Topological Defects in Anisotropic Driven Open Systems

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We study the dynamics and unbinding transition of vortices in the compact anisotropic Kardar-Parisi-Zhang equation. The combination of nonequilibrium conditions and strong spatial anisotropy drastically affects the structure of vortices and amplifies their mutual binding forces, thus stabilizing the ordered phase. We find novel universal critical behavior in the vortex-unbinding crossover in finite-size systems. These results are relevant for a wide variety of physical systems, ranging from strongly coupled light-matter quantum systems to dissipative time crystals.

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Introduction.—The celebrated theory of Kosterlitz and Thouless (KT) highlights the crucial role played by topological defects in the phase transition of U(1)-symmetric and short-range interacting two-dimensional (2D) systems in thermal equilibrium. At low temperatures, topological defects (vortices) of opposite charge form tightly bound pairs, while they are free to roam and destroy order at high temperatures. For the stability of the ordered phase, it is crucial that vortices interact like charged particles, i.e., with a Coulomb force decaying as $\sim 1/r$. In particular, faster decay at large distances would destabilize the ordered phase.

Interestingly, such a qualitative change of the vortex interaction is induced by driving the system out of thermal equilibrium. This has been studied extensively in the context of the complex Ginzburg-Landau equation (CGLE) [1], which reduces to the compact KPZ (cKPZ) equation [2,3] in the long-wavelength limit [4,5]. The extent to which these equations violate equilibrium conditions is quantified by a single parameter that determines the strength of the nonlinearity in the cKPZ equation [6–10]. Because of this nonlinearity, the vortex interaction is exponentially screened at large distances—thus, the ordered phase ceases to exist. This finding is particularly relevant, since the cKPZ equation is the long-wavelength description of a vast variety of systems, ranging from "polar active smectics" [11] to driven-dissipative condensates such as exciton polaritons [3,8–10,12–19], synchronization in oscillator arrays [20], and limit-cycle phases that emerge from a Hopf bifurcation [21–25]—such phases have attracted a lot of attention recently and could be coined dissipative time crystals [26].

In this Letter, we report that breaking *rotational* symmetry has an equally strong impact on the form of the vortex interaction and acts to *stabilize* the ordered phase. This is highly significant for the systems mentioned above, in which spatial anisotropy is either intrinsic or can be imposed deliberately. The change in the vortex interaction can be understood intuitively by considering the mere structure of a single vortex shown in Fig. 1. In the *isotropic* cKPZ equation, vortices are "radiative"; i.e., they emit waves radially away from the core, giving them a spiral

