Witnessing Optomechanical Entanglement with Photon Counting

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The ability to coherently control mechanical systems with optical fields has made great strides over the past decade, and now includes the use of photon counting techniques to detect the nonclassical nature of mechanical states. These techniques may soon be used to perform an optomechanical Bell test, hence highlighting the potential of cavity optomechanics for device-independent quantum information processing. Here, we propose a witness which reveals optomechanical entanglement without any constraint on the global detection efficiencies in a setup allowing one to test a Bell inequality. While our witness relies on a well-defined description and correct experimental calibration of the measurements, it does not need a detailed knowledge of the functioning of the optomechanical system. A feasibility study including dominant sources of noise and loss shows that it can readily be used to reveal optomechanical entanglement in present-day experiments with photonic crystal nanobeam resonators.

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Introduction.—Bell tests have initially been proposed to show that correlations between the results of measurements performed on two separated systems cannot be reproduced by classical strategies [1]. They have been used to show the limit of classical physics as a complete description of small systems involving two atoms [2,3] or two photons [4,5]. This naturally raises the question of a Bell inequality violation with larger systems. Concrete proposals have been made recently along this line to realise Bell tests with cavity opto- and electromechanical systems [6–8].

Cavity optomechanics is at the core of intense research where the cavity field is used to control the motion of a mechanical system via radiation pressure. While initial efforts have focused on the cooling of mechanical oscillators down to the ground state [9–11], impressive results including the detection of electro- [12] and optomechanical [13,14] nonclassical correlations and entanglement between two mechanical systems [15,16] are now suggesting that cavity optomechanics could serve as a building block of future quantum networks [17] for the processing and storage of quantum information [18,19]. If one is to show that cavity optomechanics can form the cornerstone of future quantum networks, it is crucial to prove that it is qualified for all possible uses of such networks. This means that the qualification must be device independent [20], that is, it cannot rely on a physical description of the actual implementation. A particular model using seemingly harmless assumptions, on the underlying Hilbert space dimension for instance, can completely corrupt the security guarantees that are offered by quantum networks for secure communications over long distances [21,22]. Device-independent schemes have been derived to certify all the building blocks of quantum networks that can be used to create, store, or process quantum information [23]. They could be directly implemented from the Bell tests proposed in Refs. [6,7]. Optomechanical Bell tests are thus not only of fundamental interest but are resources to certify the usefulness of optomechanical systems for long distance quantum communication with device-independent security guarantees.

The violation of a Bell inequality as proposed in Refs. [6–8] is, however, not trivial. Reference [6] uses a cavity optomechanical system in the resolved sideband regime where the mechanical frequency is larger than the cavity decay rate. Once cooled, the mechanical system is excited by laser light resonant with the blue sideband; see Fig. 1. Photons of the laser can decay into phonon-photon pairs, the photon being resonant with the cavity frequency and the phonon corresponding to a single excitation of the vibrational mode of the mechanical system. Energy conservation ensures that for each phononic excitation of the mechanical state, the cavity mode gets populated with a photonic excitation. These quantum correlations between phonon and photon numbers are strong enough to violate a Bell inequality [6,7]. The way to show this consists first in mapping the phononic excitations to cavity photons using laser light driving the red optomechanical sideband. This leads to a two-mode photonic state, where each mode can subsequently be detected with photon counting techniques preceded by displacement operations in phase space. By changing the amplitude and phase of the local displacements, the Bell-Clauser-Horne-Shimony-Holt (Bell-CHSH) [24] inequality can be violated as long as the global detection efficiency is higher than 67%. While several experiments have been realized combining cavity optomechanics in the revolved-sideband regime and photon counting [13–15,25], the requirement on the efficiency remains very challenging to meet.

