Hybrid Sound Modes in One-Dimensional Quantum Liquids

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We study sound in a single-channel one-dimensional quantum liquid. In contrast to classical fluids, instead of a single sound mode we find two modes of density oscillations. The speeds at which these two sound modes propagate are nearly equal, with the difference that scales linearly with the small temperature of the system. The two sound modes emerge as hybrids of the first and second sounds, and combine oscillations of both density and entropy of the liquid.

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Small oscillations of density propagate through fluids in the form of sound waves. At long wavelengths the compression and rarefaction of the fluid are adiabatic, and the speed of sound is determined by the adiabatic compressibility,

$$v_1 = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_\sigma}, \tag{1}$$

where P, ρ, and σ are the pressure, mass density, and entropy per unit mass of the fluid, respectively. Importantly, there is only one type of sound in the system.

A well-known exception to the above picture is the superfluid ^{4}He. It can be viewed as consisting of two fluids: the superfluid component and the normal one [1,2]. The latter is the gas of elementary excitations, which can move with respect to the center of mass of the fluid even in thermodynamic equilibrium. As a result, the superfluid ^{4}He supports two types of sound waves. The first sound is predominantly an oscillation of the fluid density ρ. It is similar to the sound in ordinary fluids; in the low temperature limit its velocity is given by Eq. (1). The second sound is a wave of entropy, which is for the most part decoupled from the oscillations of density. Its speed is

$$v_2 = \sqrt{\frac{\rho_s \sigma^2}{\rho_n (\partial\sigma/\partial T)_\rho}}, \tag{2}$$

where ρ_n and $\rho_s = \rho - \rho_n$ are the densities of the normal and superfluid components, respectively. The velocities of the two sound waves are different. In the most important case of low temperature T, when the excitations are phonons with linear dispersion, $v_1/v_2 = \sqrt{3}$.

The calculation of v_1/v_2 can be easily generalized to the case of space of any dimension d, resulting in $v_1/v_2 = \sqrt{d}$ at $T \to 0$. This raises an interesting question regarding the fate of the first and second sounds in one dimension, where $v_1 = v_2$. Indeed, Eqs. (1) and (2) give the speeds of the two sounds only if $v_1 \neq v_2$. Their derivation [1,2] reveals the existence of coupling between the first and second sounds, resulting in corrections to Eqs. (1) and (2), which disappear when $T/(v_1 - v_2) \to 0$. In one spatial dimension, where $v_1 - v_2 \to 0$ at $T \to 0$, the coupling between the first and second sounds and the physical nature of the resulting acoustic modes are open questions.

One-dimensional quantum fluids do not undergo superfluid transition. Nevertheless, many of their properties mirror those of superfluid ^{4}He. In particular, their low energy excitations, typically described in the framework of the Tomonaga-Luttinger liquid theory [3,4], are bosons, which are analogous to phonons in ^{4}He. Because of the absence of superfluidity, in thermodynamic equilibrium the velocity of the gas of excitations of a one-dimensional quantum fluid must equal that of the center of mass of the system. However, the relaxation processes leading to equilibration of these two velocities are exponentially slow, with the corresponding relaxation rate scaling as $\tau^{-1} \propto e^{-D/T}$ [5,6]. (Here D is the energy bandwidth of the model.) This is in contrast to the much faster rate $\tau_{\rm ex}^{-1}$ of collisions of the excitations with each other, which scales as a power of temperature [7–10]. Thus, in a broad range of frequencies

$$\tau^{-1} \ll \omega \ll \tau_{\rm ex}^{-1}, \tag{3}$$

the excitations form a gas that can move as a whole with respect to the fluid. In this regime the one-dimensional system behaves as a superfluid and supports both first and second sounds [11].

It is important to point out that the theory of one-dimensional quantum fluids developed in Ref. [11] focused on systems of fermions with spins, which support excitations of two types, corresponding to the charge and spin degrees of freedom. Because they propagate at different speeds, the aforementioned result $v_1/v_2 = \sqrt{d}$ does not apply, and in this system $v_1 \neq v_2$. On the other hand, a system of spinless bosons (or that of fermions with spins polarized by an external magnetic field) has only a single channel of bosonic excitations propagating with velocity v, which depends on the strength of interactions between the constituent particles. For such a liquid, Eqs. (1) and (2) predict $v_1 = v_2 = v$ at $T \to 0$. This leads to strong enhancement of coupling between the first and second sounds, which changes dramatically the nature of the acoustic modes in the system. The study of this phenomenon is the main goal of this Letter.