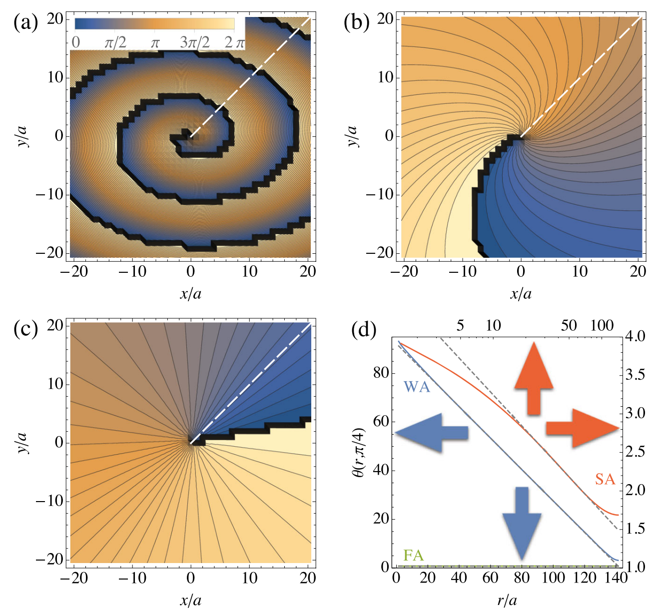

FIG. 1. Single vortices in the caKPZ equation. (a) WA regime with $\alpha_x = \lambda_x/(2D) \approx 0.9$ and $\alpha_y = \lambda_y/(2D) \approx 0.4$ in Eq. (2). The vortex has a squeezed spiral structure with a clearly visible radially emitted wave. (b) SA, $\alpha_x \approx 0.9$, $\alpha_y \approx -0.4$. The spiral structure is pronounced only at short distances from the vortex core. (c) In the FA case, with $\alpha_x = -\alpha_y \approx 0.7$, there is no radial wave. We note that (a)–(c) are equally nonlinear in the sense that $\alpha_x^2 + \alpha_y^2 = 1$. (d) Radial dependence of the vortex field $\theta(r, \phi)$ along the dashed line in (a)–(c). $\theta(r, \pi/4)$ grows linearly in the WA regime (a), logarithmically in the SA regime (b) (note the logarithmic r axis), and is constant at the FA point. The dashed lines are linear fits which agree well with the data up to finite-size effects at large distances.

structure [Fig. 1(a)]. Perturbations, e.g., due to the presence of another vortex, decay exponentially in the upstream direction of traveling waves—heuristically, this explains why the interaction between vortices is exponentially screened [2]. As we show below, the radially emitted wave decays away from the vortex core for sufficiently strong anisotropy [Fig. 1(b)] and is completely absent in a *fully anisotropic* configuration [Fig. 1(c)]. Then, vortices in the *anisotropic* cKPZ (caKPZ) are similar to those in equilibrium systems. We expect their interactions to be long range and, therefore, order to be stable, as also indicated by numerical simulations [4,5].

However, in a nonlinear theory such as the caKPZ equation, single-vortex solutions cannot be superposed to yield multivortex solutions. We present an analytical calculation of the interaction between defects, based on a recently developed mapping to a dual electrodynamics problem [3,15] and treating the nonlinearity perturbatively. This calculation is valid up to an exponentially large characteristic scale, below which the attraction between oppositely charged vortices is even enhanced as compared to the linear (i.e., thermal equilibrium) isotropic case. For this enhancement to occur, the combination of nonlinearity and anisotropy is essential: As explained above, the nonlinearity alone entails a repulsive correction to the interaction, while anisotropy in a linear theory does not affect the interaction qualitatively.

Based on the modified vortex interaction, we derive renormalization group (RG) equations describing the vortex-unbinding crossover in systems that are fully anisotropic and smaller than the characteristic scale. Since this scale is parametrically large, we expect that the universal critical behavior we find will be observed in experiments and numerical investigations of the caKPZ equation. In particular, the divergence of the correlation length is in between the essential singularity characteristic of the KT transition and true scaling behavior as in usual continuous phase transitions.

Previous studies of the caKPZ equation [8,9,11,18] assumed that vortices do not proliferate on the characteristic scale of the RG flow of the noncompact equation [11,27] and can hence be included *a posteriori* in an emergent equilibrium description. This assumption, however, breaks down close to the unbinding transition, where vortices are the dominant fluctuations. To access this region, we focus on the combined impact of nonlinearity and strong anisotropy on the vortex dynamics.

Model.—The caKPZ equation reads

$$\partial_t\theta = \sum_{i=x,y}\left(D_i\partial_i^2\theta + \frac{\lambda_i}{2}(\partial_i\theta)^2\right) + \eta, \qquad (1)$$

where η is Gaussian noise with zero mean and correlations $\langle\eta(\mathbf{r},t)\eta(\mathbf{r}',t')\rangle = 2\Delta\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')$. θ is a compact variable, i.e., one that admits topological defects. In physical realizations, θ may be the phase field in driven open condensates [3,8–10,12–18], in limit-cycle phases [21–25], or in oscillator arrays. In the latter case, Eq. (1) emerges as the continuum limit of the noisy Kuramoto-Sakaguchi model [20,28,29] with anisotropic couplings between the oscillators. θ may also represent the displacement field in polar active smectics [11].

For stability, we require $D_{x,y} > 0$, while $\lambda_{x,y}$ are unrestricted; in the following, we set $D_x = D_y = D$, which can always be achieved by an anisotropic rescaling of the units of length. For $\lambda_x = \lambda_y = 0$, Eq. (1) reduces to the (continuum limit of the) XY model with dissipative dynamics [30]. We denote $\lambda_{x,y}$ having the same and opposite signs as *weakly anisotropic* (WA) and *strongly anisotropic* (SA) regimes, respectively. In particular, we denote the configuration with $\lambda_x = -\lambda_y$ as *fully anisotropic* (FA). We shall restrict ourselves to small values of $|\lambda_{x,y}|$ in order to avoid a dynamical instability of Eq. (1) [16,20]. In systems described by Eq. (1) with $\lambda_x = \lambda_y$, the ordered phase is always destroyed in the thermodynamic limit by the proliferation of vortices [2,3,31]. Here, we investigate whether order can be stable if $\lambda_x \neq \lambda_y$.

