Robust Spin Squeezing via Photon-Mediated Interactions on an Optical Clock Transition

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Cavity QED is a promising avenue for the deterministic generation of entangled and spin-squeezed states for quantum metrology. One archetypal scheme generates squeezing via collective one-axis twisting interactions. However, we show that in implementations using optical transitions in long-lived atoms the achievable squeezing is fundamentally limited by collectively enhanced emission into the cavity mode which is generated in parallel with the cavity-mediated spin-spin interactions. We propose an alternative scheme which generates a squeezed state that is protected from collective emission, and investigate its sensitivity to realistic sources of experimental noise and imperfections.

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Introduction—Atomic clocks operated with long-lived optically excited states in large ensembles of alkaline-earth atoms have led to unprecendented advances in frequency and time standards [1–6]. This has been achieved both by taking advantage of the superior precision afforded by operating at optical rather than microwave frequencies, and by utilizing large numbers of atoms N to quickly average down quantum projection noise. However, these clocks are reaching a point where improvements in sensing capabilities based on individual particle control have limited return due to physical and practical constraints, such as difficulty increasing the number of participating atoms due to collisional shifts [7]. This presents a clear need for a new paradigm of sensors that utilize many-particle quantum correlations [8], dramatically reducing quantum noise and breaking through the standard quantum limit (SQL) on phase sensitivity, $\delta\phi \sim 1/\sqrt{N}$ rad. However, quantum correlations are difficult to create and intrinsically fragile to decoherence, and therefore the design and implementation of robust methods for entanglement generation is an important current challenge for quantum-enhanced sensors, particularly the next-generation of atomic clocks.

A canonical example of metrologically useful entanglement is squeezed states [9,10], which feature a reduction of the quantum projection noise along a particular quadrature. In atomic ensembles, spin-squeezed states have successfully been generated in proof-of-principle systems that operate on microwave-frequency transitions by projective measurement and feedback protocols [11–16], with state-of-the-art schemes reaching ~18 dB below SQL [15,16]. Deterministic production of spin-squeezed states generated by one-axis twisting (OAT) schemes has also been demonstrated on microwave transitions [17–19]. However, the best reported squeezing remains limited at 8 dB below SQL [18]. It is then desirable to understand how entanglement generated by unitary dynamics can be significantly improved via protocols applicable to optical transitions used in current state-of-the-art atomic clocks.

In this vein, recent work demonstrated the possibility of using photon-mediated spin-exchange interactions to engineer OAT in an undriven optical cavity [20,21]. The scheme is relevant to the dynamical generation of spin-squeezed states directly on the optical clock transition. However, the achievable squeezing is severely limited by intrinsic dissipative noise arising due to superradiance: the collective emission and leakage of photons from the cavity.

In this Letter, we propose to overcome this problem by generating squeezing from an unorthodox initial state composed of a pair of spin ensembles with zero mean total spin projection. This protocol, which we refer to as 'two-spin squeezing' (TSS), generates squeezing in an almost orthogonal quadrature to the noise arising from superradiance. Our theoretical calculations demonstrate this leads to robustness to collective emission and the TSS scheme consequently outperforms the conventional OAT protocol. We also examine the performance of TSS when typical single-particle decoherence is included.

Model and definitions.—We consider a system of N atoms trapped in a standing-wave optical lattice which is supported by an optical cavity [20], illustrated in Fig. 1(a). The cavity field couples the atom's ground and excited clock states with single-photon Rabi frequency $2g$. For simplicity, we assume the atom-light coupling is spatially uniform, which could be achieved by selective loading of atoms in the spatial lattice or alternatively by using a ring cavity. We describe the atomic ensemble using collective spin operators $\hat{S}^{x,y,z}_{\alpha} \equiv \sum_j \hat{\sigma}^{x,y,z}_{j,\alpha}/2$, where $\hat{\sigma}^{x,y,z}_{j,\alpha}$ denote Pauli matrices. The summation of j runs over the atomic ensemble and α indexes internal degrees of freedom, e.g., hyperfine levels.

