Probing Quantum Turbulence in ^{4}He by Quantum Evaporation Measurements

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Theory of superfluid ^{4}He shows that, due to strong correlations and backflow effects, the density profile of a vortex line has the character of a density modulation and it is not a simple rarefaction region as found in clouds of cold bosonic atoms. We find that the basic features of this density modulation are represented by a wave packet of cylindrical symmetry in which rotons with a positive group velocity have a dominant role: The vortex density modulation can be viewed as a cloud of virtual excitations, mainly rotons, sustained by the phase of the vortex wave function. This suggests that in a vortex reconnection some of these rotons become real so that a vortex tangle is predicted to be a source of nonthermal rotons. The presence of such vorticity induced rotons can be verified by measurements at low temperature of quantum evaporation of ^{4}He atoms. We estimate the rate of evaporation and this turns out to be detectable by current instrumentation. Additional information on the microscopic processes in the decay of quantum turbulence will be obtained if quantum evaporation by high energy phonons should be detected.

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A unique phenomenon takes place in liquid ^{4}He at low temperature: quantum evaporation (QE), in which an elementary excitation like a roton or a high energy phonon impinging on the surface of the superfluid causes the evaporation of a single ^{4}He atom [1,2]. This phenomenon has given important information on the properties of this strongly interacting Bose system. In addition, it has been suggested that QE can be used as a probe of other phenomena like the detection of solar neutrinos [3] and of dark matter [4]. Here we propose that QE can be very useful to uncover aspects of quantum turbulence (QT).

QT [5,6] is a paradigm of turbulence that takes place in a pure superfluid, i.e., a system in which the normal component is essentially zero like in superfluid ^{4}He at temperatures well below 1 K. In QT viscosity cannot play a role like in classical turbulence so other processes must be responsible for the experimentally observed [7,8] decay of a tangle of quantized vortex lines.

Vortex reconnections in which pairs of vortices intersect and exchange tails are relevant processes in a turbulent system to redistribute energy over different length scales in the most diverse systems, from plasmas of astrophysical or of laboratory interest, to classical or quantum fluids. Vortex reconnections have a special role in QT because in a superfluid this is the only mechanism that can change the topology of the vortex tangle generated by an initial forcing. We have now direct experimental evidence of such reconnection events in cold bosons [9] and in ^{4}He [10] as well of the generation of Kelvin waves [11], the elementary excitations of a vortex line [12]. The commonly accepted view of dissipation of energy in QT is based on vortex reconnections that excite Kelvin waves and small vortex rings [13], and of Kelvin wave cascades that lead to excitations of Kelvin waves of larger wave vectors [14,15] until they become efficient phonon emitters [16], so that the vortical energy is dissipated into heat. There is also theoretical evidence for the direct generation of phonons in a vortex reconnection [17,18]. In fact, study of vortex reconnections with the Gross-Pitaevskii equation (GPE) has shown that the local merging of the cores of two vortices and the following detachment is associated with a shortening of the length of the vortices and with the generation of a rarefaction wave that then propagates as phonons. This is a plausible picture but up to now there is no direct experimental evidence [19] of the Kelvin wave cascades, of the generation of small vortex rings, or of the rarefaction waves associated with vortex reconnections. Therefore, fundamental pieces of evidence for the decay of vorticity at very low temperatures are still missing. In the present Letter we present evidence that QE processes [1] should be induced by a vortex tangle due to vortex reconnections thus giving microscopic insight into the decay of QT. In fact, we find that the vortex core structure given by state of the art quantum many-body simulations [20] can be recovered as a cylindrically symmetric wave packet (WP) of bulk roton states, suggesting the picture of the vortex as a cloud of virtual excitations, mainly rotons, induced by the flow field. This leads us to the conjecture that part of the energy from reconnection events is in the form of nonthermal rotons. We estimate the rate of roton emission from a tangle, and show that these rotons should be detectable via processes of QE of ^{4}He atoms [1], if the liquid has a free surface. QE should also provide information on the Kelvin cascade in the high-energy phonon region.

