Quantum Critical Metrology

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Quantum metrology fundamentally relies upon the efficient management of quantum uncertainties. We show that under equilibrium conditions the management of quantum noise becomes extremely flexible around the quantum critical point of a quantum many-body system: this is due to the critical divergence of quantum fluctuations of the order parameter, which, via Heisenberg's inequalities, may lead to the critical suppression of the fluctuations in conjugate observables. Taking the quantum Ising model as the paradigmatic incarnation of quantum phase transitions, we show that it exhibits quantum critical squeezing of one spin component, providing a scaling for the precision of interferometric parameter estimation which, in dimensions $d > 2$, lies in between the standard quantum limit and the Heisenberg limit. Quantum critical squeezing saturates the maximum metrological gain allowed by the quantum Fisher information in $d = \infty$ (or with infinite-range interactions) at all temperatures, and it approaches closely the bound in a broad range of temperatures in $d = 2$ and 3. This demonstrates the immediate metrological potential of equilibrium many-body states close to quantum criticality, which are accessible, e.g., to atomic quantum simulators via elementary adiabatic protocols.

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Introduction.—Observables in extended physical systems (classical or quantum in nature) are affected by intrinsic uncertainty, which typically results from an extensive number of uncorrelated, microscopic local contributions. As a consequence, the squared uncertainty scales linearly with system size, in compliance with the central limit theorem. Yet collective phenomena, such as phase transitions, may lead to the appearance of sizable correlations among the constituents, leading to the breakdown of the central limit theorem and to superextensive scaling of fluctuations, which clearly aggravates the uncertainty of the corresponding observable. However, in quantum systems, uncertainties of noncommuting observables A and B may play complementary roles as they obey Heisenberg's inequality $\mathrm{Var}(A)\mathrm{Var}(B) \geq |\langle [A, B]\rangle|^2/4$ [where $\mathrm{Var}(A) = \langle A^2\rangle - \langle A\rangle^2$ and $\langle \ldots \rangle = \mathrm{Tr}(\rho \ldots)$ denotes the average on the state ρ—pure or mixed—of the system]. In the following, we shall focus on the physically relevant situation in which A, B, and $[A, B]$ are extensive observables. As a consequence of Heisenberg's inequality, the critical increase of fluctuations of A leads to a suppression of the lower bound for the fluctuations of B. The reduction of a lower bound is hardly constraining for the actual behavior of fluctuations, but it may be so for quantum states realizing minimal (or close to minimal) uncertainty, namely (nearly) saturating Heisenberg's inequality.

In this Letter, we show that this counterintuitive mechanism of critical suppression of fluctuations, by which the scaling of $\mathrm{Var}(B)$ becomes subextensive when the one of $\mathrm{Var}(A)$ becomes superextensive, is indeed at play at a quantum critical point (QCP) [1] occurring in the ground state of quantum many-body systems, implying that a QCP generically allows one to tune the quantum noise of extensive observables to extraordinarily low values. The redistribution of quantum noise among observables is known in the quantum optics and atomic physics literature as *squeezing* [2–4]: in the context of quantum spin systems (modeling electronic or nuclear spins in solids, or the internal states of atomic ensembles), spin squeezing [3] has both a fundamental meaning as a manifestation of entanglement [5–7], as well as an immediate application in the context of quantum metrology, leading to a fundamental gain in interferometric quantum parameter estimation [8]. In particular, we show here that paradigmatic spin models of quantum phase transitions (QPTs) exhibit quantum critical spin squeezing at the zero-temperature QCP [9,10], which generically implies the subextensive scaling of the variance of one observable; and that squeezing is also manifest in a broad region of the finite-temperature phase diagram around the QCP, making it of interest to realistic metrological protocols. Even more important, for sufficiently high dimensions, we show that equilibrium squeezing nearly saturates the maximum metrological gain dictated by the quantum Fisher information [11,12], demonstrating that a metrological protocol which exploits the equilibrium spin squeezing of thermal states in the vicinity of a QCP is (nearly) optimal.

