

Topologically Protected Landau Level in the Vortex Lattice of a Weyl Superconductor

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The question whether the mixed phase of a gapless superconductor can support a Landau level is a celebrated problem in the context of d -wave superconductivity, with a negative answer: the scattering of the subgap excitations (massless Dirac fermions) by the vortex lattice obscures the Landau level quantization. Here we show that the same question has a positive answer for a Weyl superconductor: the chirality of the Weyl fermions protects the zeroth Landau level by means of a topological index theorem. As a result, the heat conductance parallel to the magnetic field has the universal value $G = \frac{1}{2}g_0\Phi/\Phi_0$, with Φ as the magnetic flux through the system, Φ_0 as the superconducting flux quantum, and g_0 as the thermal conductance quantum.

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Introduction.—In 1998, Gor'kov and Schrieffer [1] and Anderson [2] made the remarkable prediction that the excitation spectrum in the mixed phase of a high- T_c superconductor (with massless quasiparticles at nodal points of the d -wave pair potential) has the Landau levels of the relativistic Dirac equation. This was nearly a decade before the quantum Hall effect of massless electrons was measured in graphene [3,4], and it would have marked the first appearance in the solid state of a magnetic-field independent zeroth Landau level.

It did not turn out that way: the spatially varying supercurrent in the Abrikosov vortex lattice strongly scatters the quasiparticles [5], even if the vortices overlap and produce a uniform magnetic field. Since Franz and Tešanović [6], we know that the quasiparticles in the mixed phase of a d -wave superconductor retain the zero-field Dirac cone, the main effect of the magnetic field being a renormalization of the Fermi velocity [7–16]. Recent proposals [17–19] use strain to mimic the effect of a magnetic field in a d -wave superconductor without breaking time-reversal symmetry, but the coexistence of Landau levels and a vortex lattice has remained elusive.

Here we propose that Weyl superconductors can make it happen. A Weyl semimetal with induced s -wave superconductivity has massless nodal quasiparticles in a 3D Weyl cone [20,21], with the same linear dispersion as the 2D Dirac cone of a d -wave superconductor [22,23]. We compare the band structures in Fig. 1 (see Supplemental Material [24] for a detailed calculation), where the momentum follows a path through the magnetic Brillouin zone of Fig. 2. In zero magnetic field, the gapless nodal points at the Fermi level ($E = 0$) are qualitatively the same in both superconductors. But the response to a vortex lattice is fundamentally different: while in the d -wave superconductor, the dispersive Dirac cones persist, as expected [6], in

the Weyl superconductor, a zeroth Landau level appears that is completely dispersionless in the plane perpendicular to the magnetic field.

We will return to these numerical calculations later on, but first we want to explain why the zeroth Landau level in a Weyl superconductor is not broadened by the vortex lattice, as it is in a d -wave superconductor. We have traced the origin of the difference to the topological protection of the zero mode enforced by an index theorem for Hamiltonians with chiral symmetry [34]. For this explanation, we will use an effective low-energy Hamiltonian. The numerics uses the full Hamiltonian and serves as a test of our analytics. We conclude with a discussion of the universal thermal conductance supported by the zero mode.

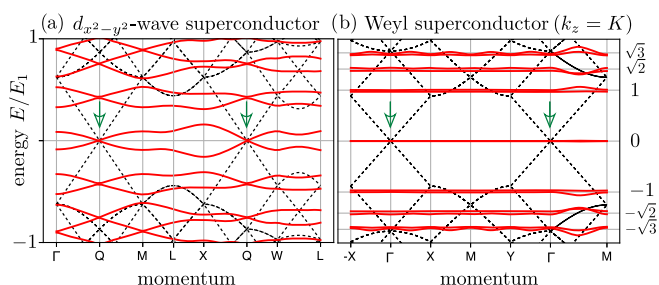


FIG. 1. Excitation spectrum of a nodal superconductor in zero magnetic field (black dashed curves) and in the mixed phase with a square lattice of Abrikosov vortices (red solid curves) [33]. (a) 2D d -wave superconductor. (b) 3D Weyl superconductor (with $k_z = \pi/3$ at the Weyl point). The momentum follows a path through the magnetic Brillouin zone of Fig. 2. The location of the zero-field Dirac and Weyl points is indicated by green arrows. The n th Landau level is expected at $E_n = \sqrt{n}E_1$, with $E_1 = 2\sqrt{\pi}v_F/d_0$. In the d -wave superconductor, the Landau levels are destroyed by the vortex lattice [6], while in the Weyl superconductor, they are protected by chiral symmetry.