The theoretical efforts to study QT are based on phenomenological Biot-Savart models [5,21] or on the mean field approximation as embodied in the GPE. While GPE gives a very accurate description of cold bosonic atoms and its predictions on vorticity in clouds of such atoms have been beautifully verified experimentally [9,22], it is known that GPE gives a very poor representation of superfluid ^{4}He. For instance, the excitation spectrum $\epsilon(q)$ given by GPE is a crossover from the phonon region at small wave vector q to a free particle q^2 behavior at large q [23], so it misses completely the maxon-roton feature so characteristic of superfluid ^{4}He [24]. The GPE static density response function $\chi_\rho(q)$ is a Lorentzian function of q centered at $q = 0$, a behavior completely different from the experimentally determined $\chi_\rho(q)$ that is characterized by a sharp peak at $q_{\chi_\rho} \simeq 2.0$ Å^{-1} [24]. For a long time it has been known from many-body computations [25,26] that the short range structure of the vortex core in ^{4}He is much more complex than the simple rarefaction region [23] given by GPE, in which the local density $\rho(r)$ vanishes at the vortex axis $r = 0$ and smoothly approaches the bulk density at large distances. As a result, we expect the GPE to provide plausible conclusions for the large scale dynamics of the vortex tangle, while phenomena like vortex reconnections, requiring the full treatment of strong correlations at atomic length scale, need further scrutiny.

The recent many-body computation [20] of a vortex line in liquid ^{4}He at $T = 0$ K is based on the fixed phase approximation: by writing the vortex wave function $\psi_v(R)$, $R = (\vec{r}_1, \ldots, \vec{r}_N)$, in term of its modulus and phase, $\psi_v(R) = |\psi_v(R)| \exp[i\Phi(R)]$, one makes an ansatz for the functional form of $\Phi(R)$, obtaining a Schrödinger-like equation for $|\psi_v(R)|$ [27]. This equation was solved [20] by the shadow path integral ground state (SPIGS) [28,29] Monte Carlo simulation, an unbiased "exact" method [30]. The resulting local density $\rho(r)$ is not a monotonic function of the distance r from the vortex axis and it approaches the bulk density ρ_0 in an oscillating way (see Fig. 1). The best vortex energy is obtained when the phase $\Phi(R)$ contains backflow terms; i.e., the phase is not a simple additive function of the phases of the particles but contains also terms depending on positions of pairs of particles in a way similar to the Feynman-Cohen theory [31] of the roton excitation. One finds three related features [32]: the density $\rho(r = 0)$ on the axis is nonzero, the velocity field $\vec{v}(\vec{r})$ at short distance deviates from the r^{-1} behavior given by GPE with $\vec{v}(\vec{r})$ being finite even at $\vec{r} = 0$ and $\nabla \times \vec{v}(\vec{r})$ is nonzero in a finite region around the vortex axis [26].

It is instructive to look not only at $\rho(r)$ but also at the Fourier transform $\Delta\tilde{\rho}(q)$, of the adimensional density variation $\Delta\rho(r) = \rho(r)/\rho_0 - 1$ (here and in the following we use the convention that momenta $\vec{q}$ lie in the xy plane). $\Delta\tilde{\rho}(q)$ multiplied by q at the equilibrium density of ^{4}He is shown in Fig. 2 for the SPIGS computation with the backflow phase [34], as well as the result for the GPE. The GPE $q\Delta\tilde{\rho}(q)$ has a rather wide minimum at q in the phonon region whereas the fixed phase $q\Delta\tilde{\rho}(q)$ is rather small in the phonon region and is dominated by a sharp minimum at a larger $q_{\min}$. At the equilibrium density $q_{\min} \simeq 2.0$ Å^{-1} is very close to q_{χ_ρ}, as expected from linear response theory, and larger than $q_R = 1.91$ Å^{-1}, the wave vector of the roton minimum. This behavior has been verified at all densities in the fluid phase [20]. Thus the spectrum of the density profile has the largest intensity in a range of q corresponding to that of R^+ rotons, i.e., rotons with $q > q_R$ for which the group velocity is positive.