We consider a Galilean invariant system of identical interacting spinless particles of mass m. The low energy properties of such quantum systems are described by the Luttinger liquid theory [4]. In this approach the excitations of the system are bosons with momenta k that are multiples of $2\pi\hbar/L$, where L is the system size, and periodic boundary conditions are assumed. The energy ε and momentum p of the system per unit length may be expressed in terms of the occupation numbers N_k of the bosonic states and two additional zero-mode variables: the total number of particles N and the integer quantum number J [12],

$$\varepsilon = \frac{mv^2}{2N_0L}(N-N_0)^2 + \frac{\pi^2\hbar^2}{2mL^3}NJ^2 + \frac{1}{L}\sum_k \epsilon_k N_k, \quad (4)$$

$$p = \frac{\pi\hbar}{L^2}NJ + \frac{1}{L}\sum_k kN_k. \quad (5)$$

In Eq. (4) v is the speed of sound at zero temperature, ϵ_k is the energy of the bosonic excitation with momentum k, and the number of particles is assumed to be near some reference value N_0.

The first term in the expression (5) for the momentum density accounts for the fact that even in the absence of bosonic excitations the system can move as a whole. Instead of the quantum number J it will be convenient to quantify the momentum associated with this motion by the velocity

$$u_0 = \frac{\pi\hbar J}{mL}. \quad (6)$$

The bosonic excitations in the Luttinger liquid theory are usually assumed to propagate with the velocity v, resulting in the energy spectrum $\epsilon_k = v|k|$. It is important to note that this is appropriate only at $J = 0$, i.e., in the reference frame moving with velocity u_0. The energy spectrum in the stationary frame,

$$\epsilon_k = v|k| + u_0 k, \quad (7)$$

can be obtained by the Galilean transformation.

To study sound modes, one has to account for the relaxation processes in the system. Collisions between the bosonic excitations occur at the relatively short time scale $\tau_{\rm ex}$. Thus at frequencies $\omega \ll \tau_{\rm ex}^{-1}$ the distribution function has the equilibrium form

$$N_k = \frac{1}{e^{(\epsilon_k - u_{\rm ex}k)/T} - 1}. \quad (8)$$

Collisions of the bosonic excitations conserve their total momentum, given by the second term in the right-hand side of Eq. (5). As a result, the equilibrium state can be realized with an arbitrary value of the velocity $u_{\rm ex}$. At the much longer time scale τ the bosonic excitations exchange momentum with the zero mode J, and only the total momentum [Eq. (5)] is conserved. This gives rise to the slow relaxation $u_{\rm ex} - u_0 \to 0$.

In the frequency range (3), in addition to the number of particles, momentum, and energy, the quantum number J is also a conserved quantity. As a result, the state of the fluid is characterized by two velocities, u_0 and $u_{\rm ex}$. This enables one to develop a two-fluid hydrodynamic description of the system [11] analogous to that of superfluid ^{4}He [1,2]. In this theory, u_0 and $u_{\rm ex}$ play the roles of the velocities of the superfluid and normal components, respectively.

Similar to the superfluid ^{4}He, one-dimensional quantum fluids support two sound modes. Equation (1) is the standard expression for the speed of sound in a fluid. Because the parameter v in the Luttinger liquid theory is the speed of sound at zero temperature, by definition $v_1 \to v$ at $T \to 0$. To obtain the speed of the second sound, we need to determine the values of the normal and superfluid densities ρ_n and ρ_s. To this end we evaluate the momentum density of the system at low temperatures using Eqs. (5) and (8) and find it to have the expected form $p = \rho_s u_0 + \rho_n u_{\rm ex}$, where

$$\rho_s = \rho - \rho_n, \qquad \rho_n = \rho\chi, \qquad \chi = \frac{\pi T^2}{3\hbar\rho v^3}. \quad (9)$$

We then evaluate the entropy per unit mass σ in the equilibrium state with $u_0 = u_{\rm ex} = 0$ and find

$$\sigma = \frac{\pi T}{3\hbar\rho v}. \quad (10)$$

Substituting these expressions into Eq. (2), in the limit $T \to 0$ we obtain the expected result $v_2 = v$.

