Speed Limit for Classical Stochastic Processes

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We consider the speed limit for classical stochastic Markov processes with and without the local detailed balance condition. We find that, for both cases, a trade-off inequality exists between the speed of the state transformation and the entropy production. The dynamical activity is related to a time scale and plays a crucial role in the inequality. For the dynamics without the local detailed balance condition, we use the Hatano-Sasa entropy production instead of the standard entropy production. Our inequalities consist of the quantities that are commonly used in stochastic thermodynamics and explicitly show underlying physical mechanisms.

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Introduction.—Obtaining a fundamental bound on the speed of state transformation is an important question relevant to broad research fields including quantum control theory and foundations of nonequilibrium statistical mechanics. This question has been investigated first in an isolated quantum system, for which the derived bounds are nowadays called quantum speed limits [1–8]. Consider a state transformation from a given initial state $\rho = \rho(0)$ to a given final state $\rho' = \rho(\tau)$ in a time interval $0 \leq t \leq \tau$ with a time-dependent Hamiltonian. Quantum speed limits claim that there is a trade-off relation between the operation time τ and the energy fluctuation. For instance, Mandelstam and Tamm derived the following inequality [1]:

$$\frac{\mathcal{L}(\rho, \rho')}{\Delta E_\tau / \hbar} \leq \tau, \tag{1}$$

where $\mathcal{L}(\rho, \rho')$ is the distance between the states ρ and ρ', and ΔE_τ and $\hbar$ are, respectively, the energy fluctuation and the Planck constant [9]. This explicitly shows the bound on the operation time τ in quantum state transformations. The quantum speed limit has recently attracted a great deal of attention, and generalization has been studied in various cases [10–15].

The quantum speed limit has its origin in the uncertainty relation between time and energy, and thus, taking a naive semiclassical limit $\hbar \to 0$ of the relation (1) fails to obtain a meaningful classical extension of the speed limits. However, recent studies have considered various extensions of the quantum speed limits to the case of classical Hamiltonian dynamics [16–18]. In other words, the concept of speed limits is not necessarily confined within the quantum regime. In addition, some attempts have been made toward further extension to classical stochastic systems [18–21]. For instance, Ref. [18] derived a speed limit inequality for time-independent Markov jump processes (i.e., relaxation processes) with the local detailed balance condition. Although these results are restricted to somewhat limited dynamics, and/or the obtained relations are given with abstract mathematical forms, they strongly suggest that there must exist a general and concise form of speed limit for classical stochastic systems such that its underlying physical mechanism becomes clear, as the original quantum speed limit does.

In this Letter, we derive two speed limit inequalities for the dynamics with and without the local detailed balance condition for the general Markovian dynamics with discrete states. These inequalities consist of several quantities that have been studied in nonequilibrium statistical physics [22]. In the first inequality, the entropy production plays the role to bound the speed of state transformation. In addition, the inequality contains the dynamical activity, which quantifies how frequently the state changes and, thus, is related to the time scale of the dynamics [23–26]. The second inequality gives a nontrivial bound even when the system has nonzero stationary heat current. In this inequality, we use a generalization of the entropy production, the Hatano-Sasa entropy production [27], instead of the conventional entropy production. It is known that the Hatano-Sasa entropy production leads to the generalized Clausius inequality for systems with a stationary current [27]. Both of the derived inequalities connect the speed limit expressions to the nonequilibrium thermodynamics, and thus, provide a clear picture of the underlying mechanism for the general Markovian dynamics with discrete states.

Setup.—Consider a classical stochastic Markov process with discrete states. Let $p_i(t)$ and $W_{ij}(t)$ be the probability distribution of the state i and the transition rate matrix element of the transition $j \to i$ at time t. The time evolution of the probability distributions is given by the following master equation:

$$\begin{aligned}\frac{d}{dt}p_i(t) &= \sum_j W_{ij}(t)p_j(t)\\ &= \sum_{j(\neq i)} W_{ij}(t)p_j(t) - W_{ji}(t)p_i(t). \end{aligned} \tag{2}$$

