Frozen Condition of Quantum Coherence for Atoms on a Stationary Trajectory

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The quantum coherence (QC) of two comoving atoms on a stationary trajectory is investigated. We develop a formalism to characterize the properties of atoms on a stationary trajectory. We give a criterion under which QC is frozen to a nonzero value. The frozen condition that vanishing super- or subradiant decay rate is not so sensitive to the initial condition of state. We show that enhanced QC and a subradiant state can be gained from the initial state.

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Introduction.—QC as a consequence of the quantum state superposition principle is one of the key features that results in nonclassical phenomena. It is a powerful resource in quantum information theory as well as entanglement and discord-type quantum correlation. Though it is important in fundamental physics, only recently have relevant steps been attempted to develop a rigorous framework to quantify QC for general states; such as the relative entropy of QC and the l_1 norm [1].

A long-standing and significant issue concerning QC is the decoherence induced by the inevitable interaction between the system and environment. In the past few years, several proposals have been suggested for fighting against the deterioration of QC, for instance, decoherence-free subspaces [2,3], quantum error correction codes [4], dynamical decoupling [5], or quantum Zeno dynamics [6,7]. Recently, the conditions of sustaining long-lived QC were investigated [8]. It was shown that QC can remain unchanged with time (freezing coherence) and the frozen condition (FC) for two qubits undergoing local identical bit-flip channels with Bell-diagonal initial states was only dependent on the initial condition of the states [9].

In a realistic physical system, atoms usually cannot be handled simply as noninteracting individual qubits when atomic spacing is small. Besides, for an ensemble of atoms, motion and temperature are also important factors which should be taken into account. Then, searching for a general FC for interacting atoms under normal conditions is necessary in practice.

In this Letter, we investigate the FC of QC for two identical two-level atoms on a stationary trajectory which has a characterization that the geodesic distance between two points on the trajectory depends only on the proper time interval [10,11]. Thus, for a stationary trajectory, field correlation function is invariance under translations in time. Besides, the stationary trajectory guarantees that the atoms have stationary states. The inertial atom in a Minkowski vacuum or thermal bath and a uniformly or circularly accelerated atom viewed by an instantaneous inertial observer are all stationary. We find that the QC for interacting atoms on a stationary trajectory can also be long lived; however, the FC is not so sensitive to the initial condition of the single excitation state but to the super- or subradiant decay rate of atoms. We develop a formalism to describe atoms on a stationary trajectory and give the general relationship between the quantities characterizing properties of atoms. Besides, we show that enhanced QC and the subradiant state can be obtained from the initial state.

Formalism.—We consider two identical two-level atoms moving on stationary trajectories $x_j(\tau) = (t(\tau), \vec{x}_j(\tau))$ in a fluctuating vacuum electromagnetic field, where $(t, \vec{x}_j)$ are the Minkowski coordinates of atom j referring to an inertial reference frame and τ denotes proper time of these two comoving atoms (see Fig. 1). The total Hamiltonian of the coupled system can be described by $H = H_s + H_f + H_I$. Here, H_s is the free Hamiltonian of atoms, and its explicit expression in the Schrödinger picture is $H_s = \sum_{j=1}^{2} \omega_0 \sigma_j^+ \sigma_j^-$, where ω_0 is the level spacing of the two-level atoms, $\sigma_j^+ = |e_j\rangle\langle g_j|$ and $\sigma_j^- = |g_j\rangle\langle e_j|$ are, respectively, the raising and lowering operators of the atom j. The free Hamiltonian H_f with respect to τ takes the form $H_f = \sum_{k\lambda} \omega_{\vec{k}\lambda} a_{\vec{k}\lambda}^\dagger a_{\vec{k}\lambda} (dt/d\tau)$ [12]. Here $a_{\vec{k}\lambda}^\dagger$, $a_{\vec{k}\lambda}$ are the creation and annihilation operators for a photon with momentum $\vec{k}$, frequency $\omega_{\vec{k}}$, and polarization λ. The Hamiltonian H_I that