The equality of the speeds of the first and second sounds enhances the effect of interaction between them. To study the consequences of this interaction, we use the equations of the two-fluid hydrodynamics, which express the conservation laws of the number of particles, momentum, energy, and J. These four equations can be either derived microscopically, following the procedure described in Ref. [11], or simply obtained by adapting the hydrodynamic equations for superfluid ^{4}He [1,2] to one spatial dimension. The state of the fluid is described by four dynamic variables, e.g., ρ, σ, u_0, and $u_{\rm ex}$. In the frequency range (3), one can neglect dissipative processes. In this case it is possible to reduce the system of four first-order differential equations of two-fluid hydrodynamics to two second-order equations by excluding the two velocities. This yields [1,2]

$$\partial_t^2 \rho = \partial_x^2 P, \tag{11a}$$

$$\partial_t^2 \sigma = \frac{\rho_s}{\rho_n} \sigma^2 \partial_x^2 T. \tag{11b}$$

In the theory of superfluid ^{4}He, the sound modes have been studied [1,2] by using T and P as independent variables in Eq. (11). In order to elucidate the nature of the sound modes in one-dimensional quantum systems, it will be more convenient to choose ρ and σ as independent variables, and treat pressure and temperature as their functions, $P(\rho, \sigma)$ and $T(\rho, \sigma)$.

We now linearize Eq. (11) in small deviations of ρ and σ from their equilibrium values. Substituting the deviations in the form $\delta\rho \cos[q(x - ut)]$ and $\delta\sigma \cos[q(x - ut)]$ into Eq. (11), we obtain

$$(u^2 - v_1^2)\delta\rho - A\delta\sigma = 0, \tag{12a}$$

$$-B\delta\rho + (u^2 - v_2^2)\delta\sigma = 0. \tag{12b}$$

Here v_1 and v_2 are given by Eqs. (1) and (2), respectively, and

$$A = \left(\frac{\partial P}{\partial \sigma}\right)_\rho, \qquad B = \frac{\rho_s}{\rho_n}\sigma^2 \left(\frac{\partial T}{\partial \rho}\right)_\sigma. \tag{13}$$

The system of linear equations (12) has nontrivial solutions at

$$u^2 = \frac{v_1^2 + v_2^2}{2} \pm \frac{1}{2}\sqrt{(v_1^2 - v_2^2)^2 + 4AB}. \tag{14}$$

This expression gives the speeds of the two sound modes in a one-dimensional quantum liquid.

Equation (14) also applies to superfluid ^{4}He. In that case, it was shown [1,2] that $AB \to 0$ at $T \to 0$, while $v_1 - v_2$ approaches a nonzero value. Therefore at low temperatures u becomes either v_1 or v_2, corresponding to the first or second sound, respectively. This is also the case for a one-dimensional system of fermions with spin considered in Ref. [11]. In contrast, for a spinless one-dimensional quantum liquid, $v_1 - v_2 \to 0$ at $T \to 0$, and the analysis of sound modes requires accounting for finite temperature corrections.

We start by determining the temperature dependence of the quantities A, B, v_1^2, and v_2^2 in Eq. (14). Using the thermodynamic Maxwell's relation $(\partial P/\partial \sigma)_\rho = \rho^2(\partial T/\partial \rho)_\sigma$ and Eqs. (9), (10), and (13), we find

$$A = \frac{\rho T}{v}\partial_\rho(\rho v), \qquad B = \frac{v^3 \chi}{\rho T}\partial_\rho(\rho v). \tag{15}$$

Both A and B scale linearly with the temperature.

To leading order in T, the temperature dependence of the free energy of a Luttinger liquid can be easily obtained from Eq. (4). This is sufficient to evaluate the correction to the pressure P, which is proportional to T^2. As a result, the correction to speed of sound [Eq. (1)] appears in the same order,

$$v_1 = v + \frac{\pi T^2}{12\hbar\rho v^3}\partial_\rho^2(\rho^2 v). \tag{16}$$

This approach yields only the leading temperature dependence for the entropy [Eq. (10)] and is thus insufficient to obtain the temperature-dependent correction to the velocity v_2 from Eq. (2).