In classical hydrodynamics of an incompressible fluid two antiparallel vortex lines form a stable object. In the quantum case the behavior is quite different as shown by GPE: due to the finite quantum compressibility a pair of antiparallel vortices approach each other until the two topological defects of opposite sign cancel each other leaving a rarefaction region [18], which expands and propagates as phonon excitations. A more general model of the reconnection dynamics, consisting of two intersecting vortex rings [17] as well for generic shape [35], indicates a strong deformation of the vortex filaments before reconnection and, after reconnection, a shortening of the vortex line length with formation of a rarefaction region. We can understand this GPE result as a way of

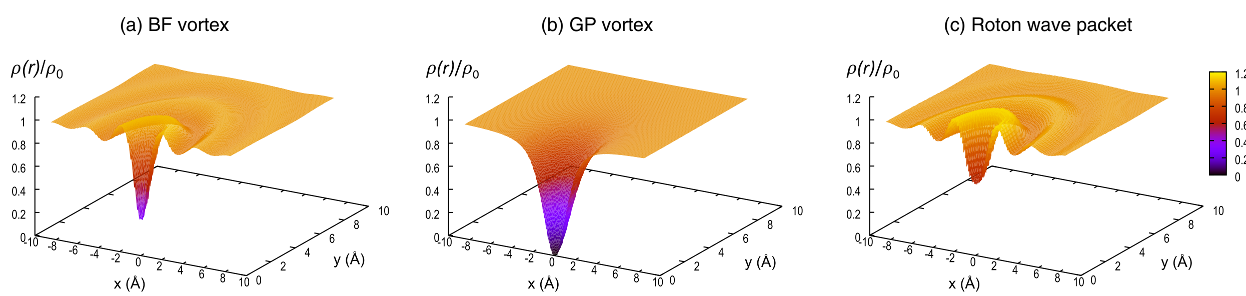

FIG. 1. Rescaled density $[\rho(x,y)/\rho_0]$ of the (cylindrically symmetric) vortex, where the vortex axis is along z, as computed from (a) BF–SPIGS [20], (b) GPE with coherence length $\xi = 0.87$ Å^{-1} [33]. In (c) the rescaled density for the wave packet (1) with parameters $A_1 = -0.8$, $\sigma_1 = 0.25$ Å^{-1}, $q_2 = 1.95$ Å^{-1}, $\sigma_2 = 0.35$ Å^{-1} is shown.