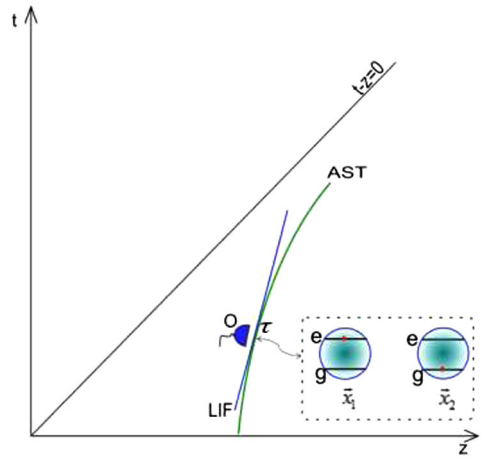

FIG. 1. Schematic illustration for two identical two-level atoms comoving on an arbitrary stationary trajectory (AST). The observer (O) located in the local inertial frame (LIF) of AST is instantaneous static with respect to the atoms. The coordinate of atoms $(t,\vec{x}_j)$ is characterized by proper time τ as $x_j(\tau)=(t(\tau),\vec{x}_j(\tau))$. The observer can also be in a laboratory reference frame, and the difference is that the total Hamiltonian we have used should be multiplied by $d\tau/dt$.

describes the atom-field interaction can be written in an electric dipole approximation in τ as $-e\sum_{j=1}^{2}\vec{r}_j\cdot\vec{E}[\vec{x}_j(\tau)]=-e\sum_{j=1}^{2}(\vec{d}_j\sigma_j^{+}+\vec{d}_j^{*}\sigma_j^{-})\cdot\vec{E}[\vec{x}_j(\tau)]$, where e is the electron electric charge, $e\vec{r}_j$ is the electric dipole moment for atom j, $\vec{d}_j=\langle e_j|\vec{r}_j|g_j\rangle$, and $\vec{E}[\vec{x}_j(\tau)]$ is the electric field strength evaluated at the position $\vec{x}_j(\tau)$. The Hamiltonian in the Schrödinger picture can be changed to the interaction picture via a unitary transformation with a unitary operator $U_0(\tau)=\exp[-i\sum_{j=1}^{2}\omega_0\sigma_j^{+}\sigma_j^{-}\tau-i\sum_{k\lambda}\omega_{\vec{k}\lambda}a_{\vec{k}\lambda}^{\dagger}a_{\vec{k}\lambda}t(\tau)]$, which is the solution to the Schrödinger equation in τ: $i(d/d\tau)U_0(\tau)=(H_s+H_f)U_0(\tau)$. Then, the atom-field interaction Hamiltonian in the interaction picture can be written as

$$H_I(\tau)=-e\sum_{j=1}^{2}(\vec{d}\sigma_j^{+}e^{i\omega_0\tau}+\vec{d}^{*}\sigma_j^{-}e^{-i\omega_0\tau})\cdot\vec{E}[x_j(\tau)],\quad(1)$$

where $\vec{E}[x_j(\tau)]=U_0^{\dagger}(\tau)\vec{E}[\vec{x}_j(\tau)]U_0(\tau)$ and we have let $\vec{d}_1=\vec{d}_2=\vec{d}$ for simplicity.

Here, we consider the polarizations of the coupling photon required by the two atoms only in the same direction, and for the convenience of calculations, we assume (i)

$$G_{ii}^{11}(u)=G_{ii}^{22}(u),\qquad G_{ii}^{12}(u)=G_{ii}^{21}(u),\qquad(2)$$

where $G_{ij}^{ab}(\tau-\tau')=\langle 0|E_i^{+}[x_a(\tau)]E_j^{-}[x_b(\tau')]|0\rangle$ is the electric field correlation function, and we have decomposed $\vec{E}[x_j(\tau)]$ in $H_I(\tau)$ into positive- and negative-frequency parts: $\vec{E}[x_j(\tau)]=\vec{E}^{+}[x_j(\tau)]+\vec{E}^{-}[x_j(\tau)]$ with $\vec{E}^{+}[x_j(\tau)]|0\rangle=0$ and $\langle 0|\vec{E}^{-}[x_j(\tau)]=0$. Under such an assumption, the correlation function is invariant under the exchange of the two atoms. Thus, there is no difference in the atom-field interaction between the atoms, and the external environment for them is the same. We further assume (ii) that the interaction between atoms and the field to be weak, so the Wigner-Weisskopf approximation can be adopted.