Before entering into the core of our Letter, we stress that our discussion of the metrological use of phase transitions differs from that offered in the literature on Hamiltonian-parameter estimation; see Refs. [13–18] for representative examples. The focus of this literature is the distinguishability among equilibrium states, which becomes maximal around phase transitions (be them thermal [19] or quantum [20]), allowing for an optimal estimation of the parameter driving the transition itself (external magnetic field, temperature, etc.). On the other hand, our study focuses on equilibrium states used as input of interferometers (namely, unitary transformations parametrized by a phase ϕ [4,11]) and their augmented ability to estimate the interferometric phase in the presence of quantum correlations.

Model.—Throughout this Letter, we focus on a paradigmatic spin model of quantum phase transitions, the transverse-field Ising (TFI) model, whose Hamiltonian on finite-dimensional systems reads

$$\mathcal{H} = -J\sum_{\langle ij\rangle} S_i^z S_j^z - \Gamma\sum_i S_i^x, \tag{1}$$

where S_i^α are $S = 1/2$ quantum spins, the sums run on nearest-neighboring pairs and sites (respectively) of a d-dimensional hypercubic lattice containing $N = L^d$ sites, and $J > 0$. In the special case of $d = \infty$ (or an infinite-connectivity model), the Hamiltonian takes rather the form

$$\mathcal{H} = -\frac{J}{N}\sum_{i<j} S_i^z S_j^z - \Gamma\sum_i S_i^x. \tag{2}$$

The TFI model is a cornerstone in the theory of QPTs [1]: a critical value g_c of the transverse field $g = \Gamma/J$ separates a low-field ferromagnetic phase with spontaneously broken symmetry from a high-field quantum paramagnetic phase lacking long-range order. Interestingly its infinite-dimensional version, Eq. (2), has been often discussed in the theory of spin squeezing [9,21–23] and implemented to dynamically generate spin squeezing in recent atomic physics experiments with spinor gases and trapped ions [24–26]. On the contrary, the metrological aspects of its finite-d versions have been far less discussed [27]. Here, we focus on the ground-state and finite-temperature properties of the above model making use of its exact solution in $d = 1$ and ∞, together with numerically exact quantum Monte Carlo (QMC) simulations [28] (see the Supplemental Material for further details [29]).

Quantum Fisher information vs squeezing.—Modeling the interferometer with a unitary transformation $e^{i\phi O}$, the minimal uncertainty on the estimation of ϕ is provided by the quantum Fisher information (QFI) of the generator $O = \sum_{i=1}^N O_i$ via the quantum Cramér-Rao bound [11]

$$(\delta\phi)^2 \geq \frac{1}{k\,\mathrm{QFI}(O)} = \frac{\chi^2}{kN}; \tag{3}$$

the QFI is defined as $\mathrm{QFI}(O) = 2\sum_{nm}(p_n - p_m)^2 \times |\langle m|O|n\rangle|^2/(p_n + p_m)$, where $|n\rangle$ ($|m\rangle$) are eigenstates of the density matrix ρ with eigenvalues p_n (p_m), and is k the number of independent measurements performed. A factor $\chi^2 = N/\mathrm{QFI}(O) < 1$ witnesses a metrological gain with respect to the shot-noise limit and the presence of entanglement [34–36]. For pure states $\mathrm{QFI}(O) = 4\mathrm{Var}(O)$; hence, choosing O as the (extensive) order parameter of a QPT, one can exploit its critical superextensive fluctuations $\mathrm{Var}(O) \sim N^{1+\zeta}$ ($\zeta > 0$) to achieve sub-shot-noise precision, namely, $(\delta\phi)^2 \sim N^{-1-\zeta}$ and $\chi^2 \sim N^{-\zeta}$. For the TFI model in dimensions $d \leq d_c = 3$, O is the z component of the collective spin $\boldsymbol{J} = \sum_i \boldsymbol{S}_i$, and $\zeta = (2 - \eta - z)/d = (1 - \eta)/d > 0$, where η and $z(= 1)$ are the anomalous dimension and dynamical critical exponent of the QPT, respectively. Above the upper critical dimension $d > d_c(= 3)$, ζ takes the above form with d_c instead of d [37], while $\eta = 0$.