Structure of a single vortex.—A vortex is a solution of Eq. (1) without noise that is stationary up to uniform oscillations with a frequency ω_0,

$$\partial_t\theta = D\nabla^2\theta + \frac{\lambda_x}{2}(\partial_x\theta)^2 + \frac{\lambda_y}{2}(\partial_y\theta)^2 - \omega_0 = 0, \qquad (2)$$

and obeys $\oint d\mathbf{l}\cdot\nabla\theta = 2\pi$ for any integration path that surrounds the vortex core. We solve Eq. (2) numerically by discretizing it on a lattice (see the Supplemental Material [32] for details). In addition, we determine analytically the asymptotic behavior of $\theta(r,\phi)$ for $r\to\infty$ at fixed polar angle ϕ, which takes the form

$$\theta(r,\phi) = k_0(\phi)r + b(\phi)\ln(r/a) + \Phi(\phi) + O(1/r), \qquad (3)$$

where $k_0(\phi)$ is the (anisotropic) asymptotic wave number, $b(\phi)$ the coefficient of the first subleading correction, and $\Phi(\phi)$ contains the topological part of the vortex field. a is a microscopic cutoff scale such as the lattice spacing in oscillator arrays or the healing length in driven open condensates. As illustrated in Figs. 1(a) and 1(d), in the WA regime, we find a squeezed spiral structure with [32]

$$k_0(\phi) = -\sqrt{\frac{2\omega_0}{\lambda_x\cos(\phi)^2 + \lambda_y\sin(\phi)^2}}, \qquad (4)$$

where ω_0 is determined by the regularization at short distances [2]. In the SA regime shown in Figs. 1(b) and 1(d), when one of $\lambda_{x,y}$ is negative, Eq. (4) implies $k_0(\phi) = \omega_0 = 0$. The leading asymptotic behavior is then given by the logarithmic term in Eq. (3) with

$b(\phi) = b_0 = \text{const}$. Finally, for FA parameters $\lambda_x = -\lambda_y$, also the coefficient b_0 vanishes. Indeed, then the exact solution takes the form $\theta(r,\phi) = \Phi_0(\phi)$ [32], and the absence of any radial dependence is evident in Figs. 1(c) and 1(d). For $\lambda_x = -\lambda_y \to 0$, this solution smoothly deforms into an "ordinary" XY vortex with $\Phi_0(\phi) = \phi$, which is in stark contrast to the isotropic case, where the transition from the linear to the nonlinear problem is highly nonanalytic. Since turning on the nonlinearity in a fully anisotropic system does not alter the *radial dependence* of the far field of a single vortex, we conclude that the interaction of vortices at large distances is not screened as in the isotropic case, and thus the ordered phase is indeed stable in the thermodynamic limit. It is an interesting open question how the logarithmic dependence of the vortex field (3) in the SA regime affects the interaction at asymptotic distances.

Electrodynamic duality and vortex interaction.—The vortex interaction can be calculated explicitly within a dual electrodynamic formalism [3,15]. This calculation treats the nonlinearity in Eq. (1) perturbatively and is valid up to a finite but exponentially large scale we determine below. The duality defines the electric field as $\mathbf{E} = -\hat{\mathbf{z}} \times \nabla\theta$. For the overdamped dynamics described by Eq. (1), fluctuations of the magnetic field are gapped and can be integrated out. The basic equations are then Gauss' law $\nabla \cdot \mathbf{E} = 2\pi n/\varepsilon$ and

$$\varepsilon\partial_t \mathbf{E} = -D\nabla \times (\nabla \times \mathbf{E}) - 2\pi \mathbf{j} - \hat{\mathbf{z}} \times \nabla\left(\sum_{i=x,y} \frac{\lambda_i}{2} E_i^2 + \eta\right). \tag{5}$$