Consider the atoms with an initial single excitation state and field as the vacuum state $|\varphi(0)\rangle=\cos(\theta/2)|e_1g_2\rangle|0\rangle+\sin(\theta/2)|g_1e_2\rangle|0\rangle$. Then, at time τ, the general form of the state vector can be written as [13,14]

$$|\varphi(\tau)\rangle=\sum_{k\lambda}b_{\vec{k}\lambda1}(\tau)|g_1g_2\rangle|1_{\vec{k}\lambda}\rangle+\sum_{k\lambda}b_{\vec{k}\lambda2}(\tau)|e_1e_2\rangle|1_{\vec{k}\lambda}\rangle+b_1(\tau)|e_1g_2\rangle|0\rangle+b_2(\tau)|g_1e_2\rangle|0\rangle,\quad(3)$$

where $|1_{\vec{k}\lambda}\rangle$ denotes one photon in the mode $(\vec{k},\lambda)$. It is worth noting that this state is observed in the local inertial reference frame of atoms.

The state probability amplitudes in (3) can be obtained (see the Supplemental Material [15]):

$$b_1(\tau)=\frac{1}{2}\left[\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)C_{+}(\tau)+\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)C_{-}(\tau)\right],$$
$$b_2(\tau)=\frac{1}{2}\left[\left(\cos\frac{\theta}{2}+\sin\frac{\theta}{2}\right)C_{+}(\tau)-\left(\cos\frac{\theta}{2}-\sin\frac{\theta}{2}\right)C_{-}(\tau)\right],\quad(4)$$

where $C_{\pm}(\tau)=e^{-(A^{11}(0)+B^{11}(0)\pm A^{12}(0)\pm B^{12}(0))\tau}$. Here, $A^{ab}(0)$ and $B^{ab}(0)$ can be decomposed as follows:

$$A^{ab}(0)=\frac{1}{2}e^2d_id_i^{*}\mathcal{G}_{ii}^{ab}(\omega_0)+ie^2d_id_i^{*}\mathcal{K}_{ii}^{ab}(\omega_0),$$
$$B^{ab}(0)=\frac{1}{2}e^2d_i^{*}d_i\mathcal{G}_{ii}^{ab}(-\omega_0)+ie^2d_i^{*}d_i\mathcal{K}_{ii}^{ab}(-\omega_0)\quad(5)$$

with

$$\mathcal{G}_{ii}^{ab}(\pm\omega_0)=\int_{-\infty}^{\infty}du e^{\pm i\omega_0u}G_{ii}^{ab}(u),$$
$$\mathcal{K}_{ii}^{ab}(\pm\omega_0)=-\frac{P}{2\pi}\int_{-\infty}^{\infty}\frac{\mathcal{G}_{ii}^{ab}(\omega)}{\omega\mp\omega_0}d\omega.\quad(6)$$

Here, P denotes the Cauchy principal value. It is worthwhile to note that $e^2d_id_i^{*}\mathcal{G}_{ii}^{11}(\omega_0)$, $e^2d_i^{*}d_i\mathcal{G}_{ii}^{11}(-\omega_0)$ are the spontaneous emission rate $\Gamma_{\downarrow}^{11}$ and spontaneous excitation rate $\Gamma_{\uparrow}^{11}$ of atom 1, respectively [16]. Also, $e^2d_id_i^{*}\mathcal{G}_{ii}^{12}(\omega_0)$ and $e^2d_i^{*}d_i\mathcal{G}_{ii}^{12}(-\omega_0)$ can be regarded as the modulation of the spontaneous emission rate $\Gamma_{\downarrow}^{12}$ [17] and spontaneous excitation rate $\Gamma_{\uparrow}^{12}$ of one atom due to the presence of another atom. Here, $\Gamma_{\downarrow}^{11}\pm\Gamma_{\downarrow}^{12}$ are actually the super- and subradiant spontaneous emission rates $\Gamma_{\downarrow\pm}$, respectively [18]; $\Gamma_{\uparrow}^{11}\pm\Gamma_{\uparrow}^{12}$