The transition rate matrix satisfies the normalization condition $\sum_i W_{ij}(t) = 0$ and non-negativity $W_{ij}(t) \geq 0$ for $i \neq j$. We consider the state transformation from $\boldsymbol{p}(0)$ to $\boldsymbol{p}(\tau)$ by changing the transition rate matrix in time for $0 \leq t \leq \tau$. We measure the distance of two probability distributions $\boldsymbol{p}$ and $\boldsymbol{p}'$ by the statistical distance with the L^1 norm, or the total variation distance [28], defined as

$$L(\boldsymbol{p}, \boldsymbol{p}') := \sum_i |p_i - p_i'|. \tag{3}$$

We consider a system attached to a single or multiple heat baths. In the latter case, the transition rate matrix consists of each contribution from independent reservoirs as $\mathbf{W}(t) = \sum_\nu \mathbf{W}^\nu(t)$, where $\mathbf{W}^\nu(t)$ is the transition rate matrix associated with the νth heat bath. We denote by $E_i(t)$ the energy of the ith state of the system. Then, the heat absorption by the bath associated with the transition $i \to j$ is given by $Q_{i\to j}(t) = E_i(t) - E_j(t)$. We also introduce the Shannon entropy of the system [28]

$$H(t) := -\sum_i p_i(t) \ln p_i(t). \tag{4}$$

Following the standard theory in the stochastic thermodynamics, we define the entropy production rate $\dot{\Sigma}$ and the total entropy production Σ by the sum of the entropy increase in the system and that in the baths as follows [22]:

$$\dot{\Sigma}(t) := \frac{d}{dt}H(t) + \sum_\nu \sum_{i\neq j} W_{ji}^\nu(t)p_i(t)\beta_\nu Q_{i\to j}(t), \tag{5}$$

$$\Sigma := \int_0^\tau dt \dot{\Sigma}(t). \tag{6}$$

Here, β_ν is the inverse temperature of the νth heat bath.

To quantify the system's time scale, we employ the dynamical activity $A(t)$ and its time average $\langle A \rangle_\tau$

$$A(t) := \sum_{i\neq j} W_{ij}(t)p_j(t), \tag{7}$$

$$\langle A \rangle_\tau := \frac{1}{\tau}\int_0^\tau dt A(t). \tag{8}$$

The dynamical activity quantifies how frequently jumps between different states occur, and thus, it characterizes the time scale of the system [23–26,29].

First main result.—First, we consider a system with the local detailed balance condition, i.e.,

$$W_{ij}^\nu(t)e^{-\beta_\nu E_j(t)} = W_{ji}^\nu(t)e^{-\beta_\nu E_i(t)}, \tag{9}$$

for any ν, i, and j. In what follows, in order to suppress the length of mathematical expressions, we drop the time dependence in quantities unless necessary.

Now, we derive the first speed limit inequality, where the entropy production and activity bound the speed of state transformation. To this end, we evaluate the entropy production rate (5) as follows:

$$\dot{\Sigma} = \frac{1}{2}\sum_\nu \sum_{i\neq j}(W_{ji}^\nu p_i - W_{ij}^\nu p_j)\ln\frac{W_{ji}^\nu p_i}{W_{ij}^\nu p_j} \tag{10}$$

$$\geq \sum_\nu \sum_{i\neq j}\frac{(W_{ji}^\nu p_i - W_{ij}^\nu p_j)^2}{W_{ji}^\nu p_i + W_{ij}^\nu p_j}. \tag{11}$$

The first line is a conventional form with the local detailed balance condition [22], and the second line follows from the inequality $(a-b)\ln(a/b) \geq 2(a-b)^2/(a+b)$, which is valid for non-negative a and b. To connect the change in the probability distribution with the entropy production rate and the instantaneous activity, we transform the master equation (2) by using the Schwarz inequality twice

$$\begin{aligned}\sum_i\left|\frac{d}{dt}p_i\right| &\leq \sum_{\nu,i}\left|\sum_{j(\neq i)}(W_{ij}^\nu p_j - W_{ji}^\nu p_i)\right|\\ &\leq \sum_{\nu,i}\sqrt{\left(\sum_{j(\neq i)}\frac{(W_{ij}^\nu p_j - W_{ji}^\nu p_i)^2}{W_{ij}^\nu p_j + W_{ji}^\nu p_i}\right)\left(\sum_{j(\neq i)}(W_{ij}^\nu p_j + W_{ji}^\nu p_i)\right)}\\ &\leq \sqrt{\left(\sum_{\nu,i\neq j}\frac{(W_{ij}^\nu p_j - W_{ji}^\nu p_i)^2}{W_{ij}^\nu p_j + W_{ji}^\nu p_i}\right)\left(\sum_{i\neq j}(W_{ij}p_j + W_{ji}p_i)\right)}\\ &\leq \sqrt{2\dot{\Sigma}A}. \end{aligned} \tag{12}$$