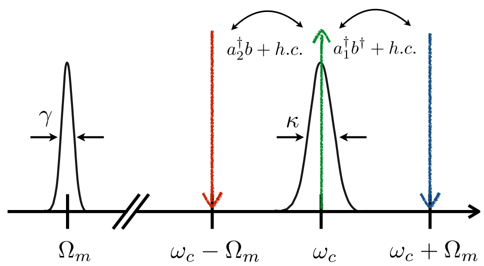

FIG. 1. A cavity optomechanical system is made with a cavity with frequency ω_c and a mechanical oscillator with frequency Ω_m. κ, and γ are the cavity and mechanical decay rates, respectively. We consider the resolved sideband regime where $\Omega_m \gg \kappa$. Starting with a cooled mechanical system, the cavity optomechanical system is first driven by a laser resonant with the blue sideband. Photon-phonon pairs are created by means of an effective squeezing operation $a_1^\dagger b^\dagger + \mathrm{H.c.}$, the bosonic operators a_1 and b corresponding to the cavity photons and mechanical phonons. The quantum nature of the correlations between the cavity photon number and the phonon number can be revealed by applying a laser resonant with the red sideband. This effectively maps the phononic state to a photonic state through a beam splitter interaction $a_2^\dagger b + \mathrm{H.c.}$ The resulting photonic state involving two temporal modes a_1 and a_2 is detected with a photon detector supplemented with a displacement operation in phase space.

Here we propose the first step of an entire research program aiming to violate a Bell inequality with optomechanical systems, that is, we propose a witness for revealing optomechanical entanglement in the same scenario. In opposition to Bell tests (see Ref. [26], Sec. A), our witness is not device independent but assumes a detailed description and correct experimental calibration of measurements. Additional measurements are also taken locally to get information about the photon number distribution. This allows us to relax the requirement on the detection efficiency, even without any assumptions about the measured state. A feasibility study shows that our witness can readily be used to reveal optomechanical entanglement in present-day experiments with photonic crystal nanobeam resonators.

Temporal evolution of the cavity field and mechanical system.—Let us recall the physics of optomechanical systems in the resolved sideband and weak coupling regime, which has been presented, at least partially, in various Refs. [6,19,29–31]. We consider the optical and mechanical modes of an optomechanical cavity with frequencies ω_c and Ω_m, respectively. The bosonic operators associated to the optical mode are called a and $a^\dagger$ while we use b and $b^\dagger$ for the mechanical mode. g_0 denotes the bare optomechanical coupling rate, κ and γ the cavity and mechanical decay rates. The cavity optomechanical system is laser driven on the lower or upper mechanical sideband with corresponding frequencies $\omega_\pm = \omega_c \pm \Omega_m$. The laser powers are labeled $P_\pm$, respectively. The full Hamiltonian includes the uncoupled cavity and mechanical systems $\mathcal{H}_0 = \hbar\omega_c a^\dagger a + \hbar\Omega_m b^\dagger b$, the optomechanical coupling $-\hbar g_0 a^\dagger a(b^\dagger + b)$, and the coupling between the cavity mode and the driving laser $\hbar(s_\pm^* e^{i\omega_\pm t} a + s_\pm e^{-i\omega_\pm t} a^\dagger)$ with $|s_\pm| = \sqrt{\kappa P_\pm/\hbar\omega_\pm}$. In the interaction picture with respect to $\mathcal{H}_0$ and focusing on the weak coupling $g_0 \ll \kappa$ and resolved-sideband $\kappa \ll \Omega_m$ regimes, the temporal evolution is given by a set of effective Langevin equations [19]

$$\frac{da}{dt} = \frac{i}{\hbar}[\mathcal{H}_\pm, a] - \frac{\kappa}{2}a + \sqrt{\kappa}a_{\mathrm{in}}, \qquad \frac{db}{dt} = \frac{i}{\hbar}[\mathcal{H}_\pm, b], \tag{1}$$