TABLE I. Relationship between the quantities characterizing atomic properties. Here, plus/minus denotes the sum or difference of the previous two terms and $D_{ii}^2 = e^2 d_i d_i^*$.

	Emission	Excitation	Plus
Spontaneous transition rates	$\Gamma_{\downarrow}^{11} = D_{ii}^2 \mathcal{G}_{ii}^{11}(\omega_0)$	$\Gamma_{\uparrow}^{11} = D_{ii}^2 \mathcal{G}_{ii}^{11}(-\omega_0)$	$\Gamma_{\downarrow}^{11} + \Gamma_{\uparrow}^{11} = \Gamma^{11}$
Corresponding modulations	$\Gamma_{\downarrow}^{12} = D_{ii}^2 \mathcal{G}_{ii}^{12}(\omega_0)$	$\Gamma_{\uparrow}^{12} = D_{ii}^2 \mathcal{G}_{ii}^{12}(-\omega_0)$	$\Gamma_{\downarrow}^{12} + \Gamma_{\uparrow}^{12} = \Gamma^{12}$
Super-/subradiant rates (plus/minus)	$\Gamma_{\downarrow\pm} = \Gamma_{\downarrow}^{11} \pm \Gamma_{\downarrow}^{12}$	$\Gamma_{\uparrow\pm} = \Gamma_{\uparrow}^{11} \pm \Gamma_{\uparrow}^{12}$	$\Gamma^{11} \pm \Gamma^{12} = \Gamma_{\downarrow\pm} + \Gamma_{\uparrow\pm}$

can be, respectively, termed as super- and subradiant spontaneous excitation rates $\Gamma_{\uparrow\pm}$. The clear definitions and the relations between the above quantities are listed in Table I. Here, $e^2 d_i d_i^* \mathcal{K}_{ii}^{aa}(\omega_0)$ and $e^2 d_i^* d_i \mathcal{K}_{ii}^{aa}(-\omega_0)$ represent the level shift of the upper state and lower state of atom a. Finally, $e^2 d_i d_i^* \mathcal{K}_{ii}^{12}(\omega_0) + e^2 d_i^* d_i \mathcal{K}_{ii}^{12}(-\omega_0)$ is the dipole-dipole interaction potential V, which results from photon exchanges between atoms.

Frozen condition.—Now we apply the previously developed formalism to investigate QC for two atoms on a stationary trajectory. For simplicity, we take l_1 norm [1] as a measure of QC, which is defined as $\mathcal{C}_{l_1}(\rho) = \sum_{i \neq j} |\rho_{i,j}|$. The reduced density matrix ρ obtained by tracing the density matrix of the total system over the field degrees of freedom can be written in the basis of the product states, $|1\rangle = |e_1 e_2\rangle$, $|2\rangle = |e_1 g_2\rangle$, $|3\rangle = |g_1 e_2\rangle$, $|4\rangle = |g_1 g_2\rangle$. Since the state $|e_1 e_2\rangle |1_{\vec{k}\lambda}\rangle$ is only an intermediate state and short lived, the off-diagonal elements ρ_{14} and ρ_{41} take effect only in short time. However, we are interested in the long time behavior of QC, so these cross terms can be neglected. Then, QC will be simply expressed as $\mathcal{C}_{l_1}(\rho) = 2|b_1(\tau) b_2^*(\tau)| = e^{-\Gamma^{11}\tau} \sqrt{\cos^2\theta \sin^2(2V\tau) + [\sin\theta \cosh(\Gamma^{12}\tau) - \sinh(\Gamma^{12}\tau)]^2}$, which depends only on ρ_{23}. Here, $\Gamma^{11} = \Gamma_{\downarrow}^{11} + \Gamma_{\uparrow}^{11}$ and $\Gamma^{12} = \Gamma_{\downarrow}^{12} + \Gamma_{\uparrow}^{12}$, see Table I.