The narrow linewidth of the clock transition γ, relative to the cavity linewidth $\kappa \gg \gamma$, allows us to adiabatically

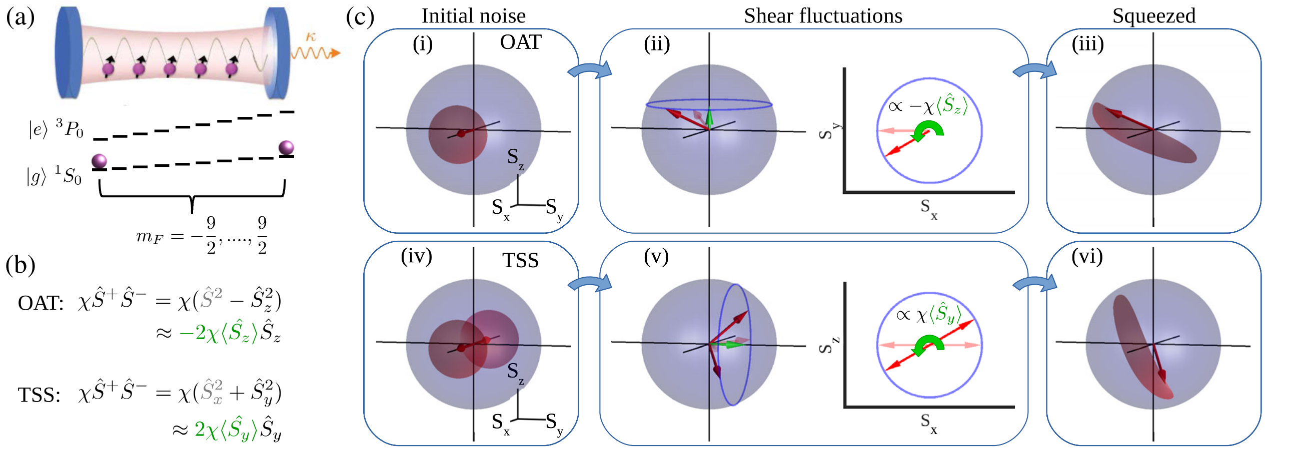

FIG. 1. (a) Proposed experimental system: An ensemble of N atoms trapped in a standing-wave lattice potential and optically coupled to the field within an optical cavity of linewidth κ. Selective population of the m_F levels of the $^1S_0$ to $^3P_0$ transition of ^{87}Sr [20] realizes independently controllable collective spins. (b) For both OAT and TSS, the Hamiltonian can be decomposed into a shearing term, $\hat{S}_z^2$ (OAT) or $\hat{S}_y^2$ (TSS), and negligible terms (gray) which do not contribute to dynamics. The shearing terms can be interpreted semiclassically as a precession driven by quantum fluctuations (green). (c) Schematic of squeezing protocols. (i)–(iii) For OAT, fluctuations along S_z (example shown by green vector) drive a precession of the Bloch vector $\vec{S}$ (red, precession indicated in blue) about the S_z axis, generating squeezing of the noise distribution [red in (i) and (iii)]. (iv)–(vi) For TSS, two back-to-back collective spins are prepared [(iv) light red], and common-mode fluctuations along S_y [(v) light red] generate a weak coherence along S_y (green), driving the precession of the Bloch vectors $\vec{S}_{1,2}$ (solid red, precession indicated in blue). Subsequent rotation about S_y of one of the spins maps this precession into squeezing of the collective distribution [(vi) red].

eliminate the cavity field such that the photons only mediate effective spin dynamics [20]. The reduced density matrix of the atomic spin then evolves according to the effective Hamiltonian [22],

$$\hat{H}_{\text{eff}} = \hbar\sum_{\alpha,\beta}\chi_{\alpha,\beta}\hat{S}_\alpha^+\hat{S}_\beta^-, \tag{1}$$

and the Lindblad jump operator $\hat{L}_\Gamma = \sum_\alpha \sqrt{\Gamma_\alpha/2}\,\hat{S}_\alpha^-$, where $\hat{S}_\alpha^\pm \equiv \hat{S}_\alpha^x + i\hat{S}_\alpha^y$ describes collectively enhanced emission into the cavity. The relative strength of the interactions $\chi_{\alpha,\beta} = 4g_\alpha g_\beta\Delta_c/(4\Delta_c^2 + \kappa^2)$ and dissipation $\Gamma_\alpha = 4g_\alpha^2\kappa/(4\Delta_c^2 + \kappa^2)$ is controlled by the detuning Δ_c of the cavity from resonance with the atomic transition. Throughout the Letter we now set $\hbar = 1$.

