Editors' Suggestion

Renormalization Group in Field Theories with Quantum Quenched Disorder

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We study the renormalization group flow in general quantum field theories with quenched disorder, focusing on random quantum critical points. We show that in disorder-averaged correlation functions the flow mixes local and nonlocal operators. This leads to a new critical exponent related to the disorder (as in classical disorder). We show that the time coordinate is rescaled at each renormalization group step, leading to anisotropic spacetime scaling at critical points. We write a universal formula for the dynamical scaling exponent z for weak disorder.

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Introduction.—Quantum critical points appear in many physical situations. These are scale-invariant continuous field theories, that appear at long distances and zero temperature upon fine-tuning some parameters, and that control many features of the corresponding systems with nearby parameters and with finite temperature [1–3].

Disorder is ubiquitous in condensed matter systems. Some quantum critical points depend on the existence of disorder, while others do not, and may (or may not) still exist when disorder is present (see Chap. 21 of [2] and references therein). We will discuss situations where the disorder is quenched (nondynamical), so that it may be viewed as a fixed background, varying in space, for the physical system. One can then assume that there is some probability distribution for the possible disorder configurations, and compute averages of physical quantities over the disorder. In some ("self-averaging") cases these will describe the typical behavior, while in other cases the variances may be large.

In this paper, we discuss the renormalization group (RG) flow of such systems. This is particularly interesting in the vicinity of random critical points, where disorder-averaged quantities have scaling properties that are captured by critical exponents. We focus on general properties of these flows, and not on specific applications. We discuss only zero temperature properties, though the same methods should be useful at finite temperature as well. We do not discuss various instabilities related to long-range fluctuations of the system, such as Griffiths-McCoy singularities [4–6], and it would be interesting to include them in our analysis.

Critical exponents are related to properties of local operators in the theory. We show that the RG evolution of these operators in disorder-averaged correlation functions is nonstandard. Operators which are nonlocal in time evolve in an independent manner from the local operators, and mix with them. This leads to new critical exponents associated with the amount of disorder, analogous to the crossover exponent ϕ in systems with classical disorder [7].

We also show that the RG flow rescales time relative to space, leading at disordered quantum critical points to an anisotropic spacetime scaling symmetry $\mathbf{x} \to \mathbf{x}/b$, $t \to t/b^z$ [8–10]. We argue that the dynamical scaling exponent z may be viewed as a (nonvanishing) beta function for a specific coupling in the action of the disordered theory, and may be computed perturbatively for weak disorder.

For some computations and for intuition we use the replica approach to disordered field theories. Perturbatively in the disorder this is just a technical trick for computing disorder-averaged quantities. Beyond perturbation theory it would be interesting to understand the implications of replica symmetry breaking for our analysis. A more detailed discussion of our assumptions and results may be found in the companion paper [11].

The setup and the replica trick.—A particular realization of disorder is described by a disorder field $h(\mathbf{x})$ which specifies the disorder configuration in the d-dimensional space parametrized by $\mathbf{x}$ (e.g., the distribution of impurities, or the strength of a background magnetic field). The disorder modifies the microscopic interactions in the system in an inhomogeneous manner. If the action of the clean system without disorder is S_0, in the presence of disorder the action becomes

$$S = S_0 + \int d^d\mathbf{x} dt h(\mathbf{x}) \mathcal{O}_0(\mathbf{x}, t), \tag{1}$$

where $\mathcal{O}_0$ stands for the leading interaction that disorder couples to (e.g., the order parameter when disorder is a background magnetic field). Euclidean signature is used in this entire Letter (using analytic continuation from Lorentzian). For simplicity, we assume that the disorder has spin zero, and that the clean theory is a relativistic critical system; the generalization to other situations is straightforward.

The disorder manifestly breaks space translations and spacetime rotations, but we assume that the probability distribution $P[h(\mathbf{x})]$ to obtain a specific disorder configuration is translationally and rotationally invariant. The averages and all higher moments of observables are then invariant under translations and space rotations, but full relativistic invariance is not restored. Disorder averages are denoted by $\bar{X} \equiv \int DhP[h]X$. We focus on short-ranged disorder correlations, such that for long distance physics the disorder range can be neglected, and the disorder at different points is statistically independent, $P[h(\mathbf{x})] = \exp[-\int d^d\mathbf{x} p(h(\mathbf{x}))]$. A commonly used distribution is the Gaussian one, in which $p(h) = h^2/(2v)$, normalized to give unit sum over probabilities.