By integrating Eq. (12) with time, the entropy production and the dynamical activity are connected to the distance of states as

$$\begin{aligned} L(\boldsymbol{p}(0), \boldsymbol{p}(\tau)) &\leq \sum_i \int_0^\tau dt\left|\frac{d}{dt}p_i\right|\\ &\leq \int_0^\tau dt\sqrt{2\dot{\Sigma}A} \leq \sqrt{2\tau\Sigma\langle A\rangle_\tau}. \end{aligned} \tag{13}$$

At the last inequality, we used the Schwarz inequality again. From this, we arrive at the following bound on the state transformation:

$$\tau_{\mathrm{I}} := \frac{L(\boldsymbol{p}(0), \boldsymbol{p}(\tau))^2}{2\Sigma\langle A\rangle_\tau} \le \tau. \tag{14}$$

This is the first main result. Remarkably, the relation (14) has a similar structure to the quantum speed limit (1). Following the interpretation on (1), the relation (14) may be interpreted as a trade-off relation between the speed of the state transformation and the entropy production. Here, the combination of the entropy production and the dynamical activity plays a central role to determine the time of state transformation. We should note that, because the time scale of stochastic systems is not universal but depends on the coefficients of transition rates, the dynamical activity also depends on specific models and is not a universal constant.

This inequality is derived for a general transition matrix $\boldsymbol{W}$ as long as it satisfies the local detailed balance condition (9). Here, we remark that the bound τ_{I} contains the entropy production in the denominator. Hence, when the system is attached to multiple heat baths with different temperatures, τ_{I} will vanish for large τ because the entropy production grows linearly in time due to finite net currents between baths. In this case, Eq. (14) falls into a trivial bound; $\tau_{\mathrm{I}} = 0 \le \tau$. On the other hand, the inequality is, in general, useful for the case with a single heat bath or the case in a relatively short time scale. To get a finite bound value for the far-from-equilibrium regime and large τ, we derive another inequality below, which is valid in general dynamics.

Second main result for general dynamics.—Here, we derive another inequality for the general dynamics, which provides a nontrivial finite bound even for systems with finite stationary currents. It is also applicable to systems without the local detailed balance condition (9).

Assuming the uniqueness of the stationary state (SS), we define the excess entropy production for general stochastic processes, which is responsible for the state transformation, as [27]

$$S^{\mathrm{ex}}_{i\to j}(t) := \ln\frac{p_j^{\mathrm{SS}}(t)}{p_i^{\mathrm{SS}}(t)}. \tag{15}$$

Here, $\boldsymbol{p}^{\mathrm{ss}}(t)$ is the instantaneous stationary probability distribution for the transition rate matrix $\boldsymbol{W}(t)$. For systems driven by baths, the excess entropy is understood as the entropy production of baths after deduction of that caused by stationary dissipation. If the stationary state is an equilibrium state, this quantity reduces to the usual expression of the entropy production in the heat bath. Using Eq. (15), we define the Hatano-Sasa (HS) entropy production rate and the total HS entropy production [27,30]

$$\dot{\Sigma}^{\mathrm{HS}}(t) := \frac{d}{dt}H(t) + \sum_{i,j} W_{ji}(t)p_i(t)S^{\mathrm{ex}}_{i\to j}(t), \tag{16}$$

$$\Sigma^{\mathrm{HS}} := \int_0^\tau dt\dot{\Sigma}^{\mathrm{HS}}(t). \tag{17}$$

We remark that the HS entropy production depends only on $\boldsymbol{W}(t)$ and not on each $\boldsymbol{W}^\nu(t)$. It is known that, even in the presence of finite stationary currents, the HS entropy production rate is infinitesimally small if the control parameter of the system is changed quasistatically [27]. This is analogous to the behavior of the conventional entropy production against the quasistatic change of the parameters in the case with a single bath. In this sense, the HS entropy production plays a similar role to the conventional entropy production in the equilibrium situation.

