

Relativistic Quantum Dynamics of Twisted Electron Beams in Arbitrary Electric and Magnetic Fields

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Relativistic quantum dynamics of twisted (vortex) Dirac particles in arbitrary electric and magnetic fields are constructed for the first time. This allows us to change the controversial contemporary situation when the nonrelativistic approximation is used for relativistic twisted electrons. The relativistic Hamiltonian and equations of motion in the Foldy-Wouthuysen representation are derived. A critical experiment for a verification of the results obtained is proposed. The new important effect of a radiative orbital polarization of a twisted electron beam in a magnetic field resulting in a nonzero average projection of the intrinsic orbital angular momentum on the field direction is predicted.

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The discovery of twisted (vortex) electron beams carrying an intrinsic orbital angular momentum (OAM) [1], whose existence has been predicted in Ref. [2], opens new possibilities in electron microscopy and in investigations of magnetic phenomena (see Refs. [3–10] and references therein). Twisted electron beams with large intrinsic OAMs (up to $1000\hbar$) have been recently obtained [11]. At present, much attention is also devoted to the interactions of such beams with atoms, nuclei, and a laser field [12]. The dynamics of the intrinsic OAM in external magnetic and electric fields has been studied in Refs. [2,13–18]. However, we note in advance that our result is different from the equation of motion of the intrinsic OAM in an electric field previously found in Ref. [2]. The general description of the relativistic dynamics of an intrinsic OAM in arbitrary electric and magnetic fields in the framework of classical physics has been made in Ref. [19]. In particular, the correction of the equation of motion of the intrinsic OAM in the electric field previously obtained in Ref. [3] has been fulfilled in this work. The methods of the manipulation of beams developed in Ref. [19] use not only the magnetic field but also the electric one. The great importance of twisted electrons requires detailed investigations of fundamental quantum-mechanical properties of twisted electron beams. Such investigations have been carried out in many publications (see Refs. [3,4] and references therein). However, only the nonrelativistic approximation has been formerly used. The correct quantum dynamics of a twisted electron in an *electric* field has not been elaborated at all. Therefore, some fundamental properties need a further inquiry. The present Letter, using a quantum-mechanical approach, differs from our recent study [19], where the classical approach had been applied.

In the present Letter, the system of units $\hbar = 1$, $c = 1$ is used. We include \hbar and c explicitly when this inclusion clarifies the problem. The curly brackets, $\{\dots;\dots\}$, denote anticommutators.

Since vortex electrons are relativistic quantum objects admitting also a semiclassical description, a construction of a *relativistic* Schrödinger-like dynamics of such particles is necessary. It is important that one mainly observes a motion of charged centroids, and it is instructive to mirror this circumstance in an appropriate quantum description. We solve this problem in the present Letter. We should also notice the previous quantum-mechanical analysis mirrored in the reviews [3,4].

A twisted electron is a single pointlike particle. However, its wave function is a superposition of states with different momentum directions. While the twisted electron in a vacuum has a nonzero intrinsic OAM and a nonzero component of the momentum in the plane orthogonal to the direction of its resulting motion, it can be described by the Dirac equation for a free particle. Quantum mechanics (QM) of the twisted electron in external electromagnetic fields are also governed by the usual Dirac equation. We can disregard the anomalous magnetic moment of the electron because its g factor is close to 2. The Schrödinger form of the relativistic QM is provided by the relativistic Foldy-Wouthuysen (FW) transformation. The relativistic extensions of the FW method [20] were first proposed in Refs. [21,22]. There are many other methods of the relativistic FW transformation as well (see Refs. [23–30] and references therein). The results obtained by different methods agree because of the uniqueness of the FW representation proven in Ref. [31]. In all previous studies of twisted particles in external fields, only the nonrelativistic

FW transformation has been fulfilled (see the reviews [3,4]). The relativistic FW transformation has been formerly used only for free twisted particles [32,33]. Thus, the nonrelativistic approximation has been used for a description of relativistic objects (the kinetic energy of twisted electrons, 200–300 keV, is comparable with their rest energy, 511 keV). As a result, the present state of the theory of twisted electron beams is controversial.