The replica trick presents quenched disorder-averages using a limit of field theories without disorder. For the disordered free energy we use the identity $\log(Z) = \lim_{n\to 0}(\partial Z^n/\partial n)$, where $Z = \int D\mu e^{-S}$ is the partition function ($D\mu$ stands for the path integral measure in the theory). The replica theory is defined by

$$\int DhP[h]Z[h]^n = \int \prod_{A=1}^{n} D\mu_A e^{-S_{\text{replica}}}, \qquad (2)$$

having n copies of the original degrees of freedom. Averaged correlation functions in the disordered theory are then related to $n \to 0$ limits of appropriate correlation functions in the replica theory.

For the Gaussian distribution the replica theory is

$$S_{\text{replica}} = \sum_{A=1}^{n} S_{0,A} - \frac{v}{2}\sum_{A,B=1}^{n}\int d^d\mathbf{x}dtdt'\mathcal{O}_{0,A}(\mathbf{x},t)\mathcal{O}_{0,B}(\mathbf{x},t'). \qquad (3)$$

As opposed to classical disordered systems, the quantum replica theory is nonlocal in time, and therefore it is not clear whether we can use RG methods. In addition, for $n \neq 0$ there are IR divergences. However, renormalization group analysis is valid for $n \to 0$ and in the disordered theory, as suggested by the local Wilsonian RG [12].

Renormalization group flow with quenched disorder.— In a clean field theory, the coupling constants λ_i run and mix along the RG flow, and this is encoded in beta functions $\beta_i(\lambda_j)$. There is a one-to-one mapping between coupling constants and local operators $\mathcal{O}_i$, such that the action includes $\lambda_i \int \mathcal{O}_i$, and the local operators also mix with each other under the RG. At a fixed point $\beta_i = 0$, and the derivatives of the beta functions encode the anomalous dimensions of the local operators, which are related to critical exponents.

A disordered theory is parameterized by the disorder distribution $P[h]$ in addition to the local couplings λ_i, and for short-range disorder this is characterized by moments κ_i which multiply different terms in $p(h(\mathbf{x}))$. In a specific realization of disorder, the couplings $h(\mathbf{x})$ flow, and this leads to a flow of the disorder distribution couplings κ_i. In general, the flow generates disorder for all coupling constants λ_i, with some general disorder distribution whose parameters we will still denote by κ_i. Under the RG flow, these parameters mix with the original uniform couplings λ_i, to the extent that this is allowed by the symmetries, so we have beta functions $\beta_{\lambda_i}(\lambda_j, \kappa_k)$ and $\beta_{\kappa_i}(\lambda_j, \kappa_k)$. Disorder-averaged correlation functions and thermodynamic quantities depend on all these couplings, and all of their beta functions have to vanish at disordered fixed points. Starting from a clean theory, the most relevant deformation associated with the disorder is the coupling v of (3), whose dimension at the fixed point is $d + 2 - 2\Delta_0$, where Δ_0 is the scaling dimension of $\mathcal{O}_0$ at the clean fixed point. Assuming that $\mathcal{O}_0$ is the lowest dimension operator allowed by the global symmetries, we have $\nu = 1/(d + 1 - \Delta_0)$, so the disorder is relevant whenever $\nu < 2/d$ (this is known as the Harris criterion [13–15]).

In the next section, we show that disordered theories have a special coupling constant related to rescalings of time and to the emergence of anisotropic scaling in spacetime. In the following section, we then discuss the general mixing of operators in disorder-averaged theories, and show that this leads to a new critical exponent.

The dynamical scaling exponent.—In this section, we show that the dynamical scaling exponent z behaves like an anomalous dimension of an operator—it runs along the RG, and converges at a fixed point to the anomalous scaling exponent. This is shown to be equivalent to a dynamical rescaling of time. The discussion is general and applies also to strongly coupled theories; in fact it holds in any system (not only with disorder) breaking relativistic invariance. Without disorder the same relation of the dynamical scaling exponent to running RG couplings was discussed in [16], with some additional assumptions that do not hold in disordered systems.