with $\mathcal{H}_+ = -\hbar g_0\sqrt{n_+}(a^\dagger b^\dagger + \mathrm{H.c.})$ and $\mathcal{H}_- = -\hbar g_0\sqrt{n_-}(a^\dagger b + \mathrm{H.c.})$ for a blue and red detuned driving laser, respectively. $n_\pm = (|s_\pm|^2)/(\Omega_m^2 + \kappa^2/4)$ is the intracavity photon number. a_{in} is the noise entering the cavity. The mechanical decay and corresponding thermal noise are neglected, that is, we focus on timescales smaller than the thermal decoherence time of the mechanical system $(\hbar\Omega_m/k_B T_{\mathrm{bath}}\gamma)$, where $k_B T_{\mathrm{bath}}$ is the Boltzmann energy.

Phonon-photon correlations in the resolved sideband regime.—Let us first focus on the initial step where a laser drives the upper sideband. We use the subscript 1 for the cavity field operators corresponding to this initial step. We proceed with an adiabatic elimination of the cavity mode $(da_1/dt) = 0$ that is, we consider a temporal evolution which is long compared to κ^{-1}. Together with the input and ouput relation, that is, $a_{1,\mathrm{out}} = -a_{1,\mathrm{in}} + \sqrt{\kappa}a_1$, we get

$$a_{1,\mathrm{out}} = a_{1,\mathrm{in}} + i\sqrt{2\tilde{g}_+}b^\dagger, \quad \frac{db_1}{dt} = \tilde{g}_+ b + i\sqrt{2\tilde{g}_+}a_{1,\mathrm{in}}^\dagger, \tag{2}$$

where $\tilde{g}_+ = (2g_0^2 n_+/\kappa)$. Integrating the previous equations and introducing the temporal modes $A_{1,(\mathrm{in/out})}(t) = \sqrt{(2\tilde{g}_+/\pm 1 \mp e^{\mp 2\tilde{g}_+ t})}\int_0^t dt' e^{\mp\tilde{g}_+ t'} a_{1,(\mathrm{in/out})}(t')$ [29] leads to $A_{1,\mathrm{out}}(t) = e^{\tilde{g}_+ t}A_{1,\mathrm{in}}(t) + i\sqrt{e^{2\tilde{g}_+ t} - 1}b^\dagger(0)$, $b(t) = e^{\tilde{g}_+ t}b(0) + i\sqrt{e^{2\tilde{g}_+ t} - 1}A_{1,\mathrm{in}}^\dagger(t)$. These two solutions can be written as $A_{1,\mathrm{out}}(t) = U_1^\dagger(t)A_{1,\mathrm{in}}U_1(t)$ and $b(t) = U_1^\dagger(t)b(0)U_1(t)$ where the propagator $U_1(t)$ is given by

$$U_1(t) = e^{i\sqrt{1-e^{-2\tilde{g}_+ t}}A_{1,\mathrm{in}}^\dagger b^\dagger} e^{-\tilde{g}_+ t(A_{1,\mathrm{in}}^\dagger A_{1,\mathrm{in}} + b^\dagger b + 1)} e^{i\sqrt{1-e^{-2\tilde{g}_+ t}}A_{1,\mathrm{in}} b}. \tag{3}$$

When $U_1(t)$ is applied on the vacuum, phonon-photon pairs are created where the phonon number equals the photon number, each of them following a thermal distribution with mean excitation number $e^{2\tilde{g}_+ t} - 1$. These correlations between the phonon and photon numbers are strong enough to violate a Bell inequality, cf. below.

Phonon-photon correlations as the basis for a Bell inequality violation.—Consider the case where a laser drives the lower sideband. We use the subscript 2 for the cavity field operators corresponding to this second step.