We show the second main result, which is valid for the general dynamics including the case without the local detailed balance condition

$$\tau_{\mathrm{II}} := \frac{c^* L(\boldsymbol{p}(0), \boldsymbol{p}(\tau))^2}{2\Sigma^{\mathrm{HS}}\langle A\rangle_\tau} \le \tau, \tag{18}$$

where $c^* := 0.896\ldots$ is a solution of

$$c^* = \min_y \frac{(1-e^y+ye^y)(1+e^y)}{(1-e^y)^2}. \tag{19}$$

The number c^* has already appeared in several contexts so far [31,32].

First, we make several remarks on the physical implication of this inequality, and next provide the proof. In the inequality, the HS entropy production is a key ingredient to characterize a bound, and the dynamical activity again plays a crucial role for determining the time scale in the dynamics. Since the total HS entropy production does not grow linearly in time in general, τ_{II} provides a finite bound even for large τ. Hence, this inequality provides a nontrivial bound for the general dynamics including the system with a finite net current attached to multiple heat baths. On the other hand, for the case with a single heat bath, the HS entropy production Σ^{HS} reduces to the standard entropy production Σ, and hence, the bound (18) becomes weaker than (14) by the factor c^*.

Now, we show the proof of Eq. (18), which is similar to that of Eq. (14). To this end, we introduce the dual matrix $\tilde{W}$ [33] given by

$$\tilde{W}_{ij} := \frac{W_{ji}p_i^{\mathrm{SS}}}{p_j^{\mathrm{SS}}}. \tag{20}$$

We can easily check that $\tilde{W}$ is indeed a transition rate matrix (i.e., $\tilde{W}$ satisfies the normalization condition and non-negativity). Note that the diagonal element of the dual matrix and the original one are equal

$$\sum_{j(\neq i)}\tilde{W}_{ji} = -\tilde{W}_{ii} = -W_{ii} = \sum_{j(\neq i)} W_{ji}. \tag{21}$$

The dual matrix provides another expression of the HS entropy production rate

$$\begin{aligned}\dot{\Sigma}^{\mathrm{HS}} &= \sum_{i\neq j} W_{ji}p_i \ln\frac{W_{ji}p_i}{\tilde{W}_{ij}p_j} \\ &= \sum_{i\neq j}\left(W_{ji}p_i\ln\frac{W_{ji}p_i}{\tilde{W}_{ij}p_j} + \tilde{W}_{ij}p_j - W_{ji}p_i\right),\end{aligned} \tag{22}$$

where we used Eq. (21). This is a generalized form of the partial entropy production [34,35]. Using an inequality $a\ln a/b + b - a \geq c^*(a-b)^2/(a+b)$ for non-negative a and b [31], we obtain the following relation:

$$\dot{\Sigma}^{\mathrm{HS}} \geq c^*\sum_{i\neq j}\frac{(W_{ji}p_i - \tilde{W}_{ij}p_j)^2}{W_{ji}p_i + \tilde{W}_{ij}p_j}. \tag{23}$$

Then, we arrive at the key relation

$$\begin{aligned}\sum_i\left|\frac{d}{dt}p_i\right| &= \sum_i\left|\sum_{j(\neq i)}(W_{ij}p_j - \tilde{W}_{ji}p_i)\right| \\ &\leq \sqrt{\left(c^*\sum_{i\neq j}\frac{(W_{ij}p_j-\tilde{W}_{ji}p_i)^2}{W_{ij}p_j+\tilde{W}_{ji}p_i}\right)\left(\frac{1}{c^*}\sum_{i\neq j}(W_{ij}p_j+W_{ji}p_i)\right)} \\ &\leq \sqrt{\dot{\Sigma}^{\mathrm{HS}}\frac{2A}{c^*}}.\end{aligned} \tag{24}$$

In the second line, we used the Schwarz inequality twice and Eq. (21). Following the same procedure as in Eq. (13), Eq. (24) leads to the desired inequality (18).