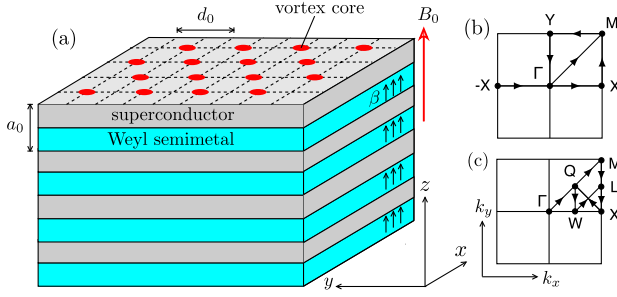


FIG. 2. Weyl superconductor in the mixed phase. (a) A Weyl semimetal-superconductor heterostructure (layers of a topological insulator, with perpendicular magnetization β , separated by s -wave superconducting spacer layers [20]). A magnetic field B_0 is applied perpendicular to the layers. The heterostructure has lattice constant a_0 , while the square vortex array has lattice constant d_0 (with two $h/2e$ vortices per unit cell). (b),(c) Two different paths through the magnetic Brillouin zone of the vortex array.

Weyl superconductor in the mixed phase.—We start quite generally from the Bogoliubov–De Gennes (BdG) Hamiltonian in the Anderson gauge [2],

$$\begin{aligned} \mathcal{H}(\mathbf{k}) &= U^\dagger \begin{pmatrix} H_0(\mathbf{k} - e\mathbf{A}) & \Delta \\ \Delta^* & -\sigma_y H_0^*(-\mathbf{k} - e\mathbf{A})\sigma_y \end{pmatrix} U \\ &= \begin{pmatrix} H_0(\mathbf{k} + \mathbf{a} + m\mathbf{v}_s) & \Delta_0 \\ \Delta_0 & -\sigma_y H_0^*(-\mathbf{k} - \mathbf{a} + m\mathbf{v}_s)\sigma_y \end{pmatrix}, \end{aligned} \quad (1)$$

with the definitions ($\hbar \equiv 1$, electron charge $+e$, mass m)

$$U = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & 1 \end{pmatrix}, \quad \mathbf{a} = \frac{1}{2}\nabla\phi, \quad m\mathbf{v}_s = \frac{1}{2}\nabla\phi - e\mathbf{A}. \quad (2)$$

The 2×2 matrix structure of H refers to electron and hole quasiparticles, with single-particle Hamiltonian H_0 and its time reverse in the diagonal blocks, coupled by the superconducting pair potential $\Delta = \Delta_0 e^{i\phi}$ in the off-diagonal blocks. The unitary transformation U removes the spatially dependent phase $\phi(x, y)$ from the pair potential into the single-particle Hamiltonian, where it combines with the vector potential $\mathbf{A}(x, y)$ in the x - y plane, corresponding to the magnetic field $\mathbf{B} = \nabla \times \mathbf{A}$ along z .

Both the gauge field $\mathbf{a}(x, y)$ and the supercurrent velocity $\mathbf{v}_s(x, y)$ wind around the positions $\mathbf{R}_n$ of the vortex cores, according to

$$\nabla \times \nabla\phi = 2\pi\hat{z} \sum_n \delta(\mathbf{r} - \mathbf{R}_n). \quad (3)$$

(For definiteness, we assume the field points in the *positive* z direction.) A spatial average over the vortices gives a vanishing supercurrent velocity, $\bar{\mathbf{v}}_s = 0$, while the average $\nabla \times \mathbf{a} = e\bar{\mathbf{B}}$ gives the average magnetic field. The field is approximately uniform, equal to B_0 , in the mixed phase

$H_{c1} \ll B_0 \ll H_{c2}$ of a type-II superconductor with overlapping vortices. In this regime, the vortex cores occupy only a small fraction $B_0/H_{c2} \ll 1$ of the volume, so the amplitude Δ_0 of the pair potential is also approximately uniform and only the phase ϕ is strongly position dependent.