For the initial subradiant state $(|e_1 g_2\rangle - |g_1 e_2\rangle)|0\rangle/\sqrt{2}$, QC will decay exponentially as $e^{-(\Gamma^{11} - \Gamma^{12})\tau}$. Then, it can be frozen to a maximum value 1 only in the case that $\Gamma^{11} - \Gamma^{12} = 0$. When the initial state is a separable state, that is, $|e_1 g_2\rangle|0\rangle$ or $|g_1 e_2\rangle|0\rangle$, QC will increase from zero and evolve as $e^{-\Gamma^{11}\tau} \sqrt{\sin^2(2V\tau) + \sinh^2(\Gamma^{12}\tau)}$. After evolving for a sufficiently long time $\tau \gg 1/\Gamma^{11}$, it is frozen to $1/2$ if $\Gamma^{11} - \Gamma^{12} = 0$. Generally, one can find that when τ is much larger than $1/\Gamma^{11}$ QC will evolve to a nonzero constant $(1 - \sin\theta)/2$ $(\theta \neq \pi/2)$ under the condition that $\Gamma^{11} - \Gamma^{12} = 0$. Such an FC can be rewritten as $\Gamma_{\downarrow -} + \Gamma_{\uparrow -} = 0$, which means that the sum of the subradiant spontaneous emission and excitation rate (namely, subradiant decay rate) should be zero. For inertial atoms in a Minkowski vacuum, there is no spontaneous excitation ($\Gamma_{\uparrow}^{11}$ and $\Gamma_{\uparrow}^{12}$ all vanish), and the FC will be simplified to $\Gamma_{\downarrow -} = 0$ such that the subradiant spontaneous emission rate is null. In the above, we only consider the case that the subradiant decay rate equals zero under which one can freeze QC except for the initial state with $\theta = \pi/2$. Because this state is a super-radiant state, the QC decays as $e^{-(\Gamma^{11} + \Gamma^{12})\tau}$ which can be frozen only when the super-radiant decay rate vanishes. In physical implementations, when the super-radiant decay rate vanishes, the subradiant decay rate will usually vanish too. In such a case, the QC for the arbitrary initial state will all be frozen.

Thermal bath.—How would the above results have been modified if the initial state of field were not a vacuum but a thermal bath described by the density matrix $\rho = e^{-\beta H_f}$ with β being the inverse temperature? Since the thermal equilibrium state is a stationary state, the above formalism can be easily generalized to this case. Following a similar procedure, it can be found that, for atoms at rest in a thermal bath, the correlation function in (6) should be replaced by the thermal Green function $G_{ii\beta}^{ab}(t - t')$ which is expressed as $\langle E_i^+[x_a(t)] E_i^-[x_b(t')]\rangle_\beta + \langle E_i^-[x_a(t)] E_i^+[x_b(t')]\rangle_\beta = \mathrm{Tr}(\rho E_i[x_a(t)] E_i(x_b[t'])])$. Then, Γ^{11} and Γ^{12} in an initial vacuum state case are replaced by $\Gamma_\beta^{11} = \Gamma_{\downarrow\beta}^{11} + \Gamma_{\uparrow\beta}^{11}$ and $\Gamma_\beta^{12} = \Gamma_{\downarrow\beta}^{12} + \Gamma_{\uparrow\beta}^{12}$, where $\Gamma_{\downarrow\beta}^{11} = e^2 d_i d_i^* \mathcal{G}_{ii\beta}^{11}(\omega_0)$ has the meaning of a total emission rate including the spontaneous emission and stimulated radiation rates, $\Gamma_{\uparrow\beta}^{11} = e^2 d_i^* d_i \mathcal{G}_{ii\beta}^{11}(-\omega_0)$ has the meaning of the absorption rate [16] and $\Gamma_{\downarrow\beta}^{12} = e^2 d_i d_i^* \mathcal{G}_{ii\beta}^{12}(\omega_0)$, $\Gamma_{\uparrow\beta}^{12} = e^2 d_i^* d_i \mathcal{G}_{ii\beta}^{12}(-\omega_0)$ can be regarded as the corresponding modulations.