Example: two-level system.—We demonstrate our inequalities (14) and (18) with a simple solvable model. This system consists of two states, 0 and 1, whose energies are given by $E_0 = 0$ and $E_1(t)$, respectively. Suppose that the initial distribution is $p_0(0) = p_1(0) = 1/2$, and we transform it to $p_0(\tau) = 3/4$ and $p_1(\tau) = 1/4$ with the time interval τ. As we see below, even such a simple model is very instructive to understand the physical structure of our results.

First, we consider a single heat bath with inverse temperature β [See Fig. 1.(a)]. For convenience, we set the transition rate matrix as

$$W_{10} = 1, \qquad W_{01} = e^{\beta E_1(t)}$$

with

$$E_1(t) \coloneqq \frac{1}{\beta}\ln\left(\frac{4\tau+1}{2\tau-t} - 1\right), \tag{25}$$

which provides the solution $p_1(t) = 1/2 - t/(4\tau)$. Then, it is straightforward to get the dynamical activity

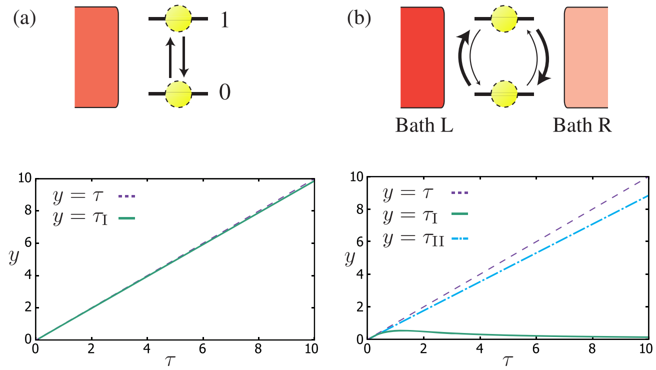

FIG. 1. (a) Demonstration of Eq. (14) for a single bath. In the bottom plot, the purple dotted line is the linear reference line $y = \tau$, and the green solid curve is $y = \tau_{\mathrm{I}}$ as a function of τ. These two lines almost agree with each other from the aspect of the smallness of the relative error $(\tau - \tau_{\mathrm{I}})/\tau$, which clearly shows the tightness of the bound (14). (b) Demonstration of Eq. (18) for two baths with $\alpha = 2/3$ (which means that a finite stationary current exists). The bottom plot shows that Eq. (14) ($\tau_{\mathrm{I}} \leq \tau$: the green solid line) is a poor bound, while Eq. (18) ($\tau_{\mathrm{II}} \leq \tau$: the blue dashed-dotted line) still provides a meaningful bound.

and the distance as $A(t) = 1 + 1/(4\tau) + t/(2\tau)$ and $L(\mathbf{p}(0), \mathbf{p}(\tau)) = 1/2$. The averaged activity $\langle A\rangle_\tau = 5/4 + 1/(4\tau)$ is a quantity of $O(1)$. The bound τ_{I} on the operation time is explicitly given by

$$\tau_{\mathrm{I}} = \frac{1}{(10 + 2/\tau)\Sigma}, \tag{26}$$

where the entropy production Σ is given through some calculations [36]:

$$\Sigma = \frac{1}{4\tau}\int_{2\tau}^{2\tau+1} dy\ln\left(1 + \frac{\tau}{y}\right). \tag{27}$$

The asymptotic behavior of Eq. (14) in large τ reads $1/(10\Sigma) \leq \tau$, which is a very good estimation since $\Sigma \simeq 1/(4\tau)\ln(3/2) = 0.101\ldots \times 1/\tau$. We show the plot of these results in Fig. 1(a).