We now specify to a Weyl superconductor, in the heterostructure configuration of Meng and Balents [20,35]: a stack in the z direction of layers of Weyl semimetal alternating with an s -wave superconductor. A magnetization β perpendicular to the layers separates the Weyl cones in the Brillouin zone along k_z . The Weyl points are at $\mathbf{k} = (0, 0, \pm K)$, $v_F^2 K^2 = \beta^2 - \Delta_0^2$, with v_F the Fermi velocity (assumed isotropic for simplicity). The Weyl cones remain gapless as long as $\Delta_0 < \beta$ [37].

In the BdG Hamiltonian (1) each Weyl cone is doubled into an electron and hole cone, mixed by the pair potential. We describe this mixing following Ref. [38], in the simplest case that the Weyl cones are close to the center $\mathbf{k} = 0$ of the Brillouin zone and we may linearize the momenta. (All nonlinearities in the full Brillouin zone are included in our numerics.) The single-particle Weyl Hamiltonian H_0 is a 4×4 matrix,

$$H_0(\mathbf{k}) = v_F \tau_z \mathbf{k} \cdot \boldsymbol{\sigma} + \beta \tau_0 \sigma_z - \mu \tau_0 \sigma_0, \quad (4)$$

with μ as the chemical potential. It is composed from Pauli matrices σ_α and τ_α that act on the spin and orbital degree of freedom, respectively. We also need a third set of Pauli matrices ν_α in the electron-hole basis. (The corresponding 2×2 unit matrices are σ_0, τ_0, ν_0 .)

A unitary transformation $\mathcal{H} \mapsto V^\dagger \mathcal{H} V$ with

$$V = \exp\left(\frac{1}{2}i\theta\nu_y\tau_z\sigma_z\right), \quad \tan\theta = -\frac{\Delta_0}{v_F k_z}, \quad \theta \in (0, \pi), \quad (5)$$

followed by a projection onto the $\nu = \tau = \pm 1$ blocks, gives for the Weyl cones an effective 2×2 low-energy Hamiltonian [39]

$$\begin{aligned} H_\pm(\mathbf{k}) &= v_F \sum_{\alpha=x,y} (k_\alpha + a_\alpha \pm \kappa m v_{s,\alpha}) \sigma_\alpha \\ &\quad + (\beta - m_{k_z}) \sigma_z \mp \kappa \mu \sigma_0, \end{aligned} \quad (6)$$

$$m_{k_z} = \sqrt{\Delta_0^2 + v_F^2 k_z^2}, \quad \kappa = -v_F k_z / m_{k_z}. \quad (7)$$

The electron- and hole-like cones have the opposite sign of the effective charge $q_{\text{eff}} = \pm \kappa e$, with $|q_{\text{eff}}| \rightarrow e\sqrt{1 - \Delta_0^2/\beta^2}$ for $|k_z| \rightarrow K$, smaller than the bare charge e due to the mixing of electrons and holes by the pair potential [40]. The velocity $v_z = \partial m_{k_z} / \partial k_z$ perpendicular to the layers is also renormalized by the superconductivity: $v_z \rightarrow v_F^2 K / \beta$ for $|k_z| \rightarrow K$.

At the Weyl point, for $\mu = 0$ and $|k_z| = K$, the Hamiltonian (6) anticommutes with σ_z . This so-called chiral symmetry gives a formal correspondence with a

problem first studied 40 years ago by Aharonov and Casher [41], as an application of an index theorem from supersymmetric quantum mechanics [34]. The problem of Ref. [41], to determine the zeroth Landau level of a two-dimensional massless electron in an inhomogeneous magnetic field, has also been studied more recently in the context of graphene [42–44]. We need to adapt the calculation here to account for the fractionally charged quasiparticles, but the basic approach carries through.