The above-cited result on the QFI already embodies the metrological interest of QCPs but remains silent about the specific measurement able to enjoy the quantum critical metrological gain witnessed by the QFI. Of much more immediate utility is instead Heisenberg's uncertainty principle for the collective spin

$$\mathrm{Var}(J^y) \geq \frac{\langle J^x\rangle^2}{4\mathrm{Var}(J^z)}, \tag{4}$$

which, at the QCP, allows one to conclude that $\mathrm{Var}(J^y) \geq \mathcal{O}(N^{1-\zeta})$; namely, the lower bound on the variance of J^y acquires a subextensive scaling at criticality. Similarly, the spin-squeezing parameter [8]

$$\xi_R^2 = \frac{N\mathrm{Var}(J^y)}{\langle J^x\rangle^2} \tag{5}$$

acquires a vanishing lower bound at criticality, $\xi_R^2 \geq N/[4\mathrm{Var}(J^z)] \sim \mathcal{O}(N^{-\zeta})$. This lower bound can also be predicted via the inequality $\xi_R^2 \geq \chi^2$ [38]. When smaller than 1, ξ_R expresses the metrological gain in Ramsey interferometry with respect to uncorrelated states, namely, $(\delta\phi)^2 = \xi_R^2/(kN)$ [8], and also witnesses entanglement [5]. The critical scaling of the lower bound on ξ_R^2 and $\mathrm{Var}(J^y)$ is suggestive of the possibility to observe quantum critical scaling of spin squeezing, but only explicit microscopic calculations can test whether critical squeezing is indeed achieved or not.

Quantum critical squeezing.—Our joint exact or numerical study of the ground-state scaling of J^y fluctuations shows a very complex and intriguing picture upon varying the number of dimensions; see Fig. 1. The case of $d = \infty$ is exactly solved by writing the Hamiltonian in the $|S;M\rangle$ basis of eigenstates of $\boldsymbol{J}^2$ and J^z and diagonalizing it in each S sector separately. There at the QCP $g_c = 1$, one observes numerically that $\xi_R^2 \simeq \chi^2 \sim N^{-1/3}$ (as already noticed in

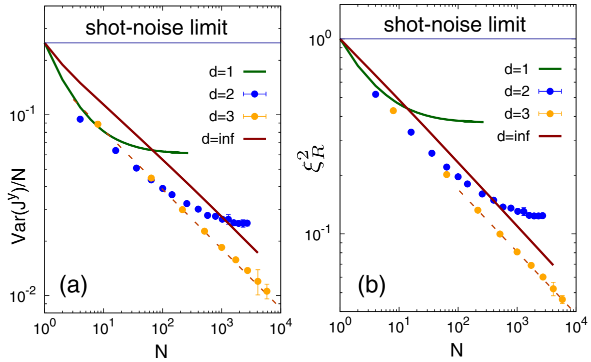

FIG. 1. Squeezing at the QCP. (a) Scaling of the variance of the collective spin component $J^y = \sum_i S_i^y$ in $d = 1, 2, 3$, and $d = \infty$. Data are taken at $g = 0.62$ and $T = 0$ from the exact solution on a system with open boundaries in $d = 1$ and at $g = g_c$ for all other dimensions. The data for $d = \infty$ are the $T = 0$ exact solution, while the data for $d = 2$ and $d = 3$ come from QMC simulations on an $N = L^d$ lattice with periodic boundaries, at temperatures sufficiently low to eliminate thermal effects [as low as $k_B T = J/(6L)$ in $d = 2$ and $k_B T = J/(5L)$ in $d = 3$]. Dashed lines are power-law fits to the form $a \times N^{\zeta'}$. The solid line indicates the shot-noise limit $\mathrm{Var}(J^y)/N = 1/4$. (b) Scaling of the spin-squeezing parameter ξ_R^2. Same significance of symbols as in panel (a).

Refs. [4,9,10,23]); namely, the ground state in $d = \infty$ is a minimal uncertainty state, realizing the maximum quantum critical squeezing authorized by Heisenberg's inequality. This can be understood using elementary quantum mechanics, as in the vicinity of the QCP (but strictly speaking, not at the QCP) a Holstein-Primakoff transformation maps the model onto a collection of harmonic oscillators, admitting a minimum-uncertainty ground state [4,29].