The dielectric constant ε accounts for screening of the electric field due to fluctuations consisting of bound vortex pairs. It takes the microscopic value $\varepsilon = 1$ and is renormalized upon coarse graining as described below. n and $\mathbf{j}$ are the vortex density and current, respectively, which obey the continuity equation $\partial_t n = -\nabla \cdot \mathbf{j}$. For $n = \mathbf{j} = 0$, Eq. (5) reduces to the noncompact anisotropic KPZ equation. The vortex density is controlled by the fugacity y. Close to the putative unbinding transition $y \ll 1$, and we restrict ourselves to consider a dipole $n(\mathbf{r}) = \sum_{\sigma=\pm} \sigma\delta(\mathbf{r} - \mathbf{r}_\sigma)$, where $\sigma = \pm$ are the charges of the vortices. They are assumed to undergo diffusive motion [31,36,37] according to

$$\frac{d\mathbf{r}_\sigma}{dt} = \mu\sigma\mathbf{E}(\mathbf{r}_\sigma) + \boldsymbol{\xi}_\sigma, \tag{6}$$

and the correlations of the zero-mean Gaussian noise sources $\boldsymbol{\xi}_\sigma$ are given by $\langle \xi_{\sigma,i}(t)\xi_{\sigma',j}(t')\rangle = 2\mu T\delta_{\sigma\sigma'}\delta_{ij}\delta(t - t')$, where the vortex "temperature" T is related to the noise strength Δ in Eq. (1) [3]. The vortex mobility μ is introduced phenomenologically, and we consider the limit of low mobility $\mu \ll D$. Then, retardation effects due to the vortices' motion are negligible, and $\mathbf{E}(\mathbf{r}_\sigma)$ in Eq. (6) can be approximated by the instantaneous electrostatic field, which is determined by Eq. (5) with $\partial_t \mathbf{E} = \mathbf{j} = \eta = 0$ [3]. Details of this calculation are given in the Supplemental Material [32], and here we only point out key features of the solution. To address the possibility of a bound state, we focus on the dynamics of the dipole moment $\mathbf{r} = \sum_{\sigma=\pm} \sigma\mathbf{r}_\sigma = \mathbf{r}_+ - \mathbf{r}_-$. In the linear (thermal equilibrium) theory, $\mathbf{r}$ is subject to the Coloumb force $\sim \mathbf{r}/r^2$. We parametrize the nonlinearity as $\alpha_\pm = (\lambda_x \pm \lambda_y)/(2D)$, with $\alpha_- = 0$ and $\alpha_+ = 0$ corresponding to isotropic and fully anisotropic systems, respectively. The Coloumb force receives corrections at second order in $\alpha_\pm$. The "isotropic" second-order correction $\propto \alpha_+^2$ was obtained previously [3]. It is a central, conservative, and, crucially, *repulsive* force. In the "anisotropic" second-order correction $\propto \alpha_-^2$, the leading contribution is also central and conservative, but *attractive*. Additionally, it features subleading terms that cannot be derived from a potential. The "mixed" correction $\propto \alpha_+\alpha_-$ includes terms $\propto (x, -y)$ that favor alignment of the dipole along one of the principal axes, in line with numerical simulations of the anisotropic CGLE [38,39].

The isotropic, anisotropic, and mixed corrections are power series in logarithms $\ln(r/a)$. Thus, perturbation theory breaks down at a scale $L_v \sim a e^{1/\alpha_{\max}}$ where $\alpha_{\max} = \max\{|\alpha_\pm|\}$. In weakly out-of-equilibrium (and thus weakly nonlinear) systems, the scale L_v can easily be much larger than any experimentally relevant system size. Then, we expect the dynamics of vortices to be described by the perturbatively obtained interaction. In the following, we discuss how the usual KT theory is modified due to nonequilibrium conditions and anisotropy. We focus on the FA configuration, where anisotropy has the most profound impact. Moreover, and as discussed in detail in the Supplemental Material [32], a vast simplification occurs in the FA case on scales $r \ll L_T = a e^{(T/\alpha_{\max})^{1/3}}$: Then, fluctuations of the orientation of the dipole lead to an angular averaging, rendering the problem effectively isotropic. Strong anisotropy is nevertheless manifest in the result of the angular average.

