Fast Preparation of Critical Ground States Using Superluminal Fronts

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We propose a spatiotemporal quench protocol that allows for the fast preparation of ground states of gapless models with Lorentz invariance. Assuming the system initially resides in the ground state of a corresponding massive model, we show that a superluminally moving "front" that *locally* quenches the mass, leaves behind it (in space) a state *arbitrarily close* to the ground state of the gapless model. Importantly, our protocol takes time $\mathcal{O}(L)$ to produce the ground state of a system of size $\sim L^d$ (d spatial dimensions), while a fully adiabatic protocol requires time $\sim\mathcal{O}(L^2)$ to produce a state with exponential accuracy in L. The physics of the dynamical problem can be understood in terms of relativistic rarefaction of excitations generated by the mass front. We provide proof of concept by solving the proposed quench exactly for a system of free bosons in arbitrary dimensions, and for free fermions in $d = 1$. We discuss the role of interactions and UV effects on the free-theory idealization, before numerically illustrating the usefulness of the approach via simulations on the quantum Heisenberg spin chain.

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Introduction.—A central challenge in harnessing the power of artificial quantum matter—for quantum computing and other technological purposes or for theoretical investigation—is that of quantum state preparation. While much progress has been made in engineering extremely isolated quantum systems—ultracold atoms in optical lattices [1–4] or traps [5,6], nitrogen vacancy centers [7–12], ion traps [13–15], superconducting qubit structures [16–19]—as these systems grow more complex, it becomes harder to devise equally elaborate tools to manipulate them while maintaining isolation from sources of decoherence. It is thus important to find theoretical answers to how efficiently specific quantum states can be prepared, and the minimum set of knobs required for this purpose.

In this regard, adiabatic evolution has served as a basis for many investigations (cf. Ref. [20]). The idea here is to prepare the system in an eigenstate of a Hamiltonian that is easily accessible and subsequently tune the Hamiltonian slowly to evolve this eigenstate into the target state. The limitation of this approach is speed; to avoid exciting the system in the process, the time taken must be of the order of the inverse square of the smallest instantaneous spectral gap between the target and excited states, a quantity that diverges in the thermodynamic limit for many systems of interest.

To achieve faster preparation, recent work has proposed engineering counterdiabatic drives [21–23] that counter the production of excitations during adiabatic evolution, or more radically, introducing "optimum-control" protocols [24–28] (including "bang-bang" protocols [29–32]) that entirely dispense with the adiabatic ansatz. As of now, a transparent theoretical prescription for diabatic protocols exists only for finite-size systems and how these insights may be extended to thermodynamically large systems is unclear. Another body of work [33,34] has proposed spatial quenches wherein a large chunk of the system serves as a bath to remove entropy from the subsystem of interest.

In this Letter, we provide a novel example of a diabatic protocol for preparing the ground state of a class of gapless systems—those with emergent Lorentz invariance—starting from the ground state of a corresponding model with an additional term that opens a gap. Such models naturally arise in the low-energy description of various condensed-matter systems, including one-dimensional quantum gases [35] and the Hubbard model at half-filling [36] in the strong coupling limit. We assume that the ground state of the massive model is easier to prepare due to the presence of a gap. Our approach differs from approaches inspired by the adiabatic ansatz in that it leads to generation of excitations; instead, our strategy is to invoke the symmetry of the model to "shepherd" excitations in a way that leaves a thermodynamically large region completely unexcited.

Specifically, we consider performing a spatiotemporal quench [37,38] wherein the *local* mass or gap is tuned to zero—abruptly, or on some timescale τ—along a superluminal trajectory $x = v_s t$ as illustrated in Fig. 1. Here $v_s > c$, where c is the speed of "light" in the emergent critical model. The quench front then serves as a source of excitations that emanate from the point $x = v_s t$, and travel onwards in all directions. Because of the motion of the front, right-moving excitations get blueshifted and are populated at higher energies, while left movers get redshifted and carry less energy. As the quench speed $v_s \to c^+$, the associated Doppler factor diverges and the left-moving modes are left entirely unpopulated.