In terms of spin operators, squeezing is characterized by the parameter $\xi^2 \equiv N\min[\langle(\delta\hat{S}_\psi)^2\rangle]/|\langle\vec{S}\rangle|^2$ [29], where $\min[\langle(\delta\hat{S}_\psi)^2\rangle]$ is the minimal variance of the state along a direction $\hat{n}_\psi$ perpendicular to $\langle\vec{S}\rangle$ (i.e., $\hat{n}_\psi\cdot\langle\vec{S}\rangle = 0$ and $\langle(\delta\hat{S}_\psi)^2\rangle \equiv \langle(\hat{n}_\psi\cdot\hat{S}_\psi)^2\rangle - \langle\hat{n}_\psi\cdot\hat{S}_\psi\rangle^2$). Squeezing $\xi^2 < 1$ indicates that the quantum noise of the state along one quadrature is reduced below the SQL (i.e., a coherent spin state).

Spin squeezing by OAT.—When a single internal level is populated, the effective Hamiltonian reduces to $\hat{H}_{\text{eff}} = \chi\hat{S}^+\hat{S}^-$, which can be rewritten as $\hat{H}_{\text{eff}} \equiv \chi(\hat{S}^2 - \hat{S}_z^2 + \hat{S}_z)$. The term $\propto \hat{S}_z^2$ generates OAT [10] while the last term generates a trivial single-particle rotation which is neglected herein. The first term $\propto \hat{S}^2 \equiv \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2$ commutes with the OAT, but is responsible for opening a many-body gap between Dicke manifolds with different eigenvalues $S(S+1)$ $(S = 0, \ldots, N/2)$ of $\hat{S}^2$, which can protect the collective dynamics from slow single-particle decoherence [20]. Here, we assume the unitary dynamics is restricted to the $S = N/2$ manifold and thus consider $\hat{H}_{\text{eff}}$ equivalent to the OAT Hamiltonian $\hat{H}_{\text{OAT}} = \chi\hat{S}_z^2$.

The OAT can be understood semiclassically, illustrated in Fig. 1(b), in terms of the mean-field Hamiltonian $\hat{H}_{\text{MF}} \equiv 2\chi\langle\hat{S}_z\rangle\hat{S}_z$, which generates rotations about the S_z axis at a rate dependent on the atomic inversion. Under $\hat{H}_{\text{MF}}$ the isotropic noise distribution of an initial spin coherent state along x, $|\Phi^{\text{OAT}}\rangle = |N/2\rangle_x$ with $\hat{S}_x|N/2\rangle_x = N/2|N/2\rangle_x$, shears into an anisotropic distribution with reduced noise along one quadrature and increased noise along the other. As the spin-spin interactions responsible for the OAT dynamics are mediated by a macroscopically populated cavity field, they are also accompanied by superradiant collective emission from the cavity mode. Leakage of photons from the cavity carries away information at the rate $\kappa\langle\hat{a}^\dagger\hat{a}\rangle \sim \kappa N^2$, and correspondingly introduces excess dissipative noise $\propto N^2\Gamma t$ to the S_z quadrature, degrading the spin squeezing.