Now, the FC of QC has to be changed to $\Gamma_\beta^{11} - \Gamma_\beta^{12}$ (or $\Gamma_\beta^{11} + \Gamma_\beta^{12}$) $= 0$. Next, we will simplify this condition. For the thermal Green function, it can be verified that $G_{ii\beta}^{ab}(t - t') = G_{ii\beta}^{ba}(t' - t - i\beta)$. Actually, this is Kubo-Martin-Schwinger (KMS) condition, and the thermal state is a KMS state. Taking Fourier transforms of the KMS condition gives

$$\Gamma_{\downarrow\beta}^{ab} = e^{\omega_0 \beta} \Gamma_{\uparrow\beta}^{ab}. \tag{7}$$

Here, we have utilized $G_{ii\beta}^{ab}(u) = G_{ii\beta}^{ba}(u)$ which is equivalent to the assumption (i). Because of the fact that the commutator of field is a c number, its expectation values should be independent of the field state:

$$\langle \{E_i[x_a(t)], E_i[x_b(t')]\}\rangle_\beta = \langle 0|\{E_i[x_a(t)], E_i[x_b(t')]\}|0\rangle. \tag{8}$$

TABLE II. Quantities characterizing the properties of atoms in thermal bath, where $n = 1/(e^{\omega_0\beta} - 1)$.

	Total emission	Absorption	Plus
Transition rates of atom 1	$\Gamma^{11}_{\downarrow\beta} = (1+n)\Gamma^{11}_{\downarrow}$	$\Gamma^{11}_{\uparrow\beta} = n\Gamma^{11}_{\downarrow}$	$\Gamma^{11}_{\beta} = (1+2n)\Gamma^{11}_{\downarrow}$
Corresponding modulations	$\Gamma^{12}_{\downarrow\beta} = (1+n)\Gamma^{12}_{\downarrow}$	$\Gamma^{12}_{\uparrow\beta} = n\Gamma^{12}_{\downarrow}$	$\Gamma^{12}_{\beta} = (1+2n)\Gamma^{12}_{\downarrow}$
Plus/minus	$(1+n)\Gamma_{\downarrow\pm}$	$n\Gamma_{\downarrow\pm}$	$\Gamma^{11}_{\beta} \pm \Gamma^{12}_{\beta} = (1+2n)\Gamma_{\downarrow\pm}$

Here $\{\}$ is used to denote commutator. The Fourier transforms of this equation leads to

$$\Gamma^{ab}_{\downarrow\beta} - \Gamma^{ab}_{\uparrow\beta} = \Gamma^{ab}_{\downarrow}. \tag{9}$$

Note that we have omitted the term $\Gamma^{ab}_{\uparrow}$ since there is no spontaneous excitation for static atoms in a Minkowski thermal bath. Equations (7) and (9) give

$$\Gamma^{ab}_{\downarrow\beta} = \frac{e^{\omega_0\beta}}{e^{\omega_0\beta}-1}\Gamma^{ab}_{\downarrow}, \qquad \Gamma^{ab}_{\uparrow\beta} = \frac{1}{e^{\omega_0\beta}-1}\Gamma^{ab}_{\downarrow}. \tag{10}$$

Thus, the FC can be simplified to $\Gamma_{\downarrow-}$ (or $\Gamma_{\downarrow+}$) $= 0$. This FC is irrelevant to temperature; thus, the presence of a thermal bath or not does not change the FC of QC for static atoms. However, if FC is not satisfied, QC will decay faster as temperature increases since the decay rate is enhanced $(1+2n)$ times compared with the zero temperature case (see Table II).