Consider the replica action (3). Whenever t' is close to t, we are allowed to use the operator product expansion (OPE), replacing the product $\mathcal{O}_0 \times \mathcal{O}_0$ by a series of local operators. There is one particularly interesting operator appearing universally in this OPE, which is the energy-momentum tensor $T_{\mu\nu}$. Let x stand for the spacetime coordinate. Around the clean critical theory, the coefficient of this operator in the OPE is given by

$$\mathcal{O}_{0,A}(x)\mathcal{O}_{0,B}(0) \supset \frac{c_{\mathcal{O}\mathcal{O}T}\delta_{AB}}{c_T}\frac{x^\mu x^\nu}{x^{2\Delta_0-d+1}}T_{\mu\nu,A}(0), \qquad (4)$$

where c_T and $c_{\mathcal{O}\mathcal{O}T}$ are the coefficients in the two-point function $\langle TT\rangle$ and the three-point function $\langle \mathcal{O}_0\mathcal{O}_0 T_{\mu\nu}\rangle$, respectively [11]. Performing the integration over t' in (3), this leads to a UV cutoff-dependent term proportional to $\int d^d\mathbf{x}dtT_{00,A}(\mathbf{x},t)$, which is no other than the Hamiltonian H integrated over time. Along the RG flow the UV cutoff changes, and this term flows (in particular, it is generated along the RG even if it was not there to begin with). The effective action for Gaussian disorder is then given by

$$S_{\mathrm{replica}} = \sum_A S_{0,A} + h_{00} \sum_A \int dt H_A - \frac{v}{2} \sum_{A,B} \int d^d\mathbf{x} dt dt' \mathcal{O}_{0,A}(\mathbf{x},t) \mathcal{O}_{0,B}(\mathbf{x},t') \quad (5)$$

with a running coupling denoted by h_{00}. The new term is equivalent to adding $h_{00} \int dtH$ to the original disordered theory.

The integrated Hamiltonian term is actually equivalent to a stretching of time. Indeed, by Noether's theorem, under an infinitesimal transformation $x'_\mu = x_\mu + \epsilon_\mu$, the variation of the action is $\delta S = -\int d^d\mathbf{x} dt \partial_\mu \epsilon_\nu T^{\mu\nu}$. Therefore, for an infinitesimal time dilation $t' = t(1+\epsilon)$, we get precisely the new generated term $\delta S = -\epsilon \int dtH$. As a result, we can either think about the RG as a flow including the coupling h_{00}, or alternatively we can rescale time to get rid of this term, with no h_{00} coupling. For instance, for scalar operators, the correlation functions in the two approaches are related by

$$\langle \mathcal{O}_1(x_1) \cdots \mathcal{O}_k(x_k) \rangle_{h_{00}} = \left(1 - h_{00} \sum_{i=1}^{k} t_i \frac{\partial}{\partial t_i}\right) \langle \mathcal{O}_1(x_1) \cdots \mathcal{O}_k(x_k) \rangle + O(h_{00}^2). \quad (6)$$

Usually, at an RG fixed point all beta functions vanish and the coupling constants flow to fixed values. However, it turns out that a constant beta function for h_{00} is also consistent with scaling, and indeed generically this is what one finds at nonrelativistic RG fixed points. At each RG step we rescale the cutoff, or, equivalently, we rescale the space and time coordinates by $\mathbf{x} \to \mathbf{x}/b$, $t \to t/b$. If h_{00} has a constant beta function $\beta_{h_{00}}$, then under an infinitesimal RG step it changes by $h_{00} \to h_{00} - \beta_{h_{00}} \log(b)$. Naively, this means that the theory is not invariant, but the discussion of the previous paragraph implies that we can equivalently keep h_{00} fixed but perform an additional rescaling of the time coordinate by $t \to t(1 + \beta_{h_{00}} \log(b)) \sim t b^{\beta_{h_{00}}}$. Thus, we find that the theory is invariant under the modified scaling transformation $\mathbf{x} \to \mathbf{x}/b$, $t \to t/b^{1-\beta_{h_{00}}}$, which is an anisotropic scaling transformation with a dynamical scaling exponent $z = 1 - \beta_{h_{00}}$. If our original clean theory has a dynamical scaling exponent z_{clean}, then the same arguments give for the new fixed point $z_{\mathrm{disorder}} = z_{\mathrm{clean}} - \beta_{h_{00}}$.