A perturbative treatment of both collective emission and OAT leads to an expression for the time evolution of the squeezing [21,22],

$$\xi^2_{\mathrm{OAT}} \approx \frac{1}{2N\beta} + \frac{2}{3}\beta^2 + \Gamma N t, \tag{2}$$

where $\beta \equiv N\chi^2 t^2/2 \ll 1$. The term $\propto 1/\beta$ describes the squeezing, while the term $\propto \beta^2$ describes oversqueezing due to the curvature of the Bloch sphere that yields a non-Gaussian distribution. The last term $\Gamma N t$ describes the collectively enhanced dissipative noise added to the squeezed quadrature. For $\Gamma = 0$, the optimal OAT squeezing is limited only by the non-Gaussian corrections, which are reached when $\beta^2 \sim 1/(N\beta)$ and for $N \gg 1$ scales as $t \sim N^{-2/3}$ [10]. In contrast, superradiant emission, $\Gamma \neq 0$, limits the optimal squeezing when $\Gamma N t \sim (1/N\beta)$ (typically a much shorter timescale than non-Gaussian effects). The impact of superradiance is clear if one minimizes Eq. (2) with respect to t, for fixed cavity parameter Γ/χ. In this case, ignoring the negligible term $\propto \beta^2$ in Eq. (2), the squeezing is bounded by

$$\xi^2_{\mathrm{OAT}}|_{\Gamma,\chi} \approx \frac{3}{2^{2/3}}\left(\frac{\Gamma}{\chi}\right)^{2/3}, \tag{3}$$

independent of atom number N.

Two-spin squeezing (TSS).—We now consider the alternative scheme and demonstrate how initiating the dynamics from a state of zero mean coherence maps to an effective Hamiltonian, which generates squeezing that is robust to collective emission. Initially the atoms are separated into two ensembles, denoted as $\alpha = 1, 2$, each composed of $N/2$ atoms, which are prepared in an incoherent state of two opposing collective spins: $|\Phi^{\mathrm{TSS}}\rangle = |N/4\rangle_{x_1} \otimes |-N/4\rangle_{x_2}$, each in a stretched eigenstate of $\hat{S}^x_{j=1,2}$, $\hat{S}^x_j|\pm N/4\rangle_{x_j} \equiv \pm(N/4)|\pm N/4\rangle_{x_j}$. This state, which could be realized using the $m_F = \pm 9/2$ hyperfine levels of ^{87}Sr [20], has coherence $\langle \hat{S}^+\rangle = 0$ and $\langle \hat{S}^+\hat{S}^-\rangle = N/2$, and as such the cavity occupation $\langle \hat{a}^\dagger \hat{a}\rangle \propto \langle \hat{S}^+\hat{S}^-\rangle \sim N$ is reduced by a factor of N to that of N independent emitting atoms. For clarity, the total collective spin operators are $\hat{S}^\alpha \equiv \hat{S}^\alpha_1 + \hat{S}^\alpha_2$ for $\alpha = x, y, z$, where the subscript denotes the ensemble internal degree of freedom.

Even though the initial state has zero mean coherence it will nevertheless nontrivially evolve under the Hamiltonian Eq. (1), $\hat{H}_{\mathrm{eff}} = \chi\hat{S}^+\hat{S}^- \equiv \chi(\hat{S}^+_1 + \hat{S}^+_2)(\hat{S}^-_1 + \hat{S}^-_2)$. Here, we assume $\chi_{\alpha,\beta} \equiv \chi$, satisfied for $m_F = \pm 9/2$ or any "symmetric" pair of hyperfine levels $\pm m_F$. To reveal squeezing, after the dynamics we perform a local spin-flip rotation of the 2nd collective spin about $\hat{y}$. For short evolution under $\hat{H}_{\mathrm{eff}}$, which is what we consider in the following, the evolved state is approximately transferred back to the fully symmetric manifold $S = N/2$ (see later discussion and Ref. [22]). For example, in the absence of evolution under $\hat{H}_{\mathrm{eff}}$ the final state would be $|N/2\rangle_x$.