On the opposite side of the spectrum lies the case of $d = 1$, whose exact solution based on Jordan-Wigner mapping onto free fermions [39] shows that $\mathrm{Var}(J^y)$ at the critical point $g_c = 1/2$ exhibits a conventional volume-law scaling. Therefore, squeezing, albeit present [namely, $\xi_R^2 < 1$ with a minimum at $g \approx 0.62 > g_c$, and a minimum $\mathrm{Var}(J^y)$ at $g \approx 0.6$], does not show any sign of quantum critical scaling, as already remarked in Ref. [27]. This observation is in stark contrast to the χ^2 factor, rapidly scaling to zero as $N^{-3/4}$ ($\eta = 1/4$). Hence, conventional Ramsey interferometry is far from being the optimal protocol exploiting the significant metrological potential of the QCP in the $1d$ TFI model.

The above results, which were already partly known in the literature [9,10,27], are interpolated in a very nontrivial way in the intermediate cases $1 < d < \infty$ (lacking an exact solution). There, our QMC results show that squeezing scales as $\xi_R^{-2} \sim N^{-\zeta'}$ with an exponent ζ', which becomes consistent with the one (ζ) dictated by the Heisenberg bound only for $d \geq 3$. Indeed, Fig. 1 shows that in $d = 2$ at $g_c = 1.52219\ldots$ [40], the squeezed fluctuations per spin $\mathrm{Var}(J^y)/N$ appear to saturate towards a very small yet finite value, indicating that $\zeta' = 0 < \zeta = 0.4818\ldots$ (using the exponents of the $3d$ Ising universality class [41]). On the other hand, at the mean-field transition in $d = 3$ ($g_c = 2.579\ldots$ [40]), a fit to the data in Fig. 1(a) gives $\zeta' = 0.329(1) \approx \zeta = 1/3$ (these results are summarized in Table I). This means that the upper critical dimension $d_c = 3$ for the Ising quantum phase transition corresponds to the lower critical dimension for arbitrarily strong spin squeezing to be observed at quantum criticality. This can be understood in that a finite value of the anomalous dimension η for $d < d_c$ represents an obstruction to the vanishing of $\mathrm{Var}(J^y)/N$, as we shall discuss in a future publication [42]. A closer inspection of Fig. 1(b) shows that, the number of spins being held fixed, the strongest squeezing is obtained at criticality in $d = 3$ (and not in $d = \infty$). Hence, the strongly correlated critical point generated by short-range interactions possesses more squeezing than the one stabilized by mean-field (infinite-range) interactions.

TABLE I. Exponent for the quantum critical scaling of the squeezing parameter ξ_R^{-2} (ζ') and of the χ^{-2} parameter (ζ); the latter corresponds to $4\mathrm{Var}(J^z)/N$ at $T = 0$.

Scaling exponent at the QCP	ζ' ($\xi_R^{-2} \sim N^{\zeta'}$)	ζ [$\mathrm{Var}(J^z)/N \sim N^{\zeta}$]
$d = 1$	0	3/4
$d = 2$	0	0.4818…
$d = 3$	≈1/3	1/3
$d = \infty$	1/3	1/3

Quantum correlations and squeezing along the QC trajectory.—We now turn to the finite-temperature case, most relevant from the perspective of potential experimental implementations. A realistic experimental situation involves the system being prepared with $g \gg 1$, namely, in a coherent spin state $\otimes_{i=1}^N |\uparrow_x\rangle_i$, and then adiabatically transformed by lowering the g ratio towards the critical g_c value. Inevitable deviations from adiabaticity will produce an equilibrium state at finite temperature or entropy density at the end of the g ramp. We then ask the question: how much of the remarkable metrological properties of the ground state survive at finite temperatures in the vicinity of the QCP?