To show how this affects correlation functions, we can look at their RG flow in the replica theory. The RG implies that when we change the RG scale M and simultaneously change the coupling constants (including v, h_{00}, and other possible couplings λ_i and κ_i) and allow for anomalous dimensions γ for operators, the theory remains the same. Applied to correlation functions, this is called the Callan-Symanzik equation [17,18]. The $n \to 0$ limit of the replica theory gives disorder-averaged correlation functions, and therefore we find for connected correlation functions of the lowest-dimension scalar operator $\mathcal{O}$ [19]

$$\left(M\frac{\partial}{\partial M} + \beta_v \frac{\partial}{\partial v} + \beta_{\lambda_i} \frac{\partial}{\partial \lambda_i} + \beta_{\kappa_i} \frac{\partial}{\partial \kappa_i} + \beta_{h_{00}} \frac{\partial}{\partial h_{00}} + k\gamma \right) \overline{\langle \mathcal{O}(x_1) \cdots \mathcal{O}(x_k) \rangle_{\mathrm{conn}}} = 0. \quad (7)$$

Using the relation (6), the two-point function at a quantum disordered fixed point (where $\beta = 0$ for all couplings other than h_{00}) satisfies

$$\left(M\frac{\partial}{\partial M} + \gamma_t^* t \frac{\partial}{\partial t} + 2\gamma^* \right) \overline{\langle \mathcal{O}(\mathbf{x},t) \mathcal{O}(0) \rangle_{\mathrm{conn}}} = 0, \quad (8)$$

where $\gamma_t^* \equiv -\beta_{h_{00}}$ and γ^* is the anomalous dimension of $\mathcal{O}$ at the fixed point. Let Δ be the dimension of $\mathcal{O}$ at the clean theory. The solution of (8) is determined up to a function F to be

$$\overline{\langle \mathcal{O}(\mathbf{x},t) \mathcal{O}(0) \rangle_{\mathrm{conn}}} = \frac{M^{-2\gamma^*}}{\mathbf{x}^{2\Delta + 2\gamma^*}} F\left(M^{\gamma_t^*} \frac{\mathbf{x}^{1+\gamma_t^*}}{t}\right). \quad (9)$$

This is indeed invariant under scaling $\mathbf{x} \to \mathbf{x}/b$, $t \to t/b^z$, with $\mathcal{O}$ having scaling dimension $\Delta + \gamma^*$ and with $z = 1 + \gamma_t^*$ as above. Thus, γ_t^* plays the role of an 'anomalous dimension of time'.

For weak disorder, we can give a universal formula for the dynamical scaling exponent. We assume that the Harris criterion is saturated, $\Delta_0 = (d+2)/2$, so that v is dimensionless and we can use perturbation theory. In this case, substituting (4) in the v term of (3) and performing the t' integration gives the M-dependent term

$$-\frac{v c_{\mathcal{O}\mathcal{O}T}}{c_T} \log(M) \sum_A \int dt H_A. \quad (10)$$

The flow of h_{00} should compensate for this M dependence, giving the beta function $\beta_{h_{00}} = v c_{\mathcal{O}\mathcal{O}T}/c_T + O(v^2)$. If we normalize the two-point function of $\mathcal{O}_0$ to one, and use the conformal Ward identity to compute $c_{\mathcal{O}\mathcal{O}T}$, we obtain at leading order in v

$$z \approx 1 + \frac{v}{2c_T} \frac{(d+1)(d+2)}{d} \frac{\Gamma((d+1)/2)}{2\pi^{(d+1)/2}}. \quad (11)$$

This formula is valid for any theory with weak (marginal) disorder. In particular, it reproduces the strongly coupled holographic result in [20] and the weakly coupled result in [9]. In the former, z was computed using holography, and here we see the field theory interpretation of this, with the same numerical value.

Note that (11) used (3) in which Gaussian disorder is assumed. However, it is still true for a generic disorder distribution with variance v, since for marginal disorder the corrections to (3) from higher disorder moments are irrelevant.

Operator mixings and disorder critical exponents.—For classical disorder, the replica theory (2) is local, and the moments of the disorder distribution are standard coupling constants of the replica theory. Thus the disordered RG flow is an $n \to 0$ limit of standard RG flows, with general mixings of all couplings $\{\lambda_i, \kappa_i\}$ [21,22]. In order to flow to a (disordered) fixed point, any coupling related to the disorder must be irrelevant at the fixed point. Since the couplings flow independently and mix, the associated critical exponent is no longer directly related to Δ_0 (and to ν) as it was at short distances, but rather there is a new critical exponent ϕ, determined by the dimension of the leading coupling related to the disorder.