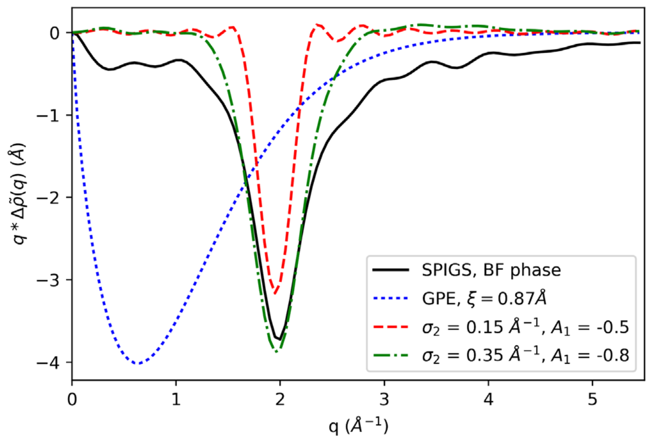

FIG. 2. Fourier transform times q, $q\int drrJ_0(qr)\Delta\rho(r)$ (J_0 being a Bessel function), of the (cylindrically symmetric) density variation $\Delta\rho(r)$ of a vortex as computed from GPE with coherence length $\xi = 0.87$ Å^{-1} (blue dotted curve), of a BF–SPIGS vortex (solid black curve) and of two wave packets with parameters $A_1 = -0.5$, $\sigma_1 = 0.25$ Å^{-1}, $q_2 = 1.95$ Å^{-1}, $\sigma_2 = 0.15$ Å^{-1} (red dashed curve) and $A_1 = -0.8$, $\sigma_1 = 0.25$ Å^{-1}, $q_2 = 1.95$ Å^{-1}, $\sigma_2 = 0.35$ Å^{-1} (green dot-dashed curve); this last one has the density profile shown in Fig. 1(c).

avoiding sharp kinks of the two vortices after reconnection because this would correspond to a very highly excited state of Kelvin waves. The process of avoiding high curvature cusps in the vortex system is expected to be generic so it should happen also in a strongly interacting system like ^{4}He. In this case, however, we propose a crucial microscopic modification: after the phase singularities of the two reconnecting vortices locally cancel out, what is left is not a rarefaction region but a density modulation dominated by wave vectors of order of 2 Å^{-1}. This modulation is no more sustained by the centrifugal force associated with the phase $\Phi(R)$ and can be efficiently described in terms of bulk excitations, which will propagate carrying away some energy. Below we will use results from GPE simulations to estimate such energy. The study of the shortening of the length of two reconnecting vortices is an important problem to address with a microscopic theory.

In order to get insight into the nature of the excitations generated in a reconnection, we pose the following question: given a wave packet $\psi(R) = \int d\vec{q}\pi(\vec{q})\psi_{\vec{q}}(R)$, built up from the *bulk* single excitation states $\psi_{\vec{q}}$, can we find an amplitude $\pi(q)$ that yields a cylindrical density modulation with similar features to those given by $|\psi_v(R)|$? A standard WP is centered around a given wave vector $\vec{q}$ and position $\vec{r}$. In order to have a packet with cylindrical symmetry with respect to the z axis $\vec{q}$ has to be normal to the vortex axis and it has to be averaged over the directions in the $q_x - q_y$ plane. In addition, one has to average also with respect to the directions of $\vec{r}$ in the x-y plane if $\vec{r}$ does not lie on the vortex axis. See the Supplemental Material [36] for such averages. At the end one can write the packet as $\psi(R) = \int d\vec{q}\pi(q)\psi_{\vec{q}}(R)$. Thus we restrict to packets of cylindrical symmetry $\pi(\vec{q}) = \pi(q)$, $q = \sqrt{q_x^2 + q_y^2}$, and as a wave function of the bulk excited states we adopt for $\psi_{\vec{q}}$ the shadow variational wave function [37–40] and explore different packets, as discussed in the Supplemental Material [36]. Density can be computed by means of a Monte Carlo sampling and search for parameters of the packets giving density profiles close to that of the vortex. In Fig. 1 one profile is shown corresponding to a double Gaussian $\pi(q)$,

$$\pi_{\{A_1,\sigma_1,q_2,\sigma_2\}}(q) = A_1 e^{-\frac{q^2}{2\sigma_1^2}} + e^{-\frac{(q-q_2)^2}{2\sigma_2^2}}, \qquad (1)$$

