Edge Singularities and Quasilong-Range Order in Nonequilibrium Steady States

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The singularities of the dynamical response function are one of the most remarkable effects in many-body interacting systems. However in one dimension these divergences only exist strictly at zero temperature, making their observation very difficult in most cold atomic experimental settings. Moreover the presence of a finite temperature destroys another feature of one-dimensional quantum liquids: the real space quasilong-range order in which the spatial correlation functions exhibit power-law decay. We consider a nonequilibrium protocol where two interacting Bose gases are prepared either at different temperatures or chemical potentials and then joined. We show that the nonequilibrium steady state emerging at large times around the junction displays edge singularities in the response function and quasilong-range order.

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Introduction.—X-ray edge singularities are one of the most spectacular phenomena of strongly correlated fermionic systems. These are divergences (in general nonanalycities) of the response functions in the vicinity of the threshold energies caused by the Fermi sea structure of the many-body ground state. The general theory of edge singularities was developed in late 1960s [1–7] and since then has been one of the hallmarks of nonperturbative quantum many-body physics.

In metals, absorption of a high energy photon (x-ray) with momentum k creates a core hole by exciting one of the electrons to the conductance band. At zero temperature, the Fermi sea is completely filled, and therefore, there is a threshold energy $\omega_-(k)$ for such a process to occur. The response of the system is then controlled by two competing processes. The created core hole for the conductance electrons leads to the orthogonality catastrophe [1], which decreases the response. On the other hand, the attractive interaction between the electron and the core hole enhances the response [2]. Both effects are nonperturbative and the result of their competition is encoded in the exponent $\mu(k)$ controlling the behavior of the dynamic structure factor (DSF) in the vicinity of $\omega_-(k)$

$$S(k,\omega) \simeq S_-(k)|\omega - \omega_-(k)|^{\mu(k)}. \qquad (1)$$

The threshold exponent $\mu(k)$ is proportional to the scattering phase of conduction electrons at the Fermi surface with the core hole. The scattering phase, that depends on the microscopic interactions, can be negative or positive resulting in either the singularity or vanishing of the DSF. Over the years, the edge singularities were observed in many electronic systems. The most direct observation is the absorption of the x-rays in metals, e.g., [8–10]. They also appear in other situations like, for example, resonant tunneling experiments [11] or a quantum dot coupled to a degenerate electron gas [12].

Phenomena of the same nature also appear in nonmetallic one-dimensional (1D) systems. In 1D systems, the presence of the interactions also leads to a formation of the Fermi sea for nonfermionic systems, such as the Lieb-Liniger gas of bosons [13]. The Fermi sea structure of the ground state is a universal feature of 1D quantum liquids, as described by the Luttinger liquids (LL) theory [14,15], which supersedes the Fermi liquid description valid in higher dimensions. The Luttinger liquid physics were very recently observed experimentally [16] and also in a number of other situations in the past years [17–19]. Like in metals, the presence of an effective Fermi sea and interactions creates suitable conditions for the appearance of the edge singularities. This intuition resulted in a full-fledged theory of nonlinear Luttinger liquids [20] (NL-LL), for which the

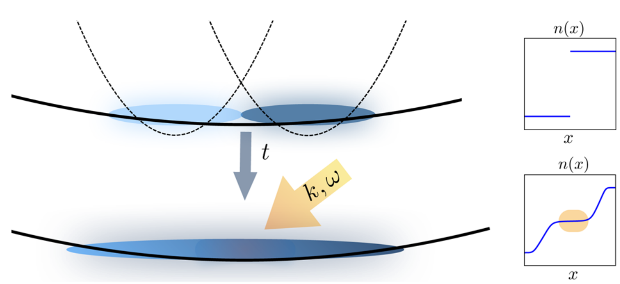

FIG. 1. Nonequilibrium protocol to observe edge singularities in the NESS. Top: two quantum gas at different density n are prepared and then joined together, Bottom. After waiting some time that the system reaches its steady state close to the junction (see plot of the density $n(x)$ on the right) a Bragg pulse is shined around the center of the cloud to probe its dynamical correlations.