Following the line of thought developed in the previous paragraph while introducing $\tilde{g}_- = (2g_0^2 n_-/\kappa)$, we can show that the cavity field and photon operators evolve according to the propagator [6,19]

$$U_2(t) = e^{i\sqrt{e^{2\tilde{g}_- t}-1}A_{2,\mathrm{in}}b^\dagger} e^{-\tilde{g}_- t(A_{2,\mathrm{in}}^\dagger A_{2,\mathrm{in}} - b^\dagger b)} e^{i\sqrt{e^{2\tilde{g}_- t}-1}A_{2,\mathrm{in}}^\dagger b}. \quad (4)$$

This corresponds to a beam splitter-type evolution, performing a conversion between the phononic and photonic modes with probability $1 - e^{-2\tilde{g}_- t}$. In the limit $\tilde{g}_- t \to \infty$, the phononic mode is perfectly mapped to the photonic mode $A_{2,\mathrm{out}}$ and the phonon-photon correlations created in the first step are mapped to two temporal photonic modes $A_{1,\mathrm{out}}$ and $A_{2,\mathrm{out}}$. If both the cavity and mechanical system are in the vacuum before the laser drive, these two photonic temporal modes are described by a vacuum squeezed state $U_2(t)^{\tilde{g}_- t\to\infty} U_1(T_1)|0\rangle = e^{-\tilde{g}_+ T_1} e^{-\sqrt{1-e^{-2\tilde{g}_+ T_1}}A_{1,\mathrm{out}}^\dagger A_{2,\mathrm{out}}^\dagger}|00\rangle$. References [32–34] have shown that such a state violates the Bell-CHSH inequality when it is measured with photon detection preceded by a displacement operation in phase space, the phase and amplitude being used to change the measurement setting. Reference [6] showed that a minimum detection efficiency of ~67% is necessary to observe a violation of the Bell-CHSH inequality. This minimum detection efficiency even increases if the mechanical system is not in its ground state initially [6]. These efficiencies include all the loss from the cavity to the detector and are thus challenging to obtain in practice. We show in the following sections a way around this requirement which consists in replacing the Bell-CHSH inequality by a witness inequality, which assumes a physical description and correct experimental calibration of the measurement devices.

Photon counting preceded by a displacement operation.—We focus on the setup described before, with which a Bell inequality is tested using photon detections preceded by a displacement operation $D(\alpha)$. Before presenting our entanglement witness, we first comment on such a measurement. We consider the realistic case where the photon detector does not resolve the photon number, that is, only two measurement results can be produced at each run. The first result corresponds to "no-detection" and is modelled by a projection on the vacuum $|0\rangle\langle 0|$. The second possible result is a conclusive detection corresponding to the projection into the orthogonal subspace, that is, $\mathbb{1} - |0\rangle\langle 0|$. If we attribute the outcome $+1$ to a no-detection and -1 to a conclusive detection, the observable including the displacement operation is given by $\sigma_\alpha = D(\alpha)^\dagger(2|0\rangle\langle 0| - \mathbb{1})D(\alpha)$. In the qubit subspace $\{|0\rangle, |1\rangle\}$, σ_0 corresponds exactly to the Pauli matrix σ_z, that is, the outcome $+1$ (-1) is associated to a projection into the state $|0\rangle$ $(|1\rangle)$. When α increases, the positive-operator valued measure (POVM) elements associated to outcomes ± 1 get closer to projections in the x-y plane of the Bloch sphere having $|0\rangle$ and $|1\rangle$ as north and south poles, respectively [35]. For $\alpha = 1$, these POVM elements are projections along nonunit vectors pointing in the x direction, while for $\alpha = i$, they are noisy projections along the y direction. This means that photon detection supplemented by a displacement operation performs noisy measurements in the qubit space $\{|0\rangle, |1\rangle\}$ whose direction in the Bloch sphere can be chosen by controlling the amplitude and phase of the displacement.