For quantum disorder the situation is different, since the disorder couplings multiply nonlocal operators in the replica theory. For Gaussian disorder there are two integrations over the time direction (3), while kth moments of the disorder distribution multiply terms with k integrations over the time direction [23]. Naively, since in such nonlocal operators the different $\mathcal{O}_0$'s are separated in time, one may expect their renormalization to be determined by that of the local operators $\mathcal{O}_0(\mathbf{x}, t)$. So one may guess that the mixings of λ_i and κ_i described above do not occur, and that the scaling dimensions of nonlocal terms in the action like (3) are determined by those of the local operators $\mathcal{O}_0(\mathbf{x}, t)$; there would then be no independent critical exponent ϕ associated with the disorder. We will show that these expectations are actually not correct.

In fact, we claim that in disorder-averaged correlation functions, general multi-local operators of the form

$$\mathcal{O}_1(\mathbf{x}, t) \int dt_2 \mathcal{O}_2(\mathbf{x}, t_2) \cdots \int dt_k \mathcal{O}_k(\mathbf{x}, t_k) \qquad (12)$$

with different values of k mix with each other. The integrals of these operators multiply the λ_i and the κ_j, so this gives rise to generic mixings of all these couplings. From the point of view of the replica theory, where disorder-averaged correlation functions of these operators are related to those of

$$\mathcal{O}_{1,A}(\mathbf{x}, t) \sum_{A_2,\ldots,A_k=1}^{n} \int dt_2 \mathcal{O}_{2,A_2}(\mathbf{x}, t_2) \cdots \int dt_k \mathcal{O}_{k,A_k}(\mathbf{x}, t_k), \qquad (13)$$

such mixings appear naturally. When two operators in (13) from the same replica $\mathcal{O}_{i,A}(\mathbf{x}, t_i)$ and $\mathcal{O}_{j,A}(\mathbf{x}, t_j)$ approach each other in time, there is a short-distance singularity, and regularizing it leads to a mixing with another operator $\mathcal{O}_{k,A}(\mathbf{x}, t_i)$; the resulting operator has one fewer time integration than the original operator (13). We already saw an example of this in the previous section. Conversely, the perturbative-in-disorder corrections to a local operator $\mathcal{O}_{k,A}(\mathbf{x}, t_0)$ can be described by bringing down a disorder interaction (3) from the action. There is then a singularity when (say) t approaches t_0 for arbitrary t', and regularizing it requires mixing this operator with $\mathcal{O}_{j,A}(\mathbf{x}, t_0) \sum_{B=1}^{n} \int dt' \mathcal{O}_{0,B}(\mathbf{x}, t')$.

From the point of view of the disordered theory, such mixings seem very surprising, since this theory is local, and operators at different times cannot mix. This is true, but in disorder-averaged correlation functions, the fact that the disorder distribution is independent of time reproduces this mixing effect, whenever the mixing of local operators depends on the disordered couplings $h(\mathbf{x})$. For instance, consider a situation where some operator $\mathcal{O}_i(\mathbf{x}, t)$ mixes under the RG flow in a specific realization of disorder with $h(\mathbf{x})\mathcal{O}_j(\mathbf{x}, t)$, and suppose that the disorder distribution is Gaussian. In such a situation, a disorder-averaged correlation function of $\mathcal{O}_i$,

$$\overline{\langle \mathcal{O}_i(\mathbf{x}, t) \cdots \rangle} = \int DhP[h] \langle \mathcal{O}_i(\mathbf{x}, t) \cdots \rangle_{h(\mathbf{x})} \qquad (14)$$

mixes with

$$\begin{aligned} &\int DhP[h] h(\mathbf{x}) \langle \mathcal{O}_j(\mathbf{x}, t) \cdots \rangle_{h(\mathbf{x})} \\ &= \int Dh \left(-v \frac{\delta P[h]}{\delta h(\mathbf{x})} \right) \langle \mathcal{O}_j(\mathbf{x}, t) \cdots \rangle_{h(\mathbf{x})} \\ &= v \int DhP[h] \frac{\delta}{\delta h(\mathbf{x})} \langle \mathcal{O}_j(\mathbf{x}, t) \cdots \rangle_{h(\mathbf{x})}. \end{aligned} \qquad (15)$$

But the derivative with respect to $h(\mathbf{x})$ brings down from the path integral of the disordered theory an operator $\int dt' \mathcal{O}_0(\mathbf{x}, t')$. So we find that correlation functions of $\mathcal{O}_i(\mathbf{x}, t)$ mix with those of $\mathcal{O}_j(\mathbf{x}, t) \int dt' \mathcal{O}_0(\mathbf{x}, t')$. One can show that such arguments account for all the nonlocal mixings seen in the replica theory, so these are not just artifacts of the replica description.