Lieb-Liniger model served as the main playground [21]. The theory of NL-LL predicts the edge singularities in the response functions to be a universal feature of the ground states of quantum liquids in 1D [20,22–26].

However, the edge singularities in 1D have avoided experimental observation so far, except for few qualitative results as in [27]. The main reason being that the ground state physics of gapless systems are obscured by the usual presence of finite temperature fluctuations that hide the quantum correlations. Indeed, while in 3D metals the Fermi temperature is of order of 10^3 kelvins, for ultracold gases in typical experimental settings this is of the order of nano-kelvins. On the other hand, the edge singularities physics are not limited to equilibrium states. Indeed, the main ingredient necessary for their appearance is a discontinuity in the fermionic occupation number. Nonequilibrium states of matter displaying edge singularities were theoretically proposed in past, like a state with two or more Fermi seas with different chemical potentials [28–31]. Another example is a defected Fermi sea (Moses state), which was introduced both in 3D [32] and 1D systems [33,34]. The past few years have witnessed huge developments in studies of low dimensional systems out of equilibrium [35–40] and the possibility of creating exotic states of matter via out-of-equilibrium protocols [41–47]. Among them, and interesting from the edge singularities point of view, are the so-called bipartite quench protocols [48–70]. They consist of two extended and independent systems at thermal equilibrium, albeit at different temperatures, T_L and T_R, and/or densities, n_L and n_R. At time $t = 0$, the two systems are connected, see Fig. 1, and at late times close to the junction, a translationally invariant nonequilibrium steady state (NESS) emerges. Given the (quasi-) particles of the model with momentum $k(\lambda)$ [with $\lambda \in \mathbb{R}$, the rapidity variable that parametrizes the particle momenta $k(\lambda)$] and their (dressed) dispersion relation $\varepsilon(\lambda)$, their momentum distribution $\vartheta(\lambda) \in [0,1]$ in the NESS takes the following form in terms of the Heaviside Θ function

$$\vartheta_{\mathrm{NESS}}(\lambda) = \Theta(v_{\mathrm{NESS}}(\lambda))\vartheta_L(\lambda) + \Theta(-v_{\mathrm{NESS}}(\lambda))\vartheta_R(\lambda). \tag{2}$$

Here, the velocity $v_{\mathrm{NESS}}(\lambda)$ is the group velocity of the quasiparticles in the NESS state. For free models and relativistic invariant field theories, this is a global parameter (or function) of the theory [52,71–73]. For interacting models, this is instead a nontrivial functional of the distribution $\vartheta_{\mathrm{NESS}}(\lambda)$ [64,65], and therefore, Eq. (2) has to be solved recursively. While the NESS state (2) requires the presence of stable (or at least long-lived) quasiparticle excitations, recently it was shown that at low temperature, the form (2) is also valid for generic 1D interacting models [74].

The presence of the discontinuity in (2), at λ_0 such that $v_{\mathrm{NESS}}(\lambda_0) = 0$, suggests, according to the general theory introduced above, that the DSF of the NESS state might exhibit edge singularities. In this Letter, we show that this is indeed the case, and we characterize the threshold energies and threshold exponents. To achieve this, we use a recently developed approach to the DSF for the Lieb-Liniger model introduced in [75,76].

Dynamical correlations of the NESS.—We focus here on the Lieb-Liniger model [13] for interacting bosons with repulsive coupling $c > 0$. The model is experimentally relevant for cold atomic physics [77–88], and its non-equilibrium properties, especially after a quantum quench [85,89–101], have attracted a lot of attention in recent years. Its Hamiltonian density is given by

$$\mathcal{H}(x) = -\psi^\dagger(x)\partial_x^2\psi(x) + c\psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x), \tag{3}$$

with $\psi(x)$ representing the canonical Bose field. The group velocity of the quasiparticles is given in terms of $\vartheta(\lambda)$ by an integral equation [102]

$$v(\lambda) = 2\lambda + \int_{-\infty}^{+\infty}\frac{d\mu}{2\pi}k'(\mu)\theta(\lambda-\mu)\vartheta(\mu)\big(v(\mu)-v(\lambda)\big), \tag{4}$$

with the scattering phase of the model $\theta(\lambda) = 2\arctan(\lambda/c)$ and with the momentum given by $k(\lambda) = \lambda + \int_{-\infty}^{+\infty}d\mu\theta(\lambda-\mu)\vartheta(\mu)$. If we specialize to the NESS state, the filling function $\vartheta(\lambda)$ is given by Eq. (2), and it displays a discontinuity at λ_0. We denote the height of the discontinuity by $\Delta\vartheta = \vartheta_L(\lambda_0) - \vartheta_R(\lambda_0)$.