one centered at $q = 0$ and one at $q = q_2$. The value of q_2 controls the position of the main minimum of $\Delta\tilde{\rho}(q)$ and σ_2 its width. In order to have a wave packet with suppression of the local density at the vortex core and a negative minimum of $\Delta\tilde{\rho}(q)$ at large q, A_1 has to be negative. One can understand this contribution at $q \simeq 0$ as due to an interference effect between two roton states of almost antiparallel momenta. See Supplemental Material [36] for further discussion. As it can be seen from Figs. 1 and 2, when q_2 is in the roton region the shape of the vortex density profile is well reproduced by our WPs, for a broad range of A_1, σ_1, and σ_2; the deviations at large wavelength in Fig. 2 are presumably due to multiple excitations not included in our model (see Supplemental Material [36]). The amplitude of the density oscillations of the WPs depends on the length L_z of the simulation box. By changing L_z the shape of $\Delta\tilde{\rho}(q)$ remain essentially unchanged but its amplitude scale roughly as $1/L_z$ because the effect of our single excitation WP is spread over a region proportional to L_z (see Supplemental Material [36]). In order that the amplitude of the density oscillation of the WP matches that of the vortex as in Fig. 2 one excitation per about 25–30 Å is needed. There is a nice consistency check of this because the contribution of $|\psi_v(R)|$ to the vortex energy is 0.4 K/Å (see Supplemental Material [36]) so that a length of order of 25 Å corresponds to the energy of a roton. Computing the energy density as $q|\pi(q)|^2\epsilon(q)$, we find that R^+ rotons with wave vector $q \in [1.93, 2.15]$ account for the 30%–50% of total energy. The roton contribution might even be higher due to multiple excitation contributions (see Supplemental Material [36]). Using WP with only phonons (maxons), we obtain density profiles with very weak modulations (different oscillation wavelengths). The shown results are robust to changing Gaussian into Lorentzian or to adding a third peak so we conjecture that the dissipation waves emitted in a reconnection event have a low-energy phonon component plus an energetically relevant roton contribution.

The presence of energetic rotons in the superfluid even at very low T due to vortex reconnections can be experimentally detected because such rotons will be able to cause QE of ^{4}He atoms [1]: QE can take place under the conditions that the excitation propagates ballistically in the bulk, that its energy is larger than $E_b = 7.15$ K, the binding energy of ^{4}He in the liquid, with conservation of momentum parallel to the interface and of energy, $\epsilon_q = E_b + \hbar^2 k^2/2m$, where $\hbar\vec{k}$ is the momentum of the ^{4}He evaporated atom. QE has been detected for phonons of energy above about 10 K, for R^- and for R^+ rotons, with R^+ rotons having the largest efficiency for QE, of order of 0.3. At $T \ll 1$ K the number of thermal rotons and high energy phonons is negligible so no evaporation of ^{4}He atoms should take place. On the basis of our conjecture, a vortex tangle is a source of nonthermal rotons, so that one should observe processes of QE even at low T, as long as the tangle is present in a superfluid that has a free surface [see Fig. 3(a)]. In order to prove that QE from a vortex tangle is due to rotons, one needs to focalize the excitations and to perform time-of-flight measurements, so that one can use the conservation laws to verify the dynamics of the process. A possible experimental setup is shown in Fig. 3(b) with one chamber where vorticity is generated and an evaporation upper chamber only partially filled with ^{4}He and with the detecting bolometers [41]. The two chambers are connected by a periodically opened duct or the generation of the vortex tangle is periodic in time.