Calculation of the zero modes.—To study the effect of chiral symmetry on the Landau level spectrum, we set $\mu = 0$, $|k_z| = K$, and focus our attention on the chiral Hamiltonian

$$H_{\text{chiral}} = v_F \begin{pmatrix} 0 & \mathcal{D} \\ \mathcal{D}^\dagger & 0 \end{pmatrix}, \quad \mathcal{D} = \Pi_x - i\Pi_y, \\ \Pi = -i\nabla + e\mathcal{A}, \quad e\mathcal{A} = \mathbf{a} \pm \kappa m v_s. \quad (8)$$

(We omit the $\pm$ subscript for ease of notation.) The effective vector potential $\mathcal{A}$ describes the effective magnetic field

$$\mathcal{B} = \partial_x \mathcal{A}_y - \partial_y \mathcal{A}_x = \Phi_0 (1 \pm \kappa) \sum_n \delta(\mathbf{r} - \mathbf{R}_n) \mp \kappa B \quad (9)$$

felt by the Weyl fermions in the vortex lattice.

For what follows, it is convenient to choose a gauge such that $\nabla \cdot \mathcal{A} = 0$ and to assume that the external magnetic field B_0 is imposed on a large but finite area S . Because there are $N_{\text{vortex}} = B_0 S / \Phi_0$ vortices in that area (with $\Phi_0 = h/2e$ the superconducting flux quantum), the flux $\Phi = \int d\mathbf{r} \mathcal{B} = B_0 S$ through the system corresponding to the effective field equals the real flux. (The κ dependence of $\mathcal{B}$ drops out upon spatial integration.)

A zero mode ψ of H_{chiral} is either a spinor $\begin{pmatrix} u \\ 0 \end{pmatrix}$ with $\mathcal{D}^\dagger u = 0$ or it is a spinor $\begin{pmatrix} 0 \\ v \end{pmatrix}$ with $\mathcal{D}v = 0$. The general solution of these two differential equations has the form [41,43,45]

$$u = f(\zeta)e^W, \quad v = f(\zeta^*)e^{-W}, \quad \zeta = x + iy, \\ W(\mathbf{r}) = \frac{1}{2\Phi_0} \int dx' \int dy' \mathcal{B}(\mathbf{r}') \ln |\mathbf{r} - \mathbf{r}'|. \quad (10)$$

The difference $\mathcal{N} = N_u - N_v$ in the number of normalizable solutions for u and v is called the index of H_{chiral} . The absolute value $|\mathcal{N}|$ is a lower bound on the degeneracy of the zero mode and the sign of $\mathcal{N}$ determines the chirality: whether the zero mode is an eigenstate of σ_z with eigenvalue $+1$ or -1 .

To determine the index of H_{chiral} , we proceed as follows. In the absence of vortices, the function $f(\zeta)$ is analytic in the entire complex plane and we can use a basis of polynomials. A polynomial $f(\zeta)$ of degree $N - 1$ then produces N linearly independent zero modes—provided u or v is normalizable, $\int r dr |\psi|^2 < \infty$. For large r , one has asymptotically

$$W \rightarrow \frac{1}{2}(\Phi/\Phi_0) \ln |\mathbf{r}| \Rightarrow e^W \rightarrow |\mathbf{r}|^{N_{\text{vortex}}/2}, \quad (11)$$

so, if only the decay at infinity would be an issue, we would conclude that $N_u = 0$, $N_v = \text{Int}[N_{\text{vortex}}/2]$. This is the answer in the absence of vortices [41], when the degeneracy of the zero mode is determined by the enclosed flux in units of $h/e = 2\Phi_0$, while the chirality is set by the sign of the magnetic field (which we have assumed positive). As we will now show, the presence of vortices introduces a dependence of the chirality on the sign of the fractional charge $q_{\text{eff}} = \pm \kappa e$ of the quasiparticles, while the degeneracy remains given by the bare electron charge e .