The *exact* relativistic Hamiltonian in the FW representation (the FW Hamiltonian) for a Dirac particle in a magnetic field was first obtained in Ref. [21], and its validity has been confirmed in other works [25,34,35]. It is given by

$$\mathcal{H}_{FW} = \beta \sqrt{m^2 + \boldsymbol{\pi}^2} - e \boldsymbol{\Sigma} \cdot \mathbf{B}, \quad (1)$$

where $\boldsymbol{\pi} = \mathbf{p} - e\mathbf{A}$ is the kinetic momentum, $\mathbf{B} = \nabla \times \mathbf{A}$ is the magnetic induction, and β and $\boldsymbol{\Sigma}$ are the Dirac matrices. This Hamiltonian is valid for a twisted and a untwisted particle. The spin angular momentum operator is equal to $s = \hbar \boldsymbol{\Sigma} / 2$. The magnetic field is, in general, nonuniform but time independent. Unlike all previous investigations, our quantum description of a twisted Dirac particle is fully relativistic and correctly defines both electric and magnetic interactions. Our preceding study [19] has shown the importance of the electric field for the manipulation of twisted electron beams.

The FW Hamiltonian acts on the bispinor wave function $\Psi_{FW} = \begin{pmatrix} \phi \\ 0 \end{pmatrix}$. Since both the nonrelativistic and the relativistic FW Hamiltonians commute with the operators $\boldsymbol{\pi}^2$ and s_z , their eigenfunctions coincide. In the *uniform* magnetic field $\mathbf{B} = B\mathbf{e}_z$, they have the form of nondiffracting Laguerre-Gauss beams [13,36,37]. It is convenient to present the exact energy spectrum obtained by different methods [21,34,35,38–41] as follows:

$$E = \sqrt{m^2 + (2n + 1 + |l_z| + l_z + 2s_z)|e|B}, \quad (2)$$

where $n = 0, 1, 2, \dots$ is the radial quantum number and $l = \mathbf{r} \times \mathbf{p} = \mathbf{L} + \mathbf{L}^{(e)}$ is the total OAM operator being the sum of the intrinsic (\mathbf{L}) and extrinsic ($\mathbf{L}^{(e)}$) OAMs. The same energy spectrum but for the squared energy operator has been obtained in Refs. [15,16]. We consider the conventional canonical OAMs. The difference between the dynamics of the canonical and kinetic (mechanical) OAMs has been investigated in Ref. [42]. All energy levels of twisted particles in the uniform magnetic field belong to the Landau levels. The relativistic approach (unlike the nonrelativistic one) demonstrates that the Landau levels are not equidistant for any field strength. This property has not been mentioned in Refs. [15,16,21,34,35,38–41], while it has been noted in Ref. [39] that the energy spectrum becomes quasicontinuous when the quantum number n becomes very

large. A nonzero intrinsic OAM increases a degeneracy multiplicity of the Landau levels (cf. Refs. [3,13]).

As a rule, we can use the weak-field approximation and suppose that the de Broglie wavelength, \hbar/p , is much smaller than the characteristic size of the nonuniformity region of the external field. In this case, the Hamiltonian (1) takes the form

$$\begin{aligned} \mathcal{H}_{FW} &= \beta \sqrt{m^2 + \boldsymbol{\pi}^2} - \frac{e}{4} \left\{ \frac{1}{\epsilon'}, \boldsymbol{\Pi} \cdot \mathbf{B} \right\} \\ &= \beta \epsilon' - \beta \frac{e}{4} \left(\frac{1}{\epsilon'} (\mathbf{l} + \boldsymbol{\Sigma}) \cdot \mathbf{B} + \mathbf{B} \cdot (\mathbf{l} + \boldsymbol{\Sigma}) \frac{1}{\epsilon'} \right), \\ \epsilon' &= \sqrt{m^2 + \mathbf{p}^2}, \end{aligned} \quad (3)$$

where $[\epsilon', \mathbf{l}] = 0$. Equation (3) agrees with the result obtained in Ref. [37]. The small term $m\omega_L^2 r^2 / 2$ presented in Eq. (2) of Ref. [37] expresses the rotational energy of the centroid *in the magnetic field*. This term originates from $e^2 A^2 / (2m)$ and is omitted in the present Letter because it is proportional to B^2 .