The overall protocol can be recast as evolution under a Hamiltonian in a rotated reference frame, acting on an initially collective state ($S = N/2$) with all spins aligned together along $\hat{x}$:

$$|\psi(t)\rangle = \hat{R}^y_2(-\pi)e^{-i\hat{H}_{\mathrm{eff}}t}\hat{R}^y_2(\pi)|N/2\rangle_x \equiv e^{-i\hat{\tilde{H}}t}|N/2\rangle_x.$$

Here, $\hat{R}^y_j(\phi) = e^{-i\phi\hat{S}^y_j}$ is a collective rotation acting on the $j = 1, 2$ internal state and $\hat{\tilde{H}} \equiv \chi[(\hat{S}^x_1 - \hat{S}^x_2)^2 + (\hat{S}^y_1 + \hat{S}^y_2)^2]$. The second term of $\hat{\tilde{H}}$ induces OAT about the $\hat{y}$ axis, leading to an approximate azimuthally ("phase") squeezed state. The first term is more complex and can lead to degradation of squeezing. However, as the initial state in this reference frame is an eigenstate of $\hat{S}^x_1$ and $\hat{S}^x_2$, then at the relevant short timescale of squeezing this term can be ignored and the dynamics essentially remains in the $S = N/2$ manifold. Our scheme is not overly sensitive to this assumption and can tolerate number fluctuations $\lesssim N^{1/3}$ in the prepared ensembles [22].

Physical intuition is gained by a semiclassical description in the original frame of the back-to-back spins $|\Phi^{\mathrm{TSS}}\rangle$, illustrated in Fig. 1(c). For this initial state [panel (iv)], the mean-field Hamiltonian corresponds to a precession of each of the individual spins about the S_y projection of the total collective spin, $\hat{H}_{\mathrm{MF}} \approx 2\chi\langle\hat{S}^y_1 + \hat{S}^y_2\rangle(\hat{S}^y_1 + \hat{S}^y_2)$. Shearing is induced by common-mode fluctuations of the initial states $\sim\sqrt{N}$ along S_y (i.e., phase noise), which generate a weak coherence about which the opposing classical Bloch vectors precess [panel (v)]. After application of the π pulse to the 2nd collective spin this precession yields net shearing of the collective ensemble about $\hat{y}$ [panel (vi)].

In contrast to OAT, the TSS dynamics are generated by a cavity field with a $\sqrt{N}$-fold reduced amplitude—a consequence of the field being induced by quantum fluctuations of the atomic coherence $\langle\hat{a}\rangle_{\mathrm{TSS}} \sim \sqrt{\langle\hat{S}^2_y\rangle} \sim \sqrt{N}$. Regardless of this relatively weak cavity field, the TSS protocol achieves a similar level of shearing relative to OAT. This is reconciled by understanding that in OAT the Bloch vector precesses at a rate $\propto N$—related to the cavity field amplitude $\langle\hat{a}\rangle_{\mathrm{OAT}} \sim \langle\hat{S}^+\rangle \sim N$—about a rotation axis that is nearly aligned to said Bloch vector, up to fluctuations associated with atomic projection noise $\propto \sqrt{N}$. Conversely, in TSS the rotation is slower by a factor of $\sqrt{N}$ relative to OAT. However, the Bloch vectors associated with the individual ensembles are nearly perpendicular to the rotation axis. Thus, in TSS the component of the Bloch vectors perpendicular to the axis of rotation is $\sqrt{N}$ larger than OAT, compensating for the $\sqrt{N}$ smaller cavity field compared to OAT.

We now shift our focus to explaining why TSS is robust against collective emission. While the reduced atomic coherence of the initial state leads to a reduction in the rate of photon leakage from the cavity, $\propto \kappa N$, dissipative noise is still added to the S_z quadrature of the final state at a rate identical to OAT, $\propto N^2\Gamma t$. This is reconciled by noting that the S_z quadrature of the measured state *after* the rotation about $\hat{y}$ actually corresponds to the inversion difference $\hat{S}_1^z - \hat{S}_2^z$ during the squeezing dynamics. In contrast to OAT, the dissipative noise in the measured S_z quadrature is then not driven by the usual atomic coherence $\langle \hat{S}^+\hat{S}^-\rangle$ (i.e., cavity occupation $\langle \hat{a}^\dagger \hat{a}\rangle$), but rather the *differential* atomic coherence $\langle (\hat{S}_1^+ - \hat{S}_2^+)(\hat{S}_1^- - \hat{S}_2^-)\rangle \sim N^2$. While this dissipative noise thus remains large, unlike OAT it now contributes predominantly to the *antisqueezed* quadrature of a *phase-squeezed* state. We illustrate this contrast to OAT in Fig. 2(b). We emphasize that the reduced atomic coherence $\langle \hat{S}^+\hat{S}^-\rangle$ and suppression of superradiance remains an important ingredient for TSS: Superradiant decay of the atomic inversion would generate fluctuations in $\hat{S}_x$ and thus degrade correlations via the term $\chi(\hat{S}_1^x - \hat{S}_2^x)^2$ in $\hat{\tilde{H}}$.