A crucial aspect is if the rate of evaporated atoms is large enough to be detected. We can evaluate this rate starting from the frequency of reconnections per unit volume in a random tangle [42], $f_{\rm rec} = (\kappa/6\pi)L^{5/2}\ln(L^{-1/2}/a_0)$, where $\kappa = h/m \simeq 10^{-3}$ cm^2/s is the quantum of circulation, a_0 is of order 1 Å, and L is the total length of vortex lines per unit volume. Typical experimental values of L are in the range 10^2–10^6 cm^{-2}. For such values of L, the volume of the cores of vortices is negligible, so rotons should propagate ballistically through the tangle. The logarithmic term in $f_{\rm rec}$ depends very weakly on L in this range and $\ln(L^{-1/2}/a_0)/6\pi$ is close to unity. For example for $L = 10^2$, 10^4, and 10^6 cm^{-2} we get, respectively, $f_{\rm rec} \simeq 10^2$, 10^7, and 10^{12} cm^{-3} s^{-1}. Next we need an estimate of how many rotons are emitted in a reconnection. In GPE the energy ΔE transferred from vortex flow energy to the rarefaction wave depends on the geometry of the reconnecting vortices and $\Delta E = 10$ K is the typical value [43]. As an order of magnitude estimate we can assume this GPE value for ΔE also for ^{4}He because the GPE vortex energy with coherence length 0.87 Å as in Ref. [33] is in good agreement with the many-body computation also at short distance (see Supplemental Material [36]). Estimating which percentage of this energy is dissipated in rotons rather than in phonons is a main goal of the experiment. Taking 10% as a lower bound for roton emission, using the known probability 0.3 for quantum evaporation by R^+ rotons [44,45] and the about 5% probability that the roton impinges on the surface within 25° from the vertical so that it can give QE [4], we get that $f_{\rm ev}$, the rate of evaporated atoms per unit time and unit volume of the tangle, is $f_{\rm ev} \simeq 10^{-1}$, 10^4, and 10^9 cm^{-3} s^{-1} for $L = 10^2$, 10^4, and 10^6 cm^{-2}, respectively. A bolometer of sensitivity 10^{-11} erg [46] is able to detect the energy of about 10^4 rotons so the estimated number of evaporated atoms should be detectable with current instrumentation [47], at least for $L > 10^4$ cm^{-2}.

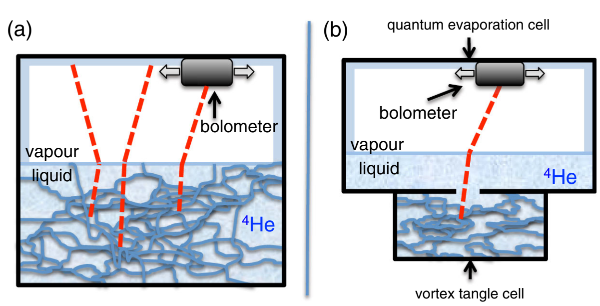

FIG. 3. Quantum evaporation setup to detect excitations generated in vortex reconnections. Red dashed lines represent excitations (rotons) transforming into evaporated ^{4}He. Scheme (a) is suited for measuring the dependence of the signal on the amount of turbulence L, while (b) is for an energy-resolved detection of particles.

The scaling of measured evaporated ^{4}He versus L, which can be independently measured [8], allows a consistency check for $f_{\rm rec}$ with the fact that the measured evaporated atoms originated from reconnections. An additional reason of interest for performing QE experiments in presence of a vortex tangle is to assess the presence of high energy phonons in quantum turbulence. By high energy phonons we mean phonons with $q > k_c \simeq 0.55$ Å^{-1} ($\epsilon_q \gtrsim 10$ K), where at k_c the dispersion changes from anomalous to normal in the liquid at s.v.p. [48], so that such phonons do not decay spontaneously but propagate ballistically and can produce QE. Present theories of dissipation in QT predict that phonons should be emitted with q well below k_c. For ^{4}He there is really no quantitative microscopic theory of Kelvin waves and of their interaction with phonons at large q, so that detection of processes of QE by nonthermal high-q phonons from a vortex tangle would add important information on such aspects.

In summary, advanced quantum simulations of a vortex line in ^{4}He and of roton in the bulk show that the vortex core structure can well be represented by a cylindrical WP of rotons so we can view the vortex core mainly as a cloud of virtual rotons sustained by the phase of the vortex wave function. In a natural way this leads to the conjecture that in a vortex reconnection some of the virtual rotons become real and propagating. We estimate the number of such nonthermal rotons and of the expected rate of quantum evaporation processes that should be large enough to be detected with current instrumentation. Additional information on quantum vorticity will be obtained if the quantum evaporation measurement should detect also the presence of high energy phonons. With such an experiment one could shed light on some of the microscopic processes that are important in the evolution of a vortex tangle in a pure superfluid. On the theory side, GPE has been generalized [49] and extended [50] in the spirit of the time dependent density functional and it will be interesting to study vortex reconnections with this last theory that is known [51] to give a good description of the vortex structure in ^{4}He.

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