It is necessary to take into account that a twisted electron is a charged centroid [2,3]. To describe observable quantum-mechanical effects, we need to present the Hamiltonian in terms of the centroid parameters. The centroid as a whole is characterized by the center-of-charge radius vector \mathbf{R} and by the kinetic momentum $\boldsymbol{\pi}' = \mathbf{P} - e\mathbf{A}(\mathbf{R})$, where $\mathbf{P} = -i\hbar\partial / (\partial \mathbf{R})$. The intrinsic motion is defined by the kinetic momentum $\boldsymbol{\pi}'' = \mathbf{p} - e[\mathbf{A}(\mathbf{r}) - \mathbf{A}(\mathbf{R})]$. Here $\mathbf{p} = -i\hbar\partial / (\partial \mathbf{r})$, $\mathbf{r} = \mathbf{r} - \mathbf{R}$, $\boldsymbol{\pi}' + \boldsymbol{\pi}'' = \boldsymbol{\pi}$, $\mathbf{P} + \mathbf{p} = \mathbf{p}$. Since

$$\mathbf{A}(\mathbf{r}) = \mathbf{A}(\mathbf{R}) + \frac{1}{2} \mathbf{B}(\mathbf{R}) \times \mathbf{r},$$

the operator $\boldsymbol{\pi}^2$ takes the form

$$\boldsymbol{\pi}^2 = \boldsymbol{\pi}'^2 + \mathbf{p}^2 - \frac{e}{2} [\mathbf{L} \cdot \mathbf{B}(\mathbf{R}) + \mathbf{B}(\mathbf{R}) \cdot \mathbf{L}] + \boldsymbol{\pi}' \cdot \boldsymbol{\pi}'' + \boldsymbol{\pi}'' \cdot \boldsymbol{\pi}'.$$

After summing over partial waves with different momentum directions, $\langle \boldsymbol{\pi}' \cdot \boldsymbol{\pi}'' + \boldsymbol{\pi}'' \cdot \boldsymbol{\pi}' \rangle = 0$. More precisely, the operator $\boldsymbol{\pi}' \cdot \boldsymbol{\pi}'' + \boldsymbol{\pi}'' \cdot \boldsymbol{\pi}'$ has zero expectation values for any eigenstates of the operator $\boldsymbol{\pi}^2$ and, therefore, it can be omitted. It can be added that this summing can be performed for the squared Hamiltonian \mathcal{H}_{FW}^2 .

The FW Hamiltonian summed over the partial waves [3] takes the form

$$\begin{aligned} \mathcal{H}_{FW} &= \beta \epsilon - \beta \frac{e}{4} \left[\frac{1}{\epsilon} \boldsymbol{\Lambda} \cdot \mathbf{B}(\mathbf{R}) + \mathbf{B}(\mathbf{R}) \cdot \boldsymbol{\Lambda} \frac{1}{\epsilon} \right], \\ \epsilon &= \sqrt{m^2 + \boldsymbol{\pi}'^2 + \mathbf{p}^2}, \quad \boldsymbol{\Lambda} = \mathbf{L} + \boldsymbol{\Sigma}. \end{aligned} \quad (4)$$

The momentum and the intrinsic OAM can have different mutual orientations in different Lorentz frames [19,43].

Such a geometry of twisted waves has been described in Ref. [44].

The acceleration of the twisted electron in a uniform magnetic field does not depend on Λ . However, such a dependence takes place in a nonuniform magnetic field due to the Stern-Gerlach-like force defined by the operator

$$\mathbf{F}_{\text{SGI}} = \beta \frac{e}{4} \left(\frac{1}{e} \nabla [\Lambda \cdot \mathbf{B}(\mathbf{R})] + \nabla [\mathbf{B}(\mathbf{R}) \cdot \Lambda] \frac{1}{e} \right), \quad (5)$$

where $\nabla \equiv \partial/(\partial \mathbf{R})$. This force is an analog of the Stern-Gerlach one affecting the spin but it is much (approximately, $2L$ times) stronger. The operators of the magnetic and electric dipole moments, $\boldsymbol{\mu}$ and \mathbf{d} , are defined by

$$\mathcal{H}_{\text{FW}}^{(\text{int})} = -\frac{1}{2} [\boldsymbol{\mu} \cdot \mathbf{B}(\mathbf{R}) + \mathbf{B}(\mathbf{R}) \cdot \boldsymbol{\mu} + \mathbf{d} \cdot \mathbf{E}(\mathbf{R}) + \mathbf{E}(\mathbf{R}) \cdot \mathbf{d}], \quad (6)$$

where $\mathcal{H}_{\text{FW}}^{(\text{int})}$ is the interaction Hamiltonian. The operator of the magnetic dipole moment of a moving centroid obtained from Eq. (4) is given by

$$\boldsymbol{\mu} = \beta \frac{e(\mathbf{L} + 2\mathbf{s})}{2e}. \quad (7)$$

This equation agrees with Refs. [15,19,45,46]. When the weak-field approximation is not used, it can be obtained that the magnetic dipole moment of a twisted particle in a magnetic field is proportional to the kinetic OAM $\mathbf{r} \times \boldsymbol{\pi}$ and is affected by the additional current. This current is proportional to $-eA$ and leads to a weak diamagnetic effect [2,3,47].

