportional to $L$ is difficult, or in the thermodynamic limit, it is impossible. Moreover, even in the finite systems, the case when $U_{\mathrm{int}}T \simeq 1$ is considered, thus the theorem is not applicable.
[44] There are four ground states of the symmetry-breaking Hamiltonian $H_{\mathrm{SSB}}$, but we can choose any of them as the initial state for observing TTSB due to Eq. (8).
[45] H. Bernien, S. Schwartz, A. Keesling, H. Levine, A. Omran, H. Pichler, S. Choi, A. S. Zibrov, M. Endres, M. Greiner, V. Vuleti, and M. D. Lukin, Nature (London) **551**, 579 (2017).
[46] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information* (Cambridge University Press, Cambridge, England, 2010).
[47] IBM, "Quantum experience," URL: https://www.research.ibm.com/ibm-q/.
[48] I. M. Georgescu, S. Ashhab, and F. Nori, Rev. Mod. Phys. **86**, 153 (2014).
[49] G. Wendin, Rep. Prog. Phys. **80**, 106001 (2017).
[50] T. Hensgens, T. Fujita, L. Janssen, X. Li, C. J. V. Diepen, C. Reichl, W. Wegscheider, S. D. Sarma, and L. M. K. Vandersypen, Nature (London) **548**, 70 (2017).
[51] F. Machado, G. D. Meyer, D. V. Else, C. Nayak, and N. Y. Yao, arXiv:1708.01620.