Discussion.—Because of the fact that the Hamiltonian used above is invariant in form under the presence of boundaries in space, our results are applicable to not only atoms in free space but also those in bounded space, as long as assumptions (i) and (ii) are satisfied.

Why can the QC be frozen at a nonzero value? For the state (3), after a long time evolution, it is usually a single-photon state, then the measure of QC will be finally zero. But in an extreme condition, such as when only a subradiant decay rate vanishes, the state will be a superposition of $|e_1 g_2\rangle|0\rangle$ with a probability of $(1-\sin\theta)/4$, $|g_1 e_2\rangle|0\rangle$ with the same probability and single-photon state. The QC is thus preserved. Actually, in such a case, the system as a whole does not decay any more and evolves into a steady state. So QC can be frozen.

When the atomic separation is much less than the resonant radiation wavelength of atoms, that is, the Dicke limit [19], the subradiant spontaneous emission and excitation rates will all approach zero, as can be seen from their definitions. The FC is thus satisfied. For implementation, the long-wavelength atoms or molecules, such as Rydberg atoms [20], are appropriate choices.

Another optional strategy is to place the atoms very near the surface of plates. Because of the fact that the tangential component of a fluctuating vacuum electric field on the boundary is null, when the distance from atoms to the boundary is much less than the resonant radiation wavelength of atoms and the polarization direction of atoms is in the surface, the electric field correlation function will go to zero. Thus, in this case, super- and subradiant decay rates will all tend to zero, and FC is satisfied.

Next, we take static atoms in free space with polarizations along their separation as an example and plot in Fig. 2. It is shown that when interatomic distance is small enough compared with the resonant radiation wavelength, QC is approximately frozen to 1/2. The shorter the atomic separation r is, the longer the QC lives. If we use LiH with $\omega_0 = 4.21 \times 10^{13}$ Hz [21], the frozen case illustrated (blue dotted line) can be fulfilled by taking $r = 1$ μm. For RbCs with $\omega_0 = 1.48 \times 10^{12}$ Hz [21], r is as large as 28 μm. If we instead use Rydberg atoms, the required separation will be largely increased since the wave emitted by Rydberg atoms can be radio frequency or microwave $(3 \times 10^8 - 3 \times 10^{11}$ Hz). Then, by shortening the distance between such atoms, QC will be more precisely frozen and maintained for a longer time; see the case of $R = 0.014$.

Now, we consider the situation that these two atoms are with an acceleration a perpendicular to their separation. When $(a/\omega_0 c) \ll 1$, in its first-order approximation, super- and subradiant decay rates are all enhanced $1 + (a/\pi\omega_0 c)$ times, and V is approximately unchanged. So the FC is the same as the static case. Low acceleration does not affect FC, as the function of a thermal bath. If QC is not totally frozen, coherence will deteriorate more severely as acceleration increases. Taking the initial separable state as an example, when $R = 0.14$, $\Gamma^{11}_{0\downarrow}\tau = 10$ ($\Gamma^{11}_{0\downarrow}$ is the spontaneous emission rate of a static atom) and $(a/\pi\omega_0 c) = 0.01$,

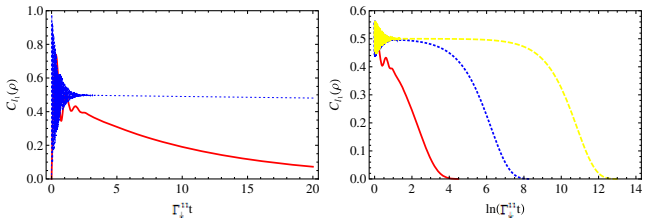

FIG. 2. Measure of coherence for static atoms with an initial separable state in free space as a function of $\Gamma^{11}_{\downarrow}t$ and $\ln(\Gamma^{11}_{\downarrow}t)$, respectively. In such a case, the modulation $\Gamma^{12}_{\downarrow} = 3\Gamma^{11}_{\downarrow}(\sin R - R\cos R)/R^3$ and the interaction potential $V = -3\Gamma^{11}_{\downarrow}(\cos R + R\sin R)/2R^3$ with $R = r\omega_0/c$ and r being atomic separation. The blue dotted line is the $R = 0.14$ case. For comparison, the red solid line with $R = 1$ and yellow dashed line with $R = 0.014$ are plotted.