To perform a general quantum description of a twisted Dirac particle in external fields, we need to add terms dependent on the electric field. For a pointlike Dirac particle, these terms have been obtained in Refs. [24,29,48]. If one disregards spin effects, one needs to add only the term $e\Phi(\mathbf{r})$. It has been proven in Ref. [49] that the passage to the classical limit in the FW representation reduces to a replacement of the operators in quantum-mechanical Hamiltonians and equations of motion with the corresponding classical quantities. The motion of the intrinsic OAM causes the electric dipole moment \mathbf{d} which interaction with the electric field should be taken into account. Certainly, $\mathbf{d}^{(0)} = 0$ in the rest frame of the twisted electron. The results obtained in Refs. [19,50] show that in the classical limit

$$\mathbf{d} = \boldsymbol{\beta} \times \boldsymbol{\mu}, \quad \boldsymbol{\beta} = \frac{\mathbf{V}}{c} \equiv \frac{\dot{\mathbf{R}}}{c}. \quad (8)$$

The centroid velocity operator, \mathbf{V} , can be obtained from the FW Hamiltonian:

$$\mathbf{V} = i[\mathcal{H}_{\text{FW}}, \mathbf{R}] = \frac{\beta}{2} \left\{ \frac{1}{e}, \boldsymbol{\pi}' \right\}.$$

Since we use the weak-field approximation, we neglect the correction to this formula proportional to the electric field. A comparison with Refs. [24,29,48] allows us to obtain the formula for the operator of the electric dipole moment:

$$\begin{aligned} \mathbf{d} &= \beta \frac{e}{4} \left(\frac{1}{e} \boldsymbol{\beta} \times \mathbf{L} - \mathbf{L} \times \boldsymbol{\beta} \frac{1}{e} \right) \\ &= \frac{e}{4} \left(\frac{1}{e^2} \boldsymbol{\pi}' \times \mathbf{L} - \mathbf{L} \times \boldsymbol{\pi}' \frac{1}{e^2} \right). \end{aligned} \quad (9)$$

Thus, the general FW Hamiltonian for a relativistic twisted particle in electric and magnetic fields is given by

$$\begin{aligned} \mathcal{H}_{\text{FW}} &= \beta e + e\Phi - \beta \frac{e}{4} \left[\frac{1}{e} \mathbf{L} \cdot \mathbf{B}(\mathbf{R}) + \mathbf{B}(\mathbf{R}) \cdot \mathbf{L} \frac{1}{e} \right] \\ &+ \frac{e}{4} \left\{ \frac{1}{e^2} \mathbf{L} \cdot [\boldsymbol{\pi}' \times \mathbf{E}(\mathbf{R})] - [\mathbf{E}(\mathbf{R}) \times \boldsymbol{\pi}'] \cdot \mathbf{L} \frac{1}{e^2} \right\}. \end{aligned} \quad (10)$$

In this equation, spin effects are disregarded because they can be neglected on the condition that $L \gg 1$. The term $e\Phi$ does not include the interaction of the intrinsic OAM with the electric field.

Equation (10) exhaustively describes the quantum dynamics of the intrinsic OAM in the general case of a twisted Dirac particle in arbitrary electric and magnetic fields. In particular, the equation of motion of the intrinsic OAM has the form

$$\begin{aligned} \frac{d\mathbf{L}}{dt} &= i[\mathcal{H}_{\text{FW}}, \mathbf{L}] = \frac{1}{2} (\boldsymbol{\Omega} \times \mathbf{L} - \mathbf{L} \times \boldsymbol{\Omega}), \\ \boldsymbol{\Omega} &= -\beta \frac{e}{4} \left\{ \frac{1}{e}, \mathbf{B}(\mathbf{R}) \right\} \\ &+ \frac{e}{4} \left[\frac{1}{e^2} \boldsymbol{\pi}' \times \mathbf{E}(\mathbf{R}) - \mathbf{E}(\mathbf{R}) \times \boldsymbol{\pi}' \frac{1}{e^2} \right]. \end{aligned} \quad (11)$$

When $\mathbf{E}(\mathbf{R}) = 0$, we obtain the relativistic quantum-mechanical equation for the Larmor precession. A comparison with results obtained in the Dirac representation shows strengths of the FW one. In Refs. [15,16], a relativistic description of twisted electron beams in a uniform magnetic field has been given in the former representation. However, the corresponding equation of motion of the OAM has not been obtained in these works.