We consider the two-point correlation function $S(x,t) = \langle\mathrm{NESS}|\hat\rho(x,t)\hat\rho(0,0)|\mathrm{NESS}\rangle$ of the density operator $\hat\rho(x) = \psi^\dagger(x)\psi(x)$. Our main interest is its Fourier transform, the DSF

$$S(k,\omega) = \int_{-\infty}^{+\infty}dt\int_{-\infty}^{+\infty}dx e^{i(kx-\omega t)}S(x,t). \tag{5}$$

In the Lieb-Liniger gas, being an interacting model, the density operator can create any number of pairs of particle-hole excitations on the reference state. Each particle-hole pair corresponds to a local (small) modification of the filling function $\vartheta(\lambda)$ at positions of the particle p and hole h. We denote a state with a single particle-hole pair as $|\mathrm{NESS}, h\to p\rangle$. The momentum k and the energy ω of this excited state with respect to the $|\mathrm{NESS}\rangle$ is [103,104]

$$k = k(p) - k(h), \qquad \omega = \varepsilon(p) - \varepsilon(h), \tag{6}$$

where $k(\lambda)$ was defined above and $\varepsilon(\lambda)$ is the dressed energy $\varepsilon(\lambda) = \lambda^2 + 2\int_{-\infty}^{+\infty}\mathrm{d}\alpha\,\alpha F(\alpha|\lambda)\vartheta(\alpha)$, such that the group velocity is given by $v(\lambda) = \partial\varepsilon(\lambda)/\partial k(\lambda)$. The back-flow function (the dressed scattering phase) obeys an integral equation

$$2\pi F(\lambda|\alpha) = \theta(\lambda-\alpha) + \int_{-\infty}^{+\infty}d\lambda'\vartheta(\lambda')F(\lambda'|\alpha)K(\lambda-\lambda'), \tag{7}$$

with the scattering kernel given by $K(\lambda) = d\theta(\lambda)/d\lambda$.

The spectral representation of $S(k,\omega)$ can be organized in the sum over the number of created particle-hole excitations and expressed through the form factor of the density operator $\langle\text{NESS}|\hat{\rho}(0)|\text{NESS}, h\to p\rangle$. These form factors for the Lieb-Liniger model on a finite line of length L were derived in [105], and since then, they were studied and used in the computation of the correlation functions [106–109]. In [75,76,110], we have studied a general expression for the thermodynamic limit of such form factors and shown that their form depends strongly on the analyticity of the distribution function $\vartheta(\lambda)$ and on the momentum carried by the excitation, see Supplemental Material [111]. When the state is characterized by a discontinuous $\vartheta(\lambda)$, the form factors contain singularities that strongly complicate the computation of the DSF, see, e.g., [106,107]. However, if the external momentum k is small enough compared with the scale k^*, which is due to the discontinuity $\Delta\vartheta$, the form factors take a manageable form. One of the consequences of this is that at a small momentum, the leading order of the DSF is given by the form factors of a single particle-hole pair, which reads [76]

$$|\langle\text{NESS}|\hat{\rho}(0)|\text{NESS}, h\to p\rangle| \simeq k'(p)\left|\frac{\lambda_0-h}{\lambda_0-p}\right|^{\Delta\vartheta F(\lambda_0)}, \qquad (8)$$

with corrections of order k. Here, $F(\lambda)=F(\lambda|p)-F(\lambda|h)$ is the backflow of the particle-hole excitation and its sign is opposite to that of the momentum k. Equation (8) holds if the momentum k of the excited state is smaller than two scales appearing in the problem: the scale k^* given by $k^*=|(k'(\lambda_0)/\partial_\lambda F(\lambda_0|\lambda)|_{\lambda=\lambda_0})|$ [76] and the interaction parameter c. We assume that $k/k^*\ll 1$ and $k/c\ll 1$.