We start addressing this question by exploring the evolution of metrologically relevant observables along the so-called quantum critical trajectory (sketch in Fig. 2), namely, by scanning the temperature at $g = g_c$. Figures 2(a)–2(d) show the temperature dependence of the squeezing parameter, along with that of the χ^{-2} parameter (when calculable, namely, for $d = 1$ [43] and ∞), as well as the quantum variance (QV) of the order parameter introduced by us in Ref. [44]. The latter is defined as

$$\mathrm{QV}(J^z) = \langle (J^z)^2 \rangle - k_B T \int_0^{(k_B T)^{-1}} d\tau \langle J^z(\tau) J^z(0) \rangle, \quad (6)$$

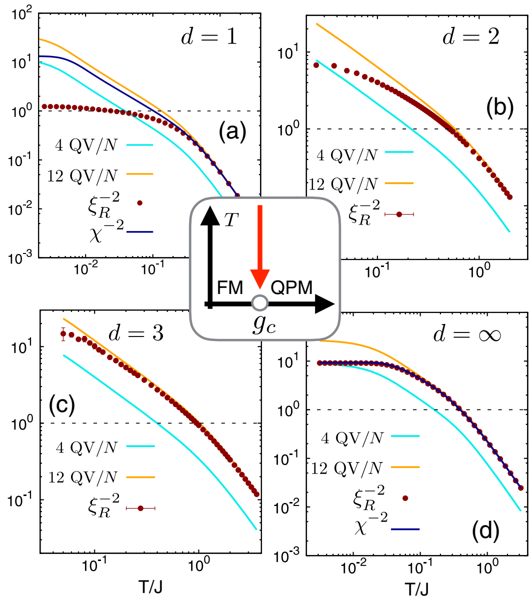

FIG. 2. Quantum correlations along the quantum critical trajectory. Squeezing parameter ξ_R^2, χ^{-2} parameter, and its bounds provided by the quantum variance for the TFI model as a function of the temperature at $g = g_c$: (a) $d = 1$, $N = 50$; (b) $d = 2$, $N = 64^2$; (c) $d = 3$, $N = 28^3$; (d) $d = \infty$, $N = 1000$.

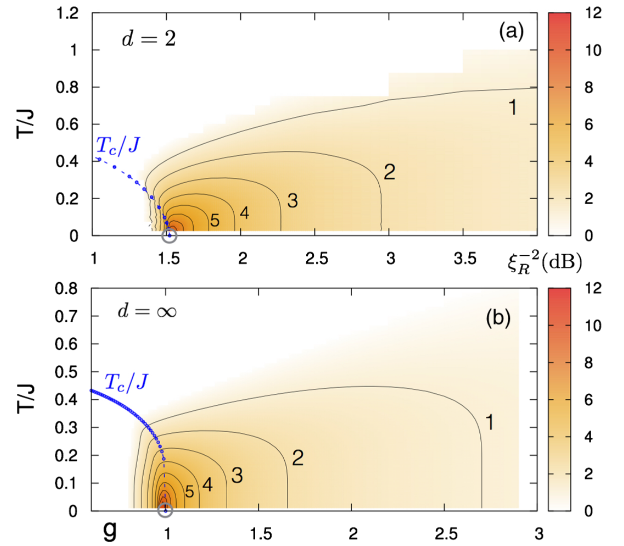

FIG. 3. Squeezing around the QCP. Squeezing parameter ξ_R^{-2} (in dB) across the phase diagram of the TFI model close to the QCP: (a) $d = 2$, $N = 64^2$; (b) $d = \infty$, $N = 500$. The gray circle marks the QCP, and the dashed blue lines indicate the critical temperatures T_c on the ordered side (T_c values for $d = 2$ from Ref. [46], and for $d = \infty$ from Ref. [47]). In the white region, $\xi_R^{-2} \leq 1$ (absence of squeezing).

where $J^z(\tau) = e^{\tau\mathcal{H}} J^z e^{-\tau\mathcal{H}}$. The QV is known [44,45] to tightly bound the QFI and, hence, the χ^2 parameter as

$$\frac{\mathrm{QV}(J^z)}{N} \leq \frac{1}{4}\chi^{-2} \leq 3\frac{\mathrm{QV}(J^z)}{N}. \quad (7)$$