With vortices, the function $f(\zeta)$ may have poles at the vortex cores $\zeta_n = x_n + iy_n$. We use this freedom to reexpress the solution (10) as

$$u = g(\zeta)e^W \prod_n (\zeta - \zeta_n)^{-1}, \quad v = f(\zeta^*)e^{-W}. \quad (12)$$

If for f and g we take polynomials of degree $N - 1$, with $N = \text{Int}[N_{\text{vortex}}/2]$, then both the functions u and v decay sufficiently rapidly at infinity. The boundary condition at the vortex cores now determines which of the two solutions is realized.

Near a vortex at position $\mathbf{r}_n$ the asymptotics is

$$|u|^2 \rightarrow |\mathbf{r} - \mathbf{r}_n|^{-1+q_{\text{eff}}/e}, \quad |v|^2 \rightarrow |\mathbf{r} - \mathbf{r}_n|^{-1-q_{\text{eff}}/e}. \quad (13)$$

Since $|q_{\text{eff}}| < e$ both solutions $\psi_u = \begin{pmatrix} u \\ 0 \end{pmatrix}$ and $\psi_v = \begin{pmatrix} 0 \\ v \end{pmatrix}$ remain square integrable at the vortex core. The boundary condition [24]

$$\sigma_z \psi = (\text{sgn} q_{\text{eff}}) \psi, \quad \text{for } \mathbf{r} \rightarrow \mathbf{r}_n \quad (14)$$

selects the most weakly divergent solution in Eq. (13): $\psi = \psi_u$ with positive chirality for $q_{\text{eff}} > 0$ and $\psi = \psi_v$ with negative chirality for $q_{\text{eff}} < 0$.

All of this was for $\mu = 0$, $|k_z| = K$, but both terms $\mu\sigma_0$ and $(\beta - m_{k_z})\sigma_z$ from Eq. (6) can be immediately reinstated since the zero mode is an eigenstate of σ_z . The resulting μ and k_z dependence of the zeroth Landau level is

$$E_{\pm}(k_z) = \mp \kappa \mu + (\text{sgn} q_{\text{eff}})(\beta - m_{k_z}). \quad (15)$$

We have thus seen how the chiral symmetry protects the zeroth Landau level from being destroyed by the vortex lattice. To complete this analytical treatment, we point out why the d -wave superconductor lacks a similar protection. In the Anderson gauge, the low-energy Hamiltonian near the nodal point of a d -wave pair potential reads [2,6,11]

$$H_{d\text{-wave}} = v_F(k_x + a_x)\sigma_z + v_\Delta(k_y + a_y)\sigma_x + m v_{s,x}\sigma_0. \quad (16)$$

There are inessential differences with H_{chiral} from Eq. (8)—the Dirac cone is anisotropic and the basis of Pauli matrices is rotated—but the essential difference is that the superfluid

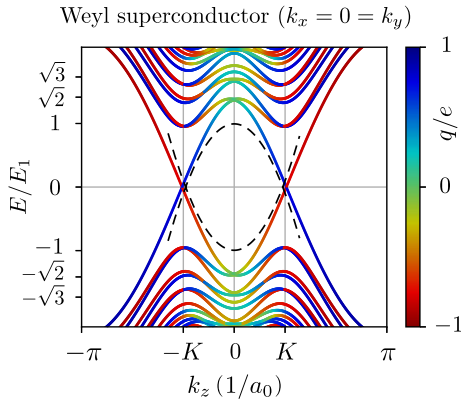


FIG. 3. Same as Fig. 1(b), but now as a function of k_z for $k_x = 0 = k_y$ at the center of the Brillouin zone [47]. The color scale indicates the charge expectation value. The dashed curve is the dispersion (15) of the zeroth Landau level, calculated analytically for $K \ll 1$ (which explains the deviation from the numerics). The effective charge at $E = 0$ is ± 0.73 , close to the analytical prediction of $\pm \kappa = \pm 1/\sqrt{2}$.

velocity breaks the chiral symmetry: $H_{d\text{-wave}}$ anticommutes with σ_y only if $v_{s,x} = 0$. In the d -wave superconductor, the superfluid velocity enters as a chirality-breaking scalar potential, while in the Weyl superconductor, it is a chirality-preserving vector potential. The former is a strong scatterer, which effectively destroys the Landau levels, while the latter cannot by force of the topological index theorem.