Witnessing phonon-photon correlations in a qubit subspace.—In order to clarify on how to witness entanglement in two-mode squeezed vacuum using local observables σ_α, we consider the state projection in the qubit subspace $1/\sqrt{1+|\epsilon|^2}(|00\rangle + \epsilon|11\rangle)$. The sum of relevant coherence terms $|00\rangle\langle 11| + |11\rangle\langle 00|$ can be measured using the ideal observable $M_{\mathrm{ideal}} = (1/2\pi)\int(\cos\varphi\sigma_x + \sin\varphi\sigma_y)\otimes(\cos\varphi\sigma_x - \sin\varphi\sigma_y)d\varphi$. Since separable states are (i) non-negative states and (ii) they stay non-negative under partial transposition [36,37], these coherence terms are upper bounded by $2\min\{\sqrt{p(0,0)p(1,1)}, \sqrt{p(0,1)p(1,0)}\}$ for two-qubit separable states. $p(i,j)$ is the probability for having i photons in mode A_1 and j photons in A_2. Any state ρ such that $\mathrm{Tr}(M_{\mathrm{ideal}}\rho) > 2\min\{\sqrt{p(0,0)p(1,1)}, \sqrt{p(0,1)p(1,0)}\}$ is thus entangled. Since $p(0,1) = p(1,0) = 0$ and $\mathrm{Tr}(M_{\mathrm{ideal}}\rho) = 2\mathrm{Re}(\epsilon)/(1+|\epsilon|^2)$ for a state of the form $1/\sqrt{1+|\epsilon|^2}(|00\rangle + \epsilon|11\rangle)$, the witness observable M_{ideal} has the potential to detect entanglement in two-mode squeezed vacuum, in the experimentally relevant regime where the squeezing is small $2\tilde{g}_+ T_1 \ll 1$, that is, when the two-mode squeezed vacuum is well approximated by its projection in the qubit subspace. This suggests that a relevant witness observable for our purpose is

$$M(\alpha,\beta) = \int_0^{2\pi} \frac{d\phi}{2\pi} \mathbf{U}_\phi^\dagger (\sigma_\alpha \otimes \sigma_\beta) \mathbf{U}_\phi, \quad (5)$$

where the unitary $\mathbf{U}_\phi = e^{i\phi A_1^\dagger A_1} \otimes e^{-i\phi A_2^\dagger A_2}$ is used to randomize the phase of displacements through the averaging over ϕ. Note that in Eq. (5), the amplitude of displacements is a free parameter. Further note that we are interested in revealing entanglement at the level of the detection. The nonunit efficiency of the detector can be

TABLE I. The witness observable here proposed is $M(\alpha,\beta)$ (see Eq. (5). The maximum value it takes on separable states is bounded by $S^\star(\alpha,\beta)$ (see [26] Eq. (9)), that is, $\max_{\rho_{\mathrm{sep}}}\mathrm{Tr}(M(\alpha,\beta)\rho_{\mathrm{sep}}) \le S^\star(\alpha,\beta)$. The observed value in an actual experiment is Q. $Q - S^\star \le 0$ thus holds for all separable state and a violation of this inequality certifies entanglement.

Witness observable	Maximum value for separable states	Observed value	Witness Inequality
$M(\alpha,\beta)$	$\le S^\star(\alpha,\beta)$	$Q(\alpha,\beta)$	$Q - S^\star \le 0$

seen as a loss operating on the state; i.e., the beam splitter modeling the detector inefficiency acts before the displacement operation whose amplitude is changed accordingly; see Ref. [26] Sec. B. This allows us to derive a witness observable with unit efficiency detection and to include the detector efficiency at the end; see Ref. [26] Sec. C.