Vortex unbinding crossover.—The noise in Eqs. (5) and (6) creates pairs of vortices and antivortices which then diffuse under the influence of their interaction and eventually recombine. Such fluctuations on short scales between the microscopic cutoff a and a running cutoff scale $a e^\ell$ renormalize the parameters that enter an effective description on larger scales. This is described by the following RG flow equations [32]:

$$\frac{d\varepsilon}{d\ell} = \frac{2\pi^2 y^2}{T}, \qquad \frac{dy}{d\ell} = \frac{1}{2}\left(4 - \frac{1}{\varepsilon T} + \frac{c\alpha_-^2}{3\varepsilon^2}\right)y,$$
$$\frac{dT}{d\ell} = \frac{c\alpha_-^2 T}{3\varepsilon^2}, \qquad \frac{dc}{d\ell} = -3, \tag{7}$$

where c is the running coefficient of the term $\propto \ln(r/a)^2$ in the effective dipole distribution with microscopic value

$c = 3/2$ (see the Supplemental Material [32]; recall that also the microscopic value $\varepsilon = 1$ is fixed). Integrating the flow equation for c yields $c = 3(1 - 2\ell)/2$; i.e., the logarithmic scale ℓ appears explicitly in the flow equations for the remaining couplings. This again necessarily invalidates the perturbative flow equations at large scales—however, the condition $r \ll L_v$, which we assumed in the derivation of the flow equations, is always more stringent. Note also that the characteristic KPZ scale on which the renormalization of the (suitably rescaled) nonlinearity in Eq. (1) due to nontopological fluctuations becomes substantial, is generically much larger than L_v, L_T [3]. Hence, analyzing Eq. (7) we can consider α_- as a fixed parameter.

The RG flow is shown in Fig. 2. Remarkably, it is qualitatively different from both the equilibrium KT flow and the RG flow in an isotropic nonequilibrium system. The most striking feature is the existence of a low-temperature phase in which vortices remain bound and fluctuations are anomalously suppressed since both y, $T \to 0$. In contrast, in isotropic systems vortices unbind at any finite temperature [3]; the low-temperature ordered phase in thermal equilibrium, on the other hand, is different in that T is conserved by the RG flow. The strong suppression of vortex fluctuations can be traced back to the dominant correction to the vortex interaction being *attractive* in the FA case. Consequently, the fundamental difference to the flow equations for isotropic systems in Ref. [3] is that here c flows to *negative* values, and therefore the terms $\propto c$ in the equations for T and y renormalize these quantities to *lower* values, thus antagonizing the unbinding of vortices. This leads to *increased* stability of the ordered phase as compared to the equilibrium case: The critical

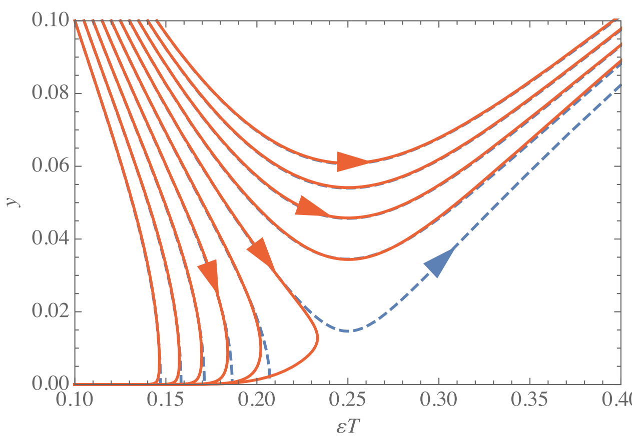

FIG. 2. RG flow (7). Dashed blue: KT flow for $\alpha_-^2 = 0$; solid red: $\alpha_-^2 = 0.01$. There are two phases with y, $\varepsilon T \to 0$ ($\varepsilon T \to$ const for $\alpha_-^2 = 0$) and y, $\varepsilon T \to \infty$, respectively. For $\alpha_-^2 = 0.01$, the critical temperature is $T_c \approx 0.13$, which is slightly larger than the KT critical temperature. The microscopic value of the fugacity is chosen as $y = 0.1$, and the temperature is varied in the range $T = 0.1, \ldots, 0.145$.

temperature T_c is higher for the same value of y. Heuristically, the lower the probability for vortex pairs to be created at a microscopic scale, the stronger noise-induced fluctuations the system can afford and still remain ordered. This is true also in equilibrium, but here we found that for $y \to 0$ the critical temperature *diverges* whereas it remains finite in KT theory.