Comparison with numerics.—To test our analytical theory, we have numerically calculated the spectrum of a Weyl superconductor with a vortex lattice, using the KWANT tight-binding code [46]. The 8×8 Hamiltonian has the BdG form (1) with [20,21,36]

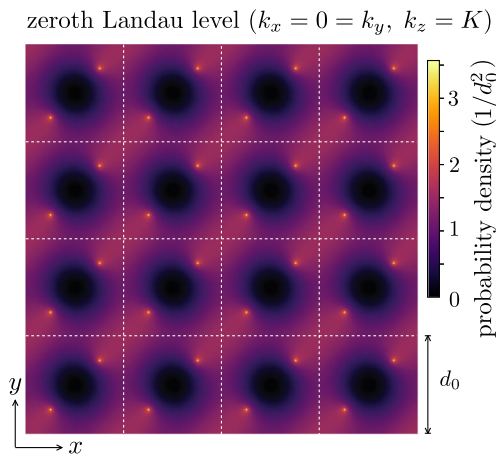


FIG. 4. Color scale plot of $|\psi(x, y)|^2$ in the zeroth Landau level of the Weyl superconductor [48]. The white dashed lines indicate the vortex array, with a pair of $h/2e$ vortices in each unit cell. On approaching a vortex core, when the separation $\delta r \rightarrow 0$, the density diverges as a power law $|\psi|^2 \propto \delta r^{1/\sqrt{2}-1}$, in accord with Eq. (13).

$$H_0(\mathbf{k}) = t_0 \sum_{\alpha=x,y,z} [\tau_z \sigma_\alpha \sin k_\alpha a_0 + \tau_x \sigma_0 (1 - \cos k_\alpha a_0)] + \beta \tau_0 \sigma_z - \mu \tau_0 \sigma_0. \quad (17)$$

Near the center of the Brillouin zone, this reduces to the linearized Hamiltonian (4), but now we will not make any linearization. Results are shown in Figs. 1(b), 3, and 4 (see Supplemental Material [24]). They are fully consistent with the analytics.

Thermal conductance.—The chiral zeroth Landau level governs the thermal transport properties of the Weyl superconductor, in the direction parallel to the magnetic field. The degeneracy $eB_0 S/h = \frac{1}{2} \Phi/\Phi_0$ of the zeroth Landau level implies a thermal conductance

$$G = \frac{1}{2} g_0 \Phi/\Phi_0, \quad g_0 = \mathcal{L} T e^2/h, \quad (18)$$

with $\mathcal{L} = \frac{1}{3} (\pi k_B/e)^2$ as the Lorenz number. In other words, each vortex contributes half a thermal conductance quantum to the heat transport—the factor $1/2$ being a reminder that the quasiparticles in the Weyl superconductor are Majorana fermions [40]. Do note that the states in the zeroth Landau level are extended over the x - y plane, the current flow is not confined to the vortex cores (see Fig. 4) [49]. We expect the universal thermal conductance (18) to be robust against nonmagnetic disorder, which in the effective Hamiltonian would enter as a term $\propto \sigma_z$ that does not couple Landau levels of opposite chirality.

Conclusion.—In this Letter, we have revisited the celebrated question [1,2] of whether quasiparticles in the vortex lattice of a gapless superconductor can condense into Landau levels. We have shown that Weyl superconductors can accomplish what d -wave superconductors could not [6]: the chirality of Weyl fermions protects the zeroth Landau level from broadening due to scattering by the vortices. We have developed the analytical argument for a simple low-energy Hamiltonian and supported it by numerical calculations for a heterostructure model of the Weyl superconductor [20]. We anticipate that the Landau levels will govern the thermodynamic and transport properties of the vortex lattice, finally allowing for the observation of quantum effects that proved elusive in the d -wave context.

We have benefited from discussions with D. I. Pikulin and J. Tworzydło. This research was supported by the Netherlands Organization for Scientific Research (NWO/OCW), an ERC Synergy Grant, and by the TÜBİTAK Grant No. 114F163.

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