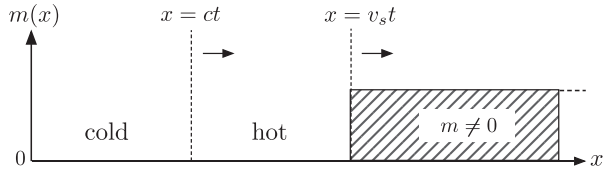

FIG. 1. The protocol: the local mass is tuned to zero along a front moving at superluminal speed $v_s > c$. As $v_s \to c^+$, right-moving waves form a shockwave carrying all the energy released in the quench, while the region $x < ct$, populated only by infinitely redshifted left-moving waves, is left unexcited.

In the one-dimensional case, for noninteracting models, this chirality has a huge consequence: the region $x < ct$ is populated only by left-moving excitations (right movers move past into the region $x \in [ct, v_s t]$) and is thus cold. These notions apply in higher dimensions albeit with minor modifications. We provide proof of concept with an exact solution of the quench for free relativistic bosons (in dimension $d \geq 1$) and fermions (in $d = 1$) for the locally instantaneous case ($\tau = 0$, finite v_s). Our protocol takes time $\sim O(L/c)$ to produce a state arbitrarily close to the ground state of the massless theory. We show that a spatially uniform, adiabatic protocol ($v_s = \infty$, finite τ) by contrast produces a state exponentially close to the ground state in time $\sim \mathcal{O}(L^2)$, parametrically slower than our protocol.

Next, we describe how our results apply to a general setting with interactions, band-curvature and ultraviolet effects. We first note that the total energy produced in the quench is, in fact, independent of v_s when $\tau = 0$—thus, cooling occurs purely due to spatial reorganization of the released energy into hot and cold regions, an effect that can be reversed by interactions favoring homogenization. It also calls into question our use of effective low-energy descriptions particularly in the limit $v_s \to c$ where the whole energy is localized in a vanishingly small region. We argue that introducing a finite τ (that does not scale as L) resolves these issues: a new timescale $\tau' = \gamma_s \tau$ emerges and controls the adiabaticity of the process. This timescale diverges in the limit $v_s \to c$ as per the Lorentz factor $\gamma_s = 1/\sqrt{1 - 1/v_s^2}$. Thus, the superluminal front enhances the timescale τ that introduces adiabaticity. We validate the effectiveness of our protocol via numerical simulations on an antiferromagnetic Heisenberg chain with a gap induced by a Néel field, as well as a classical model of phonons.

At the time of writing, we became aware of a similar proposal [39,40] in the Kibble-Zurek literature wherein a critical velocity for front propagation was proposed using scaling arguments. Our work demonstrates why this critical velocity is exactly c and the importance of relativistic effects in engendering *perfect cooling*. Moreover, Ref. [39] considers a transcritical protocol (transforming one gapped state into another, passing through a critical point) as opposed to our work focusing on the creation of the critical ground state. A description of scaling properties of correlations in such inhomogeneous protocols, in the spirit of Ref. [41] will be discussed in forthcoming work.

Model.—We study the following class of quench models described by the Lagrangian density $\mathcal{L}_b$:

$$\mathcal{L}_b = \partial_\mu \phi \cdot \partial^\mu \phi - m^2 \phi^2 f[(x - v_s t)/(v_s \tau)], \qquad (1)$$

with $f(x) = \frac{1}{2}[1 + \tanh(x)]$. We set $c \equiv 1$; $\partial_\mu \equiv (\partial_t, \boldsymbol{\nabla})$ and $\partial^\mu \equiv (\partial_t, -\boldsymbol{\nabla})$. The function f sets the local mass to m everywhere at $t = -\infty$ and 0 at $t = \infty$. The fields live in a box of linear dimension L and satisfy usual commutation relations [42]. We assume that the system is initially in the ground state of the massive theory.

The massless Lagrangian may describe the low-energy physics of a range of gapless one-dimensional systems, including many spin [43], boson, and fermion models [35], or in two dimensions, spin waves in the Hubbard model at half-filling [36], etc. A local gap in spin models may be opened by applying local magnetic fields or dimerization.