Next, we consider two heat baths, L and R, and demonstrate the validity of Eq. (18) [See Fig. 1.(b)]. The transition matrices associated with each bath are given by

$$W^L_{10} = \alpha, \qquad W^L_{01} = \frac{1}{2}\left(\frac{4\tau+1}{2\tau-t} - 1\right),$$

$$W^R_{10} = 1-\alpha, \qquad W^R_{01} = \frac{1}{2}\left(\frac{4\tau+1}{2\tau-t} - 1\right),$$

with $\alpha \neq 1/2$, which ensures the existence of the nonzero stationary current between L and R in the stationary state. The total transition rate matrix, $W_{ij} = W^L_{ij} + W^R_{ij}$, is set to be the same as the previous case, so that we can compare the second case with the first one. The probability distribution and the dynamical activity are completely the same as in the single-bath case; $p_1(t) = 1/2 - t/(4\tau)$ and $A(t) = 1 + 1/(4\tau) + t/(2\tau)$. In contrast to these quantities, the entropy production is larger than that in the single-bath case (The explicit form of the entropy production is presented in [37]). In particular, for large τ, the entropy production asymptotically behaves as $\Sigma \simeq (5\tau/8)(1/2 - \alpha)\ln[(1-\alpha)/\alpha]$, which increases in proportion to τ. Thus, the first inequality (14) falls into a trivial bound in this case (i.e., $\tau_{\rm I} \to 0$ in $\tau \to \infty$ limit).

On the other hand, the HS entropy production is given by

$$\Sigma^{\rm HS} = \frac{1}{4\tau}\int_{2\tau}^{2\tau+1} dy \ln\left(1 + \frac{\tau}{y}\right), \tag{28}$$

which is exactly the same as the entropy production (27) in the single-bath case. This coincidence is suggestive since the HS entropy production is a natural generalization of the entropy production. The bound $\tau_{\rm II}$ on the operation time is given by

$$\tau_{\rm II} = \frac{c^*}{(10 + 2/\tau)\Sigma^{\rm HS}}, \tag{29}$$

which provides a meaningful bound even for large τ; an asymptotic bound $c^*/(10\Sigma^{\rm HS}) \le \tau$. See the plot in Fig. 1(b).

Discussion.—We have established fundamental trade-off inequalities (14) and (18) in general stochastic processes which claim that quick state transformation inevitably requires large entropy production or large Hatano-Sasa entropy production. This shows clear contrast to the case of an isolated quantum system, where the energy fluctuation plays a role to bound the speed of the state transformation. The coefficient appearing in these inequalities is the dynamical activity, which determines the time scale of dynamics. These speed limit inequalities are demonstrated in a simple toy model. In the equilibrium condition, the first inequality (14) provides a very good bound, while if a stationary current exists, only the second inequality (18) provides a nontrivial bound in the long-time limit. We remark that the equalities in these inequalities are hard to hold in general setups even in systems with a single heat bath.

Since nonequilibrium thermodynamics for quantum Markov processes is also developed [38–42], and our main idea is applicable to such systems [35,43], it is natural to ask the extension of our result to open quantum systems. Although some technical difficulties exist, it is worth investigating this extension and comparing the speed limits extended from that for isolated quantum systems [10–14].

Probabilistic systems with discrete states are ubiquitous in nature, such as proteins in biosystems [44,45] and quantum dots in the classical regime [46,47], to name only a few. Thus, our inequalities can be experimentally tested at the quantitative level. In particular, quantum-dot systems have high-controllability with high accuracy nowadays [46–50], and are the most promising experimental objects.

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$$\begin{aligned}\int_0^\tau dt \ln\left(\frac{t+2\tau+1}{t+2\tau}\right) &= \int_{2\tau+1}^{3\tau+1} dx \ln x - \int_{2\tau}^{3\tau} dx \ln x \\ &= \int_{3\tau}^{3\tau+1} dx \ln x - \int_{2\tau}^{2\tau+1} dx \ln x \\ &= \int_{2\tau}^{2\tau+1} dy \ln\left(\frac{y+\tau}{y}\right)\end{aligned}$$

directly leads to the desired expression.

[37] The explicit form of the entropy production is

$$\begin{aligned}\Sigma = \frac{1}{4\tau}\int_{2\tau}^{2\tau+1} dy \ln\left(1+\frac{\tau}{y}\right) - \frac{1}{8}\ln[4\alpha(1-\alpha)] \\ + \frac{5\tau}{8}\left(\frac{1}{2}-\alpha\right)\ln\frac{1-\alpha}{\alpha},\end{aligned}$$

where the second and third terms newly appear compared to the single-bath case.

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