A perturbative treatment leads to the approximate expression for the squeezing parameter [22]:

$$\xi^2_{\mathrm{TSS}} \approx \frac{1+\Gamma N t}{2N\beta} + \frac{14}{9}\beta^2. \tag{4}$$

The contrast to OAT is signaled by the suppression of the dissipative noise by the prefactor $\sim 1/(N\beta)$, reflecting that it is added predominantly to the antisqueezed rather than squeezed quadrature. Importantly, this means that for TSS the optimal squeezing essentially remains limited only by the emergence of non-Gaussian corrections to the distribution $\sim\beta^2$. Optimizing the squeezing with respect to time for fixed cavity parameters, Eq. (4) leads in this case to

$$\xi^2_{\mathrm{TSS}}|_{\Gamma,\chi} \approx \frac{21^{1/3}}{2N^{2/3}} + \frac{7^{1/6}\Gamma}{3^{1/3}\chi N^{1/3}}. \tag{5}$$

Comparing to OAT, the key difference is that collective decoherence does not lead to a lower bound on squeezing. For large N we thus find that TSS scales with atom number as $\xi^2_{\mathrm{TSS}}|_{\Gamma,\chi} \propto N^{-1/3}$.

We directly compare the optimal squeezing generated by OAT and TSS for the case of $N = 1000$ in Fig. 2. Results are based on numerical solution of the master equation taking into account all relevant secondary effects, including decay of the spin-length $|\langle \vec{S}\rangle|$ due to quantum fluctuations and noncollective terms $\propto (\hat{S}_1^x - \hat{S}_2^x)^2$ in the TSS Hamiltonian $\hat{\tilde{H}}$. The validity thus goes beyond the perturbative analysis of Eqs. (2) and (4). They confirm that, unlike OAT, suppression of collective emission in TSS leads to squeezing limited by non-Gaussian corrections to the spin distribution at relatively long times. The strikingly different impact of superradiance in the schemes is illustrated in the inset, where we plot the numerically obtained optimal squeezing as a function of Γ/χ.

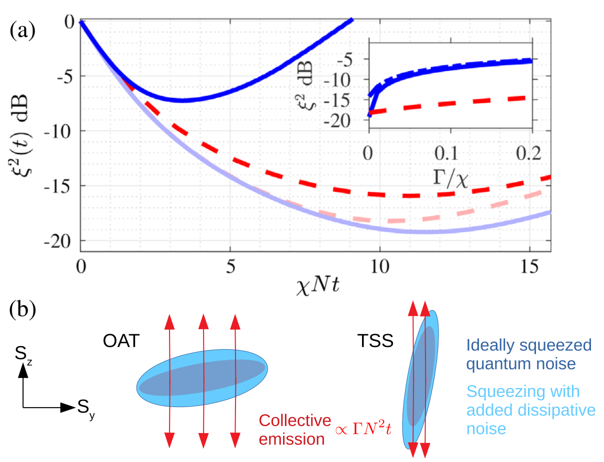

FIG. 2. (a) Comparison of squeezing with OAT and TSS protocols for $N = 1000$. Faded lines are ideal $\Gamma = 0$ results for OAT (blue) and TSS (red dashed). Solid lines are $\Gamma = 0.1\chi$ results for OAT (blue) and TSS (red dashed). Shown inset is the best squeezing as a function of Γ/χ (OAT: solid blue, TSS: dashed red). Dot-dashed blue line indicates squeezing for OAT and $N = 200$, illustrating the invariance with N. (b) Dark region indicates ideal squeezed state, light region indicates distribution with added dissipative noise due to photon leakage from the cavity (arrows), which approximately aligns with the squeezed quadrature or antisqueezed quadrature.