For each choice of (k,ω), there is the corresponding excitation (p,h). The correlation function in (5) is then equal to a single form factor contribution multiplied by the Jacobian of the change of variables from (p,h) to (k,ω) and up to corrections of order k/c is [111]

$$S(k,\omega)=\mathcal{D}(k,\omega)|\langle\text{NESS}|\hat{\rho}(0)|\text{NESS}, h\to p\rangle|^2. \qquad (9)$$

The density of states $\mathcal{D}(k,\omega)=(\vartheta(h)(1-\vartheta(p))/|v(p)-v(h)|)$ and the position of particle and hole, p, h, are given by the energy and momentum conservation. The same formula, with $|\text{NESS}\rangle$ replaced by a thermal state [104,112], holds at thermal equilibrium. Figures 2 and 3 show the dynamic structure factor computed with this formula for the NESS state and a thermal state.

The singular behavior of the DSF on the NESS is similar to the one encountered for the ground state of the Lieb-Liniger model. There, the Fermi sea structure $\vartheta_{\text{GS}}(\lambda)=\theta(\lambda+\lambda_F)\theta(\lambda_F-\lambda)$ (with λ_F the Fermi rapidity such that $k(\pm\lambda_F)=\pm k_F$) leads to two fundamental types of the particle-hole excitations [114]. The particle type describes excitation in which a hole is created at the edge of the Fermi sea and the particle is free to move. The hole type corresponds to a reversed situation when the particle position is fixed to the edge of the Fermi sea and the hole instead is free to move. The form factors of the density operator are singular for both types of excitations, which leads to well-known singularities of the correlation functions as universally described by the nonlinear Luttinger liquid [20,24,25].

Here, we face a similar situation, see Figs. 2 and 4. However, there are differences. First, there is only one discontinuity, at λ_0. Second, the NESS distribution is

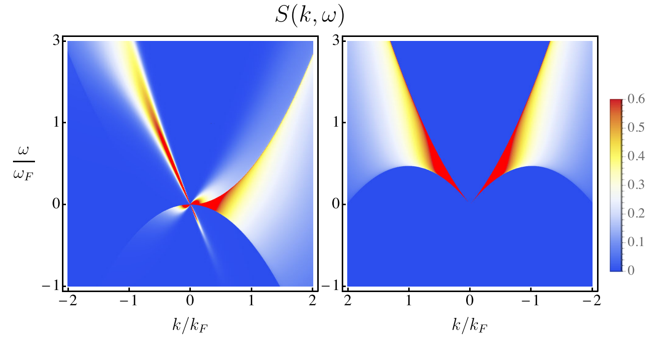

FIG. 2. Density plot of the DSF (in unit of n/ω_F) computed on the NESS (left) and on the ground state (right) of a gas, with the density $n=n_{\text{NESS}}$ and the coupling strength $c=10$ ($k_F=\pi n$, $\omega_F=k_F^2$). The NESS is obtained by joining a left gas with $T_L=1$, $n_L=1$ and a right gas with $T_R=1$, $n_R=0.1$. The coupling strength $c=10$ of both gases is the same (density $n_{\text{NESS}}=0.54$). In the NESS and for $k>0$, the DSF is divergent close to the particle excitations $\omega_+(k)\sim k^2/2m_0$, and it has a zero in proximity of the hole edge $\omega_-(k)\sim -k^2/2m_0$. For a negative k, the situation is reversed. In the ground state, the DSF has a pole (zero) around the particle (hole) edge $\omega_\pm(k)\sim v_s|k|\pm k^2/2m_F$ instead, see for example [25,113].