Solution for $\tau = 0$, finite v_s.—Here $f(x) = \Theta(x)$; the quench takes time $t_q = L/v_s$ from start to finish. The field operator at all times $t < x/v_s$ can be written in terms of a mode expansion $\phi(\boldsymbol{r}, t < x/v_s) = \sum_n [b_n v_n(x,t) + b_n^\dagger v_n^*(x,t)]$, where v_n are solutions to the massive Klein-Gordon equations, and b_n are bosonic operators associated with these modes. We work in the Heisenberg picture, fixing the initial state to $|0\rangle$ defined by the condition $b_n|0\rangle = 0 \ \forall \ n$. The condition $v_s > c$ ensures that no perturbations (traveling at speed c) due to the quench affect the space-time region $t < x/v_s$ and the mode expansion is valid.

For times $t > x/v_s$, the field operator evolves as per the massless solutions u_n: $\phi(\boldsymbol{r}, t > x/v_s) = \sum_n [b_n \gamma_n(\boldsymbol{r}, t) + b_n^\dagger \gamma_n^*(\boldsymbol{r}, t)]$, where $\gamma_n = \sum_m [\alpha_{n,m} u_m + \beta_{n,m} u_m^*]$ is determined by matching boundary conditions, that is, $D\gamma_n(\boldsymbol{r}, t = x/v_s) = D v_n(\boldsymbol{r}, t = x/v_s)$ with $D \in \{1, \partial_t, \partial_x, \boldsymbol{\nabla}_\perp\}$. The "Bogoliubov" coefficients $\alpha_{n,m}$ and $\beta_{n,m}$ can be found by evaluating appropriate Klein-Gordon norms, as described in Ref. [44]. All correlations subsequent to the quench can be evaluated using the above mode expansion, and applying Wick's theorem on the state $|0>$.

Chiral emanation.—The energy density after the quench can be evaluated as $\epsilon(\boldsymbol{r}, t) = \sum_n |\nabla \gamma_n|^2 + |\partial_t \gamma_n|^2 \approx \sum_{\boldsymbol{k}} \omega_k N_{\boldsymbol{k}} |u_{\boldsymbol{k}}(\boldsymbol{r}, t)|^2$. The approximation is valid in the infinite size limit, neglecting time-dependent terms involving products of wave functions with two different momenta or terms of the form $u_k \cdot u_{-k} \sim e^{-2i\omega_k t}$, which rapidly dephase. The population of modes is dependent on the direction $\hat{\boldsymbol{k}}$ of the mode with momentum $\boldsymbol{k} = k\hat{\boldsymbol{k}}$:

$$N_{\boldsymbol{k}} = \frac{(\Omega_{\eta(\theta)k} - \omega_{\eta(\theta)k})^2}{4\Omega_{\eta(\theta)k}\omega_{\eta(\theta)k}} \xrightarrow{\eta(\theta)\omega_k \ll m} \frac{m}{4\eta(\theta)\omega_k}, \qquad (2)$$

where $\Omega_k \equiv \sqrt{k^2 + m^2}$, $\omega_k \equiv k$ and $\eta(\theta) = \gamma_s(1 - u_s \cos\theta)$ where $u_s = 1/v_s < 1$. θ is the angle between $\hat{k}$ and $\hat{x}$,

ranging from 0 for right movers to π for left movers. For the uniform quench ($v_s = \infty$) we note that $\eta(\theta) = 1$. For $v_s \to c^+$, $\eta(0) \equiv 1/\eta_0 \to 0$, while $\eta(\pi) = \eta_0 \to \infty$; consequently, the energy $N_k \omega_k$ carried by left-moving waves vanishes $\forall k$ in this limit. In $d > 1$, most of the emission occurs in directions perpendicular to the motion of the front. Cooling in higher dimensions is based on the fact $\eta(\pi/2) = \gamma_s$ also diverges in this limit $v_s \to c^+$; this Doppler shift of orthogonally emitted radiation is a purely relativistic effect.