Another important problem is the relativistic quantum dynamics of the kinetic momentum. It is defined by the force operator:

$$\begin{aligned}
 \mathbf{F} &= \frac{d\boldsymbol{\pi}'}{dt} = \frac{\partial \boldsymbol{\pi}'}{\partial t} + i[\mathcal{H}_{\text{FW}}, \boldsymbol{\pi}'] \\
 &= e\mathbf{E}(\mathbf{R}) + \beta \frac{e}{4} \left\{ \frac{1}{\epsilon}, (\boldsymbol{\pi}' \times \mathbf{B}(\mathbf{R}) - \mathbf{B}(\mathbf{R}) \times \boldsymbol{\pi}') \right\} + \mathbf{F}_{\text{SGI}}.
 \end{aligned} \tag{12}$$

Beam splitting in nonuniform electric and magnetic fields is conditioned by the Stern-Gerlach-like force operator

$$\begin{aligned}
 \mathbf{F}_{\text{SGI}} &= \beta \frac{e}{4} \left\{ \frac{1}{\epsilon} \nabla[\mathbf{L} \cdot \mathbf{B}(\mathbf{R})] + \nabla[\mathbf{B}(\mathbf{R}) \cdot \mathbf{L}] \frac{1}{\epsilon} \right\} \\
 &\quad - \frac{e}{4} \left\{ \frac{1}{\epsilon^2} \nabla(\mathbf{L} \cdot [\boldsymbol{\pi}' \times \mathbf{E}(\mathbf{R})]) - \nabla([\mathbf{E}(\mathbf{R}) \times \boldsymbol{\pi}'] \cdot \mathbf{L}) \frac{1}{\epsilon^2} \right\}.
 \end{aligned} \tag{13}$$

The force is exerted to the center of charge of the centroid. Equations (5) and (13) demonstrate an importance of a general description which includes particle interactions with nonuniform fields. Like Eq. (5), Eq. (13) presents the Stern-Gerlach-like force in terms of the centroid parameters. It is important to mention that $\sqrt{m^2 + \mathbf{p}^2}$ is an effective mass of the twisted particle.

We can conclude that the quantum-mechanical equations obtained in this Letter agree with the corresponding classical results given in Ref. [19].

In the present Letter, we consider two important applications of the results obtained and describe new effects which experimental study allows one to ascertain fundamental properties of twisted electron beams.

Rotation of the intrinsic OAM in crossed electric and magnetic fields.—Previously fulfilled experiments used coherent superpositions of two twisted beams moving in a longitudinal magnetic field (z axis). Landau modes with equal amplitudes, the same radial index n , and opposite projections of the intrinsic longitudinal OAMs undergo the rotation with the Larmor frequency [3,13,37]. This rotation is similar to the Faraday effect in optics [37]. We propose a similar and a simpler experiment in crossed electric and magnetic fields satisfying the relation $\mathbf{E} = -\boldsymbol{\beta} \times \mathbf{B}$, where $\mathbf{E} \perp \mathbf{B} \perp \boldsymbol{\beta}$ and $\boldsymbol{\beta}$ is the normalized beam velocity. Such fields characterizing the Wien filter do not affect a beam trajectory. We suppose the fields \mathbf{E} and \mathbf{B} to be uniform.

In the considered case, the classical limit of the relativistic equation for the angular velocity of precession of the intrinsic OAM is given by

$$\boldsymbol{\Omega}^{(W)} = -\frac{e(m^2 + \mathbf{p}^2)}{2\epsilon^3} \mathbf{B}. \tag{14}$$

Since $\mathbf{p}^2 \ll m^2$, Eq. (14) agrees with the corresponding equation obtained in Ref. [19].

While the experiment proposed is similar to the above-mentioned experiment carried out in Refs. [13,37], it needs a simpler experimental setup. It is necessary to use a single

twisted electron beam possessing a standard orbital polarization collinear to the beam momentum (z axis). The direction of the magnetic field \mathbf{B} and the quasimagnetic one $\boldsymbol{\beta} \times \mathbf{E}$ is transversal (x axis). Therefore, the intrinsic OAM rotates in the yz plane with the angular frequency $\boldsymbol{\Omega}^{(W)}$ and reverses its direction with the angular frequency $2\boldsymbol{\Omega}^{(W)}$. Since twisted electron beams are relativistic, a quantitative verification of Eq. (14) can be fulfilled. Thus, the experiment proposed is a *critical experiment* for a verification of the main equations for the FW Hamiltonian and the intrinsic-OAM dynamics obtained in the present Letter.