As a consequence, $\xi_R^{-2} \leq \chi^{-2} \leq 12\mathrm{QV}(J^z)/N$. These bounds to the χ^{-2} parameter turn out to be extremely useful because (1) they are thermodynamical quantities generically computable with large-scale numerics such as the QMC method adopted here, while the QFI (contained in χ) is not, unless one has access to the exact solution of the model [43]; and (2) the joint upper bound to the squeezing parameter and the χ^{-2} one offered by the QV allows one to probe directly how close ξ_R and χ are, even if χ is unknown. Indeed, if ξ_R^{-2} approaches $12\mathrm{QV}/N$, we know for sure that χ^{-2} is tightly sandwiched in between. We observe that in all dimensions, ξ_R^{-2} saturates its upper bound (and, therefore, coincides with χ^{-2}) for sufficiently high temperatures; namely, the QFI and the squeezing parameter contain the same information. But the lower the dimension, the higher the temperature at which the two quantities start to deviate, and particularly so in $d \leq 2$, as χ^{-2} displays a power-law divergence as $T \to 0$ (consistent with QC behavior [43,45]), while ξ_R^{-2} does not diverge (see Fig. 1). For $d \geq 3$, ξ_R^{-2} is seen to exhibit QC temperature scaling consistent with its divergence at $T = 0$ but with a seemingly different power law with respect to the one of χ^{-2} and of the QV (which exhibits the same divergence as the QFI [45]). Yet, already in $d = 3$, the squeezing parameter and the χ^2 parameter remain extremely close to each other down to very low temperatures $T \sim 10^{-1}J$. Finally, for $d = \infty$, ξ_R^2 and χ^2 are seen to coincide at any temperature, and this despite the strong finite-size effects that infinite-range interactions entail.

Finite-temperature squeezing around the critical point.— Finally, to demonstrate the potential metrological utility of the equilibrium physics close to the QCP, we map out the squeezing parameter in the temperature-field plane. Figure 3 shows ξ_R^{-2} as a function of the field and temperature in the case of $d = 2$ and ∞ (analogous figures for $d = 1$ and 3 are shown in the Supplemental Material [29]). It is remarkable to observe that the very existence of squeezing $\xi_R < 1$ is essentially induced in the model by the existence of the QCP. Indeed, for $g \to \infty$, the ground state is a coherent spin state with $\xi_R = 1$, and squeezing is not produced at finite temperature either. The introduction of spin-spin interactions ($g < \infty$) produces correlations and squeezing in the ground state. A perturbative calculation [29] shows that $\xi_R^2 = 1 - d/(2g) + \mathcal{O}(g^{-2})$, and ξ_R decreases monotonically upon decreasing g towards the QCP. Such ground-state squeezing is protected at a finite temperature by the existence of the spectral gap controlled by the field (and linear in Γ at large Γ/J). Upon approaching the QCP, the gap closes, but ground-state squeezing becomes critical (in $d > 2$), and, as a consequence, it remains sizable at a finite temperature (up to $T/J \sim 0.5$) or, equivalently, at finite entropy density. As we explicitly show in the Supplemental Material [29], an adiabatic protocol ramping down the transverse field from the

$g \to \infty$ limit towards the QCP leads to *cooling*. As a consequence the squeezing which is produced at the QCP is intrinsically robust to a finite entropy density, produced by deviations from adiabaticity or by noise sources. Once the QCP is crossed, squeezing is quickly lost as one enters the ordered phase; the finite-size ground state for $g \ll g_c$ is a Schrödinger's cat state with no squeezing.

Conclusions and perspectives.—In this Letter, we have unveiled the interest of using equilibrium quantum many-body states lying in the vicinity of a QCP as input states for interferometric measurements which beat the shot-noise limit. We have revealed that extreme spin squeezing—diverging with system size—appears at the QCP of the quantum Ising model in $d \geq 3$, and that very strong squeezing associated with equally strong quantum correlations survives up to sizable temperatures or entropy densities above the QCP. In particular, the precision of standard Ramsey interferometry interrogating the collective spin of the output state nearly saturates the quantum Cramér-Rao bound down to low temperatures in $d = 3$ and higher, showing that the quantum correlations of QCPs can be potentially exploited in current metrological setups such as atomic clocks. The metrological potential of QCPs in $d < 3$ can instead only be exploited via more complex observables due to the non-Gaussian nature of the corresponding states [48]. Our findings are immediately relevant to quantum simulation setups realizing the quantum Ising model and its quantum phase transition, namely, trapped ions [26], Rydberg atoms [49,50], ultracold binary atomic mixtures [51], or superconducting circuits [52], suggesting that quantum simulators of quantum critical phenomena can potentially find an application as quantum sensors.

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