On the other hand, corrections to v_2 can be obtained in some important cases. For noninteracting spinless fermions, the Sommerfeld expansion of the free energy involves only even powers of T, resulting in

$$v_2 = v - \frac{73\pi T^2}{30\hbar\rho v^2}. \tag{17}$$

For the model of one-dimensional bosons with short-range repulsion [14], the thermodynamics can be studied by means of the thermodynamic Bethe ansatz [15]. The Sommerfeld expansion of the free energy of this system also contains only even powers of T [16,17], yielding a correction to v_2 that scales as T^2. These examples strongly suggest that the quadratic temperature dependence of the corrections to v_2 is a generic property of one-dimensional quantum liquids. For systems with interactions that decay with distance faster that $1/x^3$, this property can be demonstrated by a careful treatment of irrelevant perturbations to the Hamiltonian of the Luttinger liquid [18].

Taking into account the temperature dependences $v_1^2 - v_2^2 \propto T^2$ and $AB \propto T^2$, we approximate the square root in Eq. (14) by $\sqrt{4AB}$. This results in linear in T splitting of the sound velocities,

$$u_\pm = v \pm \frac{\sqrt{\chi}}{2}\partial_\rho(\rho v), \tag{18}$$

where we applied Eq. (15).

Importantly, the physical character of the two sound modes that propagate at the speeds (18) is very different from those of the first and second sounds. To demonstrate this, we obtain the relative magnitudes of the oscillations of ρ and σ by substituting $u_\pm$ into Eq. (12) and find $\delta\rho/\delta\sigma = \pm\sqrt{A/B}$. It is instructive to express this result in terms of variations of the particle density $n = \rho/m$ and entropy density $s = \sigma\rho$,

$$\frac{\delta s}{\delta n} = \pm \frac{\pi}{\sqrt{3K}}. \qquad (19)$$

Here $K = \pi\hbar n/mv$ is the usual Luttinger liquid parameter.

Equation (19) shows that the ratio of the amplitudes of oscillations of s and n remains finite at $T \to 0$. This is in contrast with the usual first sound, which is predominantly a density wave, with $\delta s/\delta n \propto T$, and the second sound—the wave of entropy, with $\delta n/\delta s \propto T$. Thus the two sound modes in one-dimensional quantum liquids are qualitatively different from the first and second sounds, but combine the essential characteristics of both. This hybrid nature of sound in one dimension is our main result.

To illustrate the unusual properties of the hybrid sound modes, let us discuss the evolution of a local initial perturbation of density $\delta n(x) = f(x)$, assuming that the entropy is not perturbed, $\delta s(x) = 0$. In a classical fluid, the perturbation propagates in both directions at the speed of sound, $\delta n(x,t) = \frac{1}{2}[f(x - v_1 t) + f(x + v_1 t)]$, while to first approximation the entropy density remains undisturbed. The evolution of density perturbations in the one-dimensional system of fermions with spin is qualitatively similar. The behavior of spinless quantum liquids is dramatically different: the density perturbation splits into four pulses,

$$\delta n(x,t) = \frac{1}{4}[f(x - v_+ t) + f(x + v_+ t) + f(x - v_- t) + f(x + v_- t)]. \qquad (20)$$

These density pulses are accompanied by the entropy disturbance of the form

$$\delta s(x,t) = \frac{\pi}{4\sqrt{3K}}[f(x - v_+ t) + f(x + v_+ t) - f(x - v_- t) - f(x + v_- t)]. \qquad (21)$$

Let us now comment on the evolution of an entropy perturbation created by a local heating of the system. In this case all three kinds of fluids behave very differently. In a classical fluid the entropy spreads diffusively, in accordance with Fourier's law of heat transfer. In a quantum fluid of fermions with spin, the entropy disturbance propagates as two pulses moving in opposite directions at the speed of the second sound. In a spinless quantum fluid, such a disturbance again splits into four pulses of both entropy and density, in analogy with Eqs. (20) and (21).

Propagation of density pulses in one-dimensional quantum liquids can be studied experimentally in long quantum wires [19,20] or in atomic traps [21,22]. Electron spins in a quantum wire can be polarized by applying magnetic field, whereas density pulses can be both created and detected by short gates. In atomic traps the detection of density pulses can be performed using the techniques of Refs. [23,24].

In summary, we studied sound propagation in single-channel one-dimensional quantum liquids. In the frequency range (3), the system may be described by two-fluid hydrodynamics and supports two sound modes. In contrast to superfluid ^{4}He and one-dimensional systems with spin, the two modes are qualitatively different from the first and second sounds, which correspond to predominantly density or entropy waves. Each of these hybrid modes combines comparable in magnitude oscillations of both density and entropy, Eq. (19). The difference of speeds [Eq. (18)] of the hybrid modes scales linearly with the temperature.

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