Witnessing phonon-photon correlations without dimensionality restriction.—Using the property that separable states stay positive under partial transposition, we show in Ref. [26] Sec. C that the maximum mean value $M(\alpha,\beta)$ can take if the measured state is separable is such that

$$\max_{\rho_{\rm sep}}[M(\alpha,\beta)\rho_{\rm sep}] \leq S^{\star}(\alpha,\beta), \tag{6}$$

where $S^{\star}(\alpha,\beta)$ depends on some joint probabilities $p(i,j)$ for having i photons in mode A_1 and j photons in A_2 and the marginal probabilities $p(n_{A_1}\geq 2)$ and $p(n_{A_2}\geq 2)$ to have strictly more than one photon in mode A_1 and A_2, respectively. These probabilities are bounded in two steps in practice. In the first step, the probability $P(\pm 1 \pm 1|00)$ and $P(\pm 1 \mp 1)|00)$ of having ± 1 for the outcomes of the detection of mode A_1 and A_2 without displacement ($\alpha=\beta=0$) are measured. They provide the following upper bounds $p(0,0)\leq P(+1+1|0,0)$, $p(0,1)\leq P(+1-1|0,0)$, $p(1,0)\leq P(-1+1|0,0)$ and $p(1,1)\leq P(-1-1|0,0)$. Second, two detectors after a 50/50 beam splitter are used to measure the probability to get a twofold coincidence $P_{\rm c}(A_{1/2})$ after the beam splitter for both mode A_1 and A_2. These coincidence probabilities provide the upper bounds on the missing elements, that is, $p(2,1)\leq p(n_{A_1}\geq 2)\leq 2P_{\rm c}(A_1)$ and $p(1,2)\leq p(n_{A_2}\geq 2)\leq 2P_{\rm c}(A_2)$. This results in a bound $S^{\star}(\alpha,\beta)$ whose value depends on the local displacement amplitudes α and β. Finally, the mean value $Q(\alpha,\beta)$ of $M(\alpha,\beta)$ is measured by evaluating $P(+1+1|\alpha,\beta)$, $P(+1|\alpha)$ and $P(+1|\beta)$, that is

$$Q(\alpha,\beta) = 1 - 2P(+1|\alpha) - 2P(+1|\beta) + 4P(+1+1|\alpha,\beta). \tag{7}$$

If there is a value for the couple α, β such that $Q(\alpha,\beta) - S^{\star}(\alpha,\beta) > 0$, we deduce that the photonic modes A_1 and A_2 are entangled. Since the state describing A_2 is obtained from a local operation on the phononic state, $Q(\alpha,\beta) - S^{\star}(\alpha,\beta) > 0$ also certifies photon-phonon entanglement. See Table I for a clarification of the quantities involved.

Results.—We focus on the statistics that would be collected in modes A_1 and A_2 if the upper sideband is laser driven during the time interval T_1 and the lower sideband is subsequently driven for a duration T_2. The value $Q - S^{\star}$ that would be obtained in this case when optimizing the arguments of local displacements α, β and the amount of initial squeezing $\bar{g}_+T_1$ is shown in Fig. 2 as a function of the phonon-photon conversion efficiency $T = 1 - e^{-2\bar{g}_- T_2}$ for various overall detection efficiency η; see Ref. [26], Sec. D for more details. Figure 2 shows a very favorable robustness of our witness to inefficiencies. We stress that the efficiency η represents the global detection efficiency, including all the loss from the cavity optomechanical system to the detector (except the phonon-photon conversion efficiency for mode A_2 specified by T). We here assumed that the mechanical system is prepared in its ground state. In the more realistic case where the initial mechanical cooling leads to a mechanical thermal state with nonzero mean occupation number n_0, the results presented in Fig. 2 for $\eta = 0.3$, for example, are essentially unchanged as long as $n_0 \leq 0.1$ and substantial differences between Q and $S^{\star}$ can still be observed for $n_0 \sim 1$; see Ref. [26], Sec. D. Note that in case where the marginal probabilities $p(n_{A_1}\geq 2)$ and $p(n_{A_2}\geq 2)$ are negligible, the observed quantum correlations are ultimately limited by single phonon coherence time, which could be upper bounded by recording $Q - S^{\star}$ for various delays between the pulses resonant with the upper and lower sidebands.