Sensitivity to single-particle decoherence.—Instead of operating at fixed $\Gamma/\chi \equiv \kappa/\Delta_c$ one could, in principle, remove the detrimental effect of superradiance in OAT by operating at a large detuning Δ_c. However, in reality under this condition the generation of squeezing will become sufficiently slow that other external and technical noise sources become the limiting factors for metrological sensitivity. In this vein, we now include relevant single-particle decoherence mechanisms which typically can be characterized in terms of the single-particle jump operators $\hat{L}_j^s = \sqrt{\gamma_s/2}\hat{\sigma}_j^-$ (describing, e.g., spontaneous emission or Raman light scattering) and $\hat{L}_j^{el} = \sqrt{\gamma_{el}/8}\hat{\sigma}_j^z$ (describing, e.g., Rayleigh scattering and dephasing from stray fields or collisions).

By treating single-particle and collective emission perturbatively [22] we obtain the approximate expressions

$$\xi^2_{\mathrm{TSS},\gamma_s} \approx \frac{1+\Gamma N t}{2N\beta} + \gamma_s t, \qquad \xi^2_{\mathrm{OAT},\gamma_s} \approx \frac{1}{2N\beta} + \Gamma N t + \gamma_s t,$$

where we ignore the terms $\propto \beta^2$ as irrelevant compared to the dissipative contribution. The clear difference here is that the squeezing achievable via OAT is limited by *both* collective and single-particle emission, whereas TSS suppresses the collective component. One then expects that squeezing can be generated faster with TSS to minimize single-particle decoherence, as collective decoherence is not the most relevant limitation. This is supported by optimising the achievable squeezing by varying the cavity detuning Δ_c, yielding the superior $\xi^2_{\mathrm{TSS}}|_{\gamma_s} \approx \sqrt{24/(N\eta_s)}$ when compared to $\xi^2_{\mathrm{OAT}}|_{\gamma_s} \approx 6(N\eta_s)^{-1/3}$ where $\eta_s = 4g^2/(\kappa\gamma_s)$ is an effective cavity cooperativity. We note that the scaling achievable with TSS is equivalent to that predicted by a twist-and-turn protocol in Ref. [21]. Nevertheless, this scheme requires an additional continuous drive, in contrast to TSS.

However, the robustness of TSS to collective emission comes at a tradeoff to increased sensitivity to single-particle dephasing. This decoherence intuitively adds excess noise to the squeezed quadrature of the phase-squeezed state, whereas for OAT it only contributes to the antisqueezed quadrature. The time evolution of the squeezing for $\gamma_{el}t \ll 1$ is approximately [22],

$$\xi^2_{\mathrm{TSS},\gamma_{el}} \approx \frac{1+\Gamma Nt}{2N\beta} + \gamma_{el}t, \qquad \xi^2_{\mathrm{OAT},\gamma_{el}} \approx \frac{1+2\gamma_{el}t}{2N\beta} + \Gamma Nt.$$

Here, the effectiveness of TSS and OAT become similar as the role of single-particle dephasing and collective emission is interchangeable between the schemes. This is reflected by again optimizing the cavity detuning, which yields $\xi^2 \sim (N\eta_{el})^{-1/2}$ for both protocols where $\eta_{el} = 4g^2/(\kappa\gamma_{el})$ [22]. It is thus clear that TSS is superior over OAT in the limit where single-particle decoherence is dominated by spontaneous emission, $\gamma_s \gg \gamma_{el}$. Such a regime is relevant as γ_s and Γ are fundamental sources of decoherence, due to the finite transition linewidth and engineering of the squeezing Hamiltonian respectively, whereas γ_{el} is a technical barrier.

Conclusion.—We have proposed a squeezing protocol which is intrinsically robust against the detrimental effects of superradiance and thus particularly useful for the next generation of quantum enhanced optical atomic clocks. Its implementation could open a path to deliver significant gains to sensors with real-world applications.

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