A specific implication of this RG analysis is that the renormalization of the nonlocal replica operators related to the κ_i is independent of that of the local operators. In particular, the dimension of the disorder operator multiplying v in (3) is not directly related to that of $\mathcal{O}_0$, but rather gives rise at a fixed point to an independent critical exponent ϕ, governing the effect of disorder at and near that fixed point. We will give an explicit example of this below. It would be interesting to measure this critical exponent at disordered quantum critical points. In order to flow to the disordered fixed point, this disordered coupling

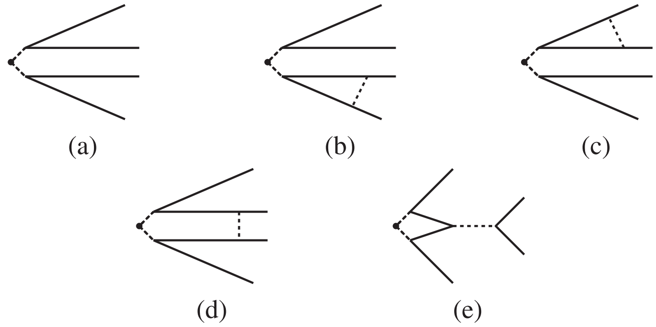

FIG. 1. The diagrams contributing to the anomalous dimension of the (nonlocal in time) disorder operator Ψ. The solid lines are propagators, and the dashed lines correspond to the nonlocal v interaction. The dot in each diagram, with two dashed lines attached, represents $\Psi(\mathbf{x})$.

must be irrelevant, but it is not directly related to ν, so from this point of view the relation $\nu > 2/d$ is not required to hold at a disordered fixed point. Nevertheless, there are independent arguments that this relation must hold [24].

An example.—We give a simple example showing explicitly that the disorder operator [multiplying v in (3)] has an independent scaling dimension, rather than being twice the dimension of $\mathcal{O}_0$. The model we use is a simple variant of the one used in [9], a real scalar field φ with disorder coupled to φ^2, related to the random-bond Ising model. But we study this model for four space dimensions, where the disorder saturates the Harris bound. The replica action is

$$S_{\text{replica}} = \frac{1}{2}\sum_{A=1}^{n}\int d^4\mathbf{x}dt\left(\sum_{i=1}^{4}(\partial_i\varphi_A)^2 + \alpha(\partial_t\varphi_A)^2\right) - \frac{v}{2}\sum_{A,B=1}^{n}\int d^4\mathbf{x}dtdt'\varphi_A(\mathbf{x},t)^2\varphi_B(\mathbf{x},t')^2. \quad (16)$$

The need for the running coupling α in this case was noticed in [9]. In fact, it is a special case of our general discussion above, since the Hamiltonian deformation in (5) reduces to this action [11] with $\alpha = 2h_{00} + 1$. The computations below are performed with this action in the $n \to 0$ limit.

A standard field theory computation gives the beta function of v and its dimension $[v] = 4v/\pi^2 + O(v^2)$. Therefore, the dimension of the operator $\Psi(\mathbf{x}) \equiv \sum_{A,B}\int dtdt'\varphi_A(\mathbf{x},t)^2\varphi_B(\mathbf{x},t')^2$ has to be $[\Psi] = 4 - 4v/\pi^2 + O(v^2)$. On the other hand, computing the dimension of φ^2 gives $[\varphi^2] = 3 - v/(2\pi^2) + O(v^2)$. We see that subtracting the dimensions of the time integrals from $2[\varphi^2]$ does not reproduce $[\Psi]$.

Instead, we should consider Ψ as an independent operator. We can alternatively compute its dimension by considering the correlation function $\langle\Psi(\mathbf{x}) \times \sum_{A_1}\varphi_{A_1}(\mathbf{x}_1,t_1)\cdots\sum_{A_4}\varphi_{A_4}(\mathbf{x}_4,t_4)\rangle$. In Fig. 1(a), we show the tree level contribution to this correlation function. At the next order we find the corrections of Figs. 1(b) and 1(c), which correspond to the anomalous dimensions of each of the φ^2 factors in Ψ. The naive claim that the dimension of Ψ is fixed by $[\varphi^2]$ corresponds to taking into account only these corrections. However, there are additional corrections shown in Figs. 1(d) and 1(e) (there are no other diagrams contributing to the anomalous dimension as $n \to 0$). The sum of these corrections gives precisely the value of $[\Psi]$ above.

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