Energy density after quench.—To calculate the space and time dependence of the energy density, slow time-dependent correlations cannot be neglected. Their effect, however, can be captured using a simple physical picture of "heat waves" as described in Ref. [38] for $d = 1$, but which we find to be valid generally. In particular, excitations emanate from the quench front, carrying an energy $\omega_k N_{\boldsymbol{k}}$, which depends on $\hat{\boldsymbol{k}}$. The energy density at the end of the quench at any point $\epsilon_{\rm th}(x)$ is given by the average energy of all excitations emanating from the quench front and ending at this point. These ideas are empirically verified in Fig. 1 of Ref. [44]. Here we focus on the aspect of "cooling" and calculate the energy density at the end of the quench. First, note that the energy carried by waves emitted in the θ direction is given by $\epsilon_\theta \propto \int^{m/\eta(\theta)} k^{d-1} dk \omega_{\boldsymbol{k}} N_{\boldsymbol{k}} \propto (m/4)[1/\eta(\theta)^{d+1}](1/L_m^d)$, where $L_m^d = (m/c)^d$ has dimensions of volume. Higher momenta modes yield a parametrically similar contribution. (UV divergences occur for $d \geq 3$ but these are eliminated using finite τ.) Summing the contribution from these chiral waves yields in $d = 1$, $\epsilon_{\rm th}(x) = (1/\eta_0^2)\Theta(-x - L/2 + ct_q) + \frac{1}{2}[\eta_0^2 + (1/\eta_0^2)]\Theta(x + L/2 - ct_q)$; thus, the energy density goes to zero for $x < L/2$ in the limit $v_s \to c^+$ and all the energy is singularly located at $x = L/2$. Similarly, for $d > 1$, using the picture in Fig. 2(a), we find (see Ref. [44] for a more elaborate derivation)

$$\epsilon_{\rm th}(x) = \frac{\int_{\theta_x}^{\pi} \sin^{d-2}\theta \epsilon_\theta + \int_{\pi-\theta_x}^{\pi} \sin^{d-2}\theta \epsilon_\theta}{\int_0^\pi d\theta \sin^{d-2}\theta},$$
$$\theta_x = \mathrm{Re}\{\cos^{-1}[(x + L/2)/(u_s L)]\}, \tag{3}$$

where $\sin^{d-2}\theta$ is the appropriate angular measure in dimension $d > 1$. Some features of $\epsilon_{\rm th}(x)$ in different dimensions are shown in Fig. 2. Importantly, a thermodynamically relevant region is seen to become infinitely cold for $v_s \to c^+$.

Infinite accuracy.—The above discussion assumed the limit $L \to \infty$ to find the energy of excitations emanated in different directions but the distinction between "left" and "right" is meaningless at momenta $\sim 1/L$. The population of these modes is instead found to scale as $N_k = (m/4\gamma_s)$. Importantly, (a) this population also goes to zero as $v_s \to c^+$, and (b) it can be shown that this result is unaffected by finite L (a technical discussion and numerical confirmation is presented in Ref. [44]). Thus, the population of the lowest momentum modes can be tuned arbitrarily close to zero in our protocol.

Adiabatic cooling: Solution for finite τ, $v_s = \infty$.—In this case the quench occurs uniformly in space, but on a timescale τ. The time-dependent equations of motion can be solved exactly for fixed momenta to find two complete sets of modes $u_{\boldsymbol{k}}^{\rm ad}$ and $v_{\boldsymbol{k}}^{\rm ad}$ that behave like the massless and massive modes $u_{\boldsymbol{k}}$ and $v_{\boldsymbol{k}}$ at $t = \infty$ and $t = -\infty$ respectively—for details, see Sec. 3.4 of Ref. [49], where an analogous problem is solved for fields evolving in a time-dependent metric. Thus, the initial state can be described as a vacuum of the quanta $b_{\boldsymbol{k}}^{\rm ad}$ of the modes $v_{\boldsymbol{k}}^{\rm ad}$. The population of the quanta $a_{\boldsymbol{k}}^{\rm ad}$ of the modes $u_{\boldsymbol{k}}^{\rm ad}$, $\langle a_{\boldsymbol{k}}^{\dagger \rm ad} a_{\boldsymbol{k}}^{\rm ad} \rangle$, can be found exactly once $\beta_{\boldsymbol{k}}$ in $a_{\boldsymbol{k}}^{\rm ad} = \alpha_{\boldsymbol{k}} b_{\boldsymbol{k}}^{\rm ad} - \beta_{\boldsymbol{k}} b_{(-k_x, \boldsymbol{k}_\perp)}^{\dagger \rm ad}$ is determined. We find