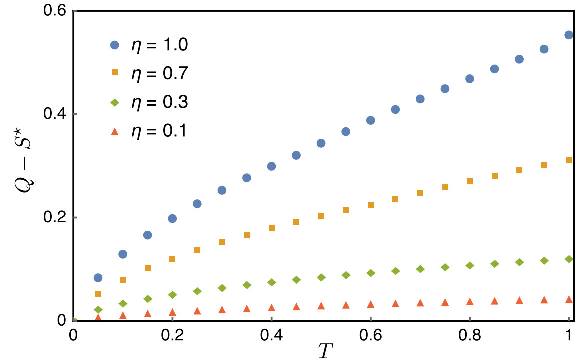

FIG. 2. Difference $Q - S^{\star}$ between the mean value of our witness observable $M(\alpha,\beta)$ that would be observed between the optical modes A_1 and A_2 and the maximum value that would be obtained with a separable state as a function of the phonon-photon conversion efficiency $T = 1 - e^{-2\bar{g}_- T_2}$ for various overall detection efficiencies η, optimized over displacement choices α, β and the amount of initial squeezing $\bar{g}_+T_1$ which is kept small. $Q - S^{\star} > 0$ witnesses entanglement.

Feasibility study.—To illustrate the feasibility, we focus on a photonic crystal nanobeam resonator [11,38,39] which distinguishes itself by a high mechanical frequency $\Omega_m/2\pi = 5.25$ GHz [14]. Together with the cavity decay rate $\kappa/2\pi = 846$ MHz [14] and the optomechanical coupling rate $g_0/2\pi = 869$ kHz [14], this resonator is placed in the deep resolved sideband and weak coupling regimes. To control the initial number of excitations, we consider the use of a dilution refrigerator, which can bring the mean phonon number down to $n_0 \sim 0.2$. Furthermore, to prevent decoherence of the phonon state we also consider pulse durations much smaller than the typical decoherence time of the oscillator, which is of the order of 10 μs [38,40]. Considering a global detection efficiency $\eta = 10\%$, an initial mean phonon number of $n_0 = 0.2$ and state-swap efficiency of $T = 30\%$ which can be realized using a pulse laser resonant with the red sideband with a duration of $T_2 = 50$ ns and intracavity photon number $n_- \approx 318$, we expect to conclude about the presence of entanglement (violation of the inequality $Q - S^\star \leq 0$ by 3 standard deviations) within 750 000 experimental runs, see Ref. [26], Sec. E. This involves the creation of a phonon-photon state using a blue-detuned pulse of duration $T_1 = 50$ ns and $n_+ \approx 298$, and the choice of displacement amplitudes $\alpha = -\beta = 2.63$. Given the experiments reported in Refs. [14,15], we conclude that our scheme appears feasible with currently available technologies.

Conclusion.—We have presented a witness tailored for the detection of optomechanical entanglement using photon countings. Our proposal is based on the measurement of single and twofold coincidence counts. It requires basic phase stabilizations and is robust to loss, see Ref. [26], Sec. F. This makes us confident that it can be used in present day experiments with photonic crystal nanobeam resonators to show directly optomechanical entanglement. Following Ref. [7], it also applies straightforwardly to electromechanical systems where it could be used to demonstrate electromechanical entanglement with non-Gaussian resources. Beyond opto- and electromechanics, our witness could find applications in nanophotonics to measure the coherence time of single phonons in any Raman-active vibrational modes using two-color pump-probe Raman scattering measurements [41]. It could also be used to detect atom-photon entanglement directly in spontaneous Raman protocols, e.g., to certify the proper functioning of photon pair sources relevant for long-distance quantum communications [42,43].

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[26] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.121.023602 for further details, which includes Refs. [27,28]. Section A explains the connection between Bell tests and device independent conclusions, and how our witness yields a state independent conclusion. Sections B–E explain technical details for our calculations and statistical analysis. Section F has a summary on how our witness can be implemented.
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