While at low temperatures the flow $T \to 0$ is presumably cut at large scales when the flow equations (7) become invalid, at high temperatures the rapid growth of ε, indicating the screening of vortex interactions, stops the flow of T. Then, at larger scales, the flow in the disordered phase is the same as in equilibrium [32] (in particular, y, $\varepsilon \to \infty$).

The existence of two distinct phases points to the existence of a fixed point that controls critical behavior at the transition. Even if the "true" critical behavior at the largest scales is not captured by the flow equations (7), they still entail the finite-size criticality that is observable up to parametrically large scales. However, in contrast to usual continuous phase transitions, the flow equations (7) cannot have a true fixed point since c grows steadily. A "flowing fixed point" can be found by the change of variables $\tilde{\varepsilon} = \varepsilon/x$, $\tilde{y} = \sqrt{x}y$, and $\tilde{T} = xT$, where $x = -c$ (note that $c < 0$ in the regime of interest at large ℓ), which recasts the flow equations as

$$\frac{d\tilde{\varepsilon}}{dx} = \frac{1}{x}\left(\frac{2\pi^2\tilde{y}^2}{3\tilde{T}} - \tilde{\varepsilon}\right), \qquad \frac{d\tilde{T}}{dx} = \frac{1}{3x}\left(3 - \frac{\alpha_-^2}{3\tilde{\varepsilon}^2}\right)\tilde{T},$$
$$\frac{d\tilde{y}}{dx} = \frac{1}{6}\left[4 - \frac{1}{\tilde{\varepsilon}\tilde{T}} + \frac{1}{x}\left(3 - \frac{\alpha_-^2}{3\tilde{\varepsilon}^2}\right)\right]\tilde{y}. \tag{8}$$

These equations have a fixed point at $\tilde{\varepsilon}_* = |\alpha_-|/3$, $\tilde{y}_* = \sqrt{3/8}(1/\pi)$, $\tilde{T}_* = 3/(4|\alpha_-|)$. The existence of this fixed point implies that the correlation length diverges at the transition: As T is tuned closer to its critical value, the RG flow stays close to the fixed point up to larger scales before it eventually runs off to the ordered or disordered phase. In the disordered high-temperature phase, the correlation length is given by the scale at which the flow trajectory departing from the vicinity of the fixed point reaches $y = 1$. It can be calculated from an asymptotic analysis of the linearized flow equations, which yields [32]

$$4\sqrt{\ln(\xi/a)} + \ln[\ln(\xi/a)]/4 \sim -\ln(t), \tag{9}$$

where $t = (T - T_c)/T_c$ is the reduced temperature. This peculiar universal divergence of ξ is stronger than conventional scaling $\xi/a \sim t^{-\nu}$ but weaker than the essential singularity $\xi/a \sim e^{C/\sqrt{t}}$ at the equilibrium KT transition. Experimentally or numerically, it will be challenging to confirm the precise type of singularity (9)—especially, since the asymptotic seems to be approached only for T very close to T_c when ξ becomes extremely large [32]. A more easily accessible feature that distinguishes the transition is the absence of the characteristic jump of the superfluid stiffness $\sim 1/\varepsilon$ with the universal value $1/(\varepsilon T) \to 4$ for T approaching T_c from below in the KT transition. Here, in contrast, the renormalized value of ε diverges at T_c, leading to a smoothly vanishing superfluid stiffness at the transition.

Conclusions.—We showed that strong spatial anisotropy stabilizes the bound phase of vortices in 2D driven open systems and gives rise to novel critical behavior. Recent progress in stabilizing condensates of exciton polaritons [40] (in which strong anisotropy is achievable [18]) paves the way towards testing our predictions experimentally. Moreover, the modification of the vortex interaction is readily amenable to numerical investigations [41] and should have directly observable effects on phase-ordering kinetics [42–45]. It remains to be seen whether strong anisotropy can also have a healing effect on the dynamical instability reported in Refs. [16,20].

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