Radiative orbital polarization of twisted electron beams in a magnetic field.—The radiation from twisted electrons is one of their fundamental properties. It is caused by their acceleration in external magnetic and electric fields and by the time dependence of their magnetic moments. The latter effect conditions the magnetic dipole radiation [51]. Its intensity is standardly much smaller than that of the electric dipole radiation due to the particle acceleration. The magnetic dipole radiation proportional to the intrinsic OAMs is important for the Cherenkov radiation and the transition one [51]. Since the particle motion in a magnetic field is accelerated, we can consider only the electric dipole radiation. For particles closed in storage rings, it is conditioned by radiative transitions between Landau levels and is called the synchrotron radiation. As a rule, magnetic focusing is used. We expect that a storage and an acceleration of twisted electrons in cyclotrons (electron rings) will be carried out in the near future. Evidently, these processes cannot vanish the intrinsic OAM due to the conservation of angular momentum. A natural process of an orbital depolarization is caused by a loss of the beam coherence and is not important for the considered problem. The initial orbital polarization of twisted electrons is longitudinal (z axis), while their expected final polarization is vertical and antiparallel to the main magnetic field (y axis). The beam incoherence does not influence the *vertical* orbital polarization.

We predict the new effect of a *radiative orbital polarization of twisted electron beams in a magnetic field*—the *orbital Sokolov-Ternov effect*. The well-known effect is the radiative spin polarization of electron or positron beams in storage rings caused by the synchrotron radiation (Sokolov-Ternov effect [39]). It consists in the radiative spin polarization that is acquired by unpolarized electrons, and it is opposite to the direction of the main magnetic field. The reason for the effect is a dependence of spin-flip transitions from the initial particle polarization. The standard analysis [39,52] can be extended on the twisted particles. It follows from the results obtained in Refs. [12,53] that quantum-electrodynamics effects are rather similar for twisted and untwisted particles. In particular, the amplitude of elastic scattering of two vortex electrons is well approximated by two plane-wave scattering amplitudes with different momentum transfers, which

interfere and give direct experimental access to the Coulomb phase [53].

However, an influence of the synchrotron radiation on the orbital polarization of twisted relativistic particles needs a detailed separate study. In this Letter, we restrict ourselves to a consideration of some aspects of this problem. First of all, we should note the evident similarity between interactions of the spin and the intrinsic OAM with the magnetic field [see Eq. (4)]. In particular, energies of stationary states depend on projections of the spin and the intrinsic OAM on the field direction. This similarity validates the existence of the effect of the radiative orbital polarization. As well as the radiative spin polarization, the corresponding orbital polarization acquired by unpolarized twisted electrons should be opposite to the direction of the main magnetic field. The effect is conditioned by transitions with a change of a projection of the intrinsic OAM. The probability of such transitions is large enough if the electron energy is not too small. Similarly to the spin polarization, the orbital one is observable when electrons are accelerated up to the energy of the order of 1 GeV. The acceleration can depolarize twisted electrons but cannot vanish L . During the process of the radiative polarization, the average energy of the electrons should be kept unchanged. For additional explanations, see Supplemental Material [54].

Thus, a discovery of the fundamental property of the radiative orbital polarization needs much higher energies than usual energies of twisted electron beams (about 300 keV). We consider this as a positive factor because the twisted (vortex) states of particles can play an important role in high-energy physics.

In this Letter, we have studied the special properties of relativistic twisted particles moving in nonuniform electric and magnetic fields. The general FW Hamiltonian has been derived and a Stern-Gerlach-like force has been presented in terms of the centroid parameters. Furthermore, a benchmark experiment has been proposed to confirm the dynamics of the intrinsic OAM of twisted electrons with a very simple experimental setup. At last, we have predicted a new orbital Sokolov-Ternov effect for a twisted electron beam in a magnetic field, which can be measured with a high-energy twisted electron beam. The present state of quantum mechanics of twisted electrons is controversial because it uses the nonrelativistic approximation for a description of relativistic objects. Our relativistic approach substantiates (and generalizes) the most of the previous results. We have correctly introduced the interaction of the intrinsic OAM with the electric field into quantum mechanical equations for the first time, and we have corrected the equation of its motion in the electric field previously obtained in Ref. [2]. The relativistic approach shows that the Landau levels in the uniform magnetic field are not equidistant.

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