0.05, respectively, the measure of coherence will be correspondingly 0.4902 and 0.4898, which are all less than 0.4903 in the static case.

One may wonder why the QC for atoms initially prepared in a separable state can be created to a nonzero value. Actually, this can be attributed to the interaction (1) between atoms and the environment, which makes the transitions $|e_1g_2\rangle|0\rangle \rightleftarrows |e_1e_2\rangle|1_{\vec{k}\lambda}\rangle \rightleftarrows |g_1e_2\rangle|0\rangle \rightleftarrows |g_1g_2\rangle|1_{\vec{k}\lambda}\rangle \rightleftarrows |e_1g_2\rangle|0\rangle$ possible. For the initial separable state $|e_1g_2\rangle|0\rangle$, at the neighborhood of initial time, the probability of appearing in the state $|g_1e_2\rangle|0\rangle$, $p_1 = e^{-\Gamma_{\downarrow}^{11}t}[\cosh(\Gamma_{\downarrow}^{12}t) - \cos(2Vt)]/2$ increases from zero. The interference term ρ_{23}, with $|\rho_{23}| = \sqrt{p_1 p_2}$ and $p_2 = e^{-\Gamma_{\downarrow}^{11}t}[\cosh(\Gamma_{\downarrow}^{12}t) + \cos(2Vt)]/2$ being the probability of appearing in the state $|e_1g_2\rangle|0\rangle$, comes up. Since then, the system evolves into a superposition state, and the QC varies with the change of the probability distribution of each state.

For a general initial state, the measure of coherence is $|\sin\theta|$. It will be less than the frozen value $(1 - \sin\theta)/2$ $(\theta \neq \pi/2)$ in the case that $\sin\theta \in (-1, 1/3)$. Thus for a initial state in this range, the QC can be enhanced by engineering the subradiant decay rate. When the subradiant decay rate is small enough and $\tau \gg 1/(\Gamma^{11} + \Gamma^{12})$, the state will be a subradiant state, which can be easily found in the coupled basis $\{|e_1e_2\rangle, |S\rangle = (|e_1g_2\rangle + |g_1e_2\rangle)/\sqrt{2}, |A\rangle = (|e_1g_2\rangle - |g_1e_2\rangle)/\sqrt{2}, |g_1g_2\rangle\}$. Thus, we can prepare a subradiant state from initial state except $|S\rangle$ which is orthogonal to $|A\rangle$.

Note that the QC enhancement results from the adjustment of probability distribution induced by interaction rather than the transformation of a basis state space. Besides, the QC is not inflated, but underestimated, especially at the beginning of states evolution. The QC depends on not only the interference term ρ_{23}, but also on ρ_{14} which is omitted due to its short time behavior. Then, if we initially prepare a superposition state of $|e_1g_2\rangle$ and $|g_1e_2\rangle$, after long time evolution of the states, the QC will still be only encoded on these two states.

What is the relation between QC being investigated and entanglement [22]? For our X form density matrix, the concurrence [23] as a measure of entanglement is $\mathrm{Max}\{0, 2(|\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}), 2(|\rho_{14}| - \sqrt{\rho_{22}\rho_{33}})\}$ [24]. We can see that the concurrence is not greater than QC, $2(|\rho_{23}| + |\rho_{14}|)$. But when time is large enough, ρ_{14} and ρ_{11} all approximate zero, concurrence will tend to QC.

Our results can be generalized to a many atoms case. Our formalism can be used to investigate the decoherence induced by gravity [25] or explore the structure of spacetime [26].

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