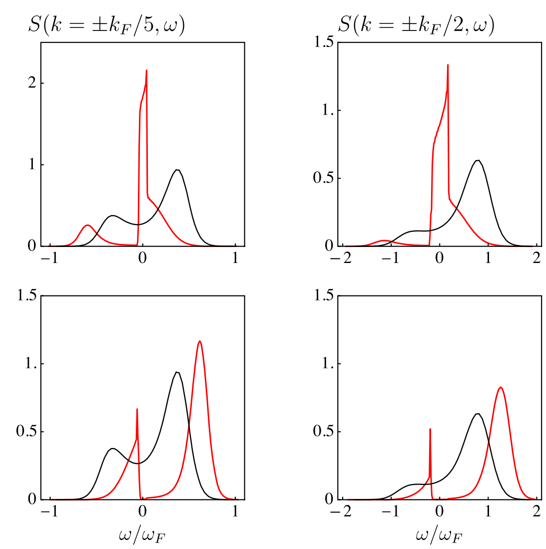

FIG. 3. DSF (9) of the NESS (red line) and on a thermal state with $T=1$, $n=n_{\text{NESS}}$, and $c=4$. Plots on top display data with $k=k_F$ and the ones on the bottom with $k=-k_F$ ($k_F=\pi n$). The NESS is the same as the one computed in Fig. 2.

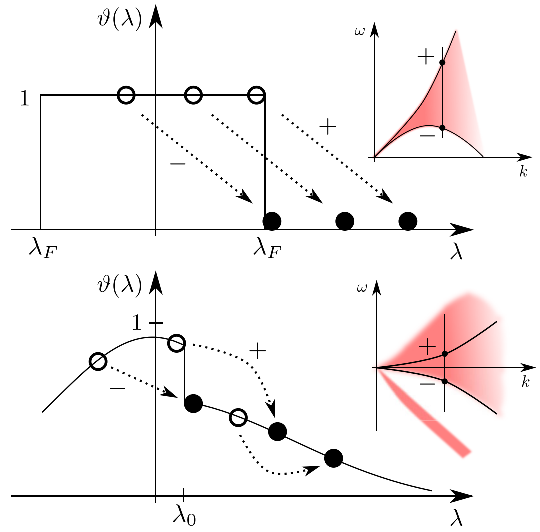

FIG. 4. The distribution function $\vartheta(\lambda)$ for the ground state (top) and the NESS state (bottom). For a fixed momentum k, a particle-hole excitation of the ground state can be created only for a finite interval of energies $\omega_+(k) - \omega_-(k)$. The edge excitations correspond to a configuration where either particle or hole is right at the edge of the Fermi sea λ_F. On the contrary, over the NESS state, particle-hole pairs of arbitrary energy can be created; however, there are again two edge modes where either particle or hole is placed at the discontinuity λ_0.

asymmetric, which implies that $S(k,\omega)$ is not a symmetric function of k, see Fig. 2. The third difference is related to the nature of the excitations. The ground state Fermi sea is completely filled up to λ_F and then empty. This means that the hole, which for the particle excitations should be placed at the edge, can be only placed just before the edge. For the NESS, the situation is different, since the distribution function is neither 1 or 0 in the vicinity of λ_0. Therefore, the hole can be created on both sides of the discontinuity. The situation is analogous to the hole type excitation where a particle can also be placed on both sides of the discontinuity. Therefore, the particle and hole excitations are themselves formed by two different microscopic configurations with the same dispersion relations.

The dispersion relations for the particle and hole excitations can be derived with a standard thermodynamic Bethe ansatz techniques [104]. At small k, they read

$$\omega_\pm(k) = \pm\frac{k^2}{2m_0} + O(k^3), \qquad m_0 = \frac{k'(\lambda_0)}{v'(\lambda_0)}. \quad (10)$$

While the ground state of quantum 1D liquids supports sound waves excitations, excitations in the NESS around the edge are massive since the velocity $v(\lambda)$ vanishes at λ_0.

The singularities in the DSF appear when the energy ω is close to $\omega_\pm(k)$. Explicitly, formula (9), in the vicinity of either singularity, is

$$S(k,\omega) \simeq S_\pm(k)\left|\frac{\omega-\omega_-(k)}{\omega-\omega_+(k)}\right|^{\mu(k)}, \quad (11)$$

with the exponent $\mu(k)$ and the momentum dependent prefactors $S_\pm(k)$ given in their leading order in k by

$$\mu(k) = -2\Delta\vartheta F(\lambda_0), \quad (12)$$

$$S_+(k) = \frac{m_0 k'(\lambda_0)^2}{|k|}(\vartheta_L(\lambda_0)+\vartheta_R(\lambda_0))(1-\vartheta_R(\lambda_0)),$$

$$S_-(k) = \frac{m_0 k'(\lambda_0)^2}{|k|}\vartheta_L(\lambda_0)(2-[\vartheta_L(\lambda_0)+\vartheta_R(\lambda_0)]). \quad (13)$$

The sum of the distribution functions in $S_\pm(k)$ reflects the aforementioned fact that each mode is made of two microscopic types of configurations. The singularity itself is controlled by the exponent $\mu(k)$. For the NESS state of Fig. 1, it results in a divergence along the particle excitations at positive k and along hole excitations for negative k.

Spatial correlations.—The presence of a discontinuity in the occupation number $\vartheta(\lambda)$ has also important consequences on the structure of the spatial density-density correlations. Static correlations in real space $S(x,0)$ can be expanded as a sum over particle-hole form factors, weighted by the momentum phase $e^{i(k(p)-k(h))x}$. For a NESS state at large x, the sum over particle-hole position accumulates around the discontinuity, and this leads to a power law decay of the correlations as

$$\frac{S(x,0)}{n_{\rm NESS}^2} \simeq 1 - \frac{A_0}{2(\pi x n_{\rm NESS})^2} + O(e^{-\beta_{L,R}|x|}), \qquad x \gg n_{\rm NESS}^{-1}. \quad (14)$$

The amplitude A_0 is given by the matrix element of a single particle-hole excitation close to the discontinuity λ_0, namely $\lim_{p,h\to\lambda_0}|\langle \mathrm{NESS}|\hat\rho|\mathrm{NESS}, h\to p\rangle|^2 = k'(\lambda_0)^2$, see [110]. We obtain $A_0 = (k'(\lambda_0)\Delta\vartheta)^2/2$. Notice that, this expression gives back the Luttinger liquid parameter $K = (k'(\pm\lambda_F))^2$ when the state is the ground state, see [104,107,109]. This shows that the NESS has much longer range density-density correlations compared to the left and right state, where the decay is instead exponential at large distances. A similar behavior was observed in the NESS states realized in d-dimensional free theories [115] and in trap-release experiments with hard-core bosons [42,116].

Beyond small k.—Until now, we have been considering the structure of singularities at small momenta. Comparing the obtained results with the NL-LL theory we can conjecture a formula for the edge exponents at an arbitrary k. The NL-LL theory predicts the threshold exponents of the ground state of the Lieb-Liniger model to be $1-\mu_L(k)-\mu_R(k)$ with $\mu_{L(R)}(k) = (1+F(\pm\lambda_F))^2$, where $L(R)$ are contributions coming from the left and the right

Fermi edges, both of height 1. The presence of a nontrivial $\vartheta(\lambda)$ (not simply equal to 0 or 1) affects the scattering phase as $F(\lambda) \to \vartheta(\lambda)F(\lambda)$, and we conjecture the threshold exponents for the NESS to be

$$\mu(k) = 1 - (1 + \Delta\vartheta F(\lambda_0))^2. \quad (15)$$

at any k. In the small momentum limit, the backflow is small and we recover the threshold exponent (12). In order to prove such a statement, one would need to formulate a NL-LL field theory for the excitations around the NESS, which is currently not known. Certain progresses in this direction recently reported on inhomogenous Luttinger liquids [99,117].

Conclusions.—We have shown that the bipartite non-equilibrium protocol leads to excited states with unusual properties. They have a finite energy density and entropy like thermal states, but despite that, they display correlations that are typical of the ground state; i.e., they exhibit edge singularities and quasilong-range order. We considered here an integrable model, but a NESS has been shown to exist for any model described by a conformal field theory [49,71] or strongly interacting theories in higher dimensions [118]. Moreover, a NESS should appear at intermediate time scales for any interacting theory sufficiently close to an critical [74,119] or integrable point [85]. We believe that our results pave the way towards a field theoretical, universal description of the NESS, similarly to the nonlinear Luttinger Liquid theory for the ground state [24] and general zero-entropy states [120].

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