$$N_{\boldsymbol{k}}^{\rm ad} = |\beta_{\boldsymbol{k}}|^2 = \frac{\sinh^2\left(\frac{\pi}{2}\tau(\Omega_{\boldsymbol{k}} - \omega_{\boldsymbol{k}})\right)}{\sinh(\pi\tau\omega_{\boldsymbol{k}}) \sinh(\pi\tau\Omega_{\boldsymbol{k}})} \underset{k > \tau^{-1}}{\overset{m\tau \gg 1}{\to}} e^{-2\pi\omega_k \tau}. \tag{4}$$

One can easily check that to obtain an energy density $\epsilon(x) \sim e^{-L}$, the time required scales as $\tau_{\rm ad} \sim \mathcal{O}(L^2)$. Thus, our proposed superluminal protocol that takes time $\sim \mathcal{O}(L)$ is more efficient than the adiabatic protocol.

Bosons vs fermions.—The fundamental conclusions above are unchanged for relativistic theories with different statistics. We examine this in the context of free

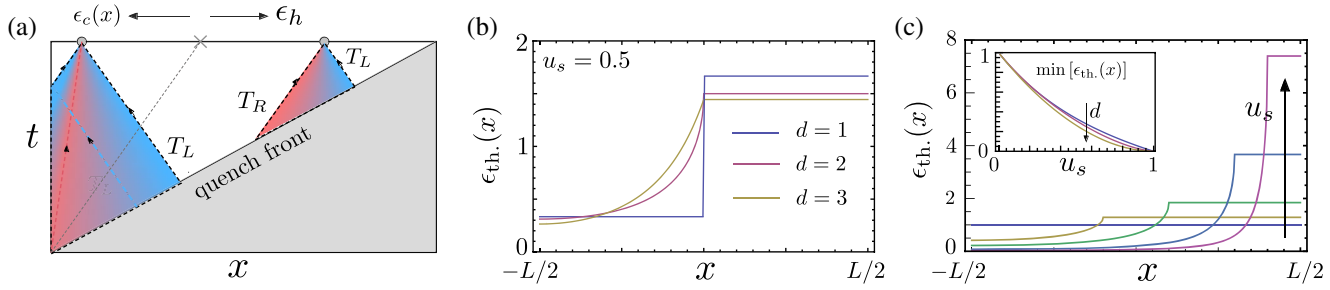

FIG. 2. (a) The energy density is determined by tracing the excitations impending on it to the quench front. At the end of the quench, $\epsilon_{\rm th}(x)$ is uniform in the region $x > -L/2 + ct_q$. (b) $\epsilon_{\rm th}(x)$ for $u_s = 0.5$ for various d. For $d > 1$, the curve is smooth for $x \leq -L/2 + ct_q = 0$. (c) $\epsilon_{\rm th}(x)$ in $d = 2$ for $u_s = \{0, 0.4, 0.6, 0.8, 0.9\}$; inset is $\min_x \epsilon_{\rm th}(x)$ as a function of u_s for $d = \{1, 2, 3\}$. The hot region grows slimmer and the transition into the cold region grows sharper as $u_s \to 1$ in any dimension.

fermions in $d = 1$, governed by the action $\mathcal{L}_f = i\bar{\psi}\partial_\mu\gamma^\mu\psi - m\bar{\psi}\psi f[(x - v_s t)/(v_s\tau)]$. The results for $\tau = 0$, finite v_s are obtained analogously to the bosonic solution; see Ref. [44]. We find occupation numbers

$$N_k^F = \frac{\Omega_{\eta(\theta)k} - \omega_{\eta(\theta)k}}{2\Omega_{\eta(\theta)k}} \xrightarrow{\eta(\theta)k \ll m} \frac{1}{2}. \tag{5}$$

Thus, for fermions, excitations are populated up to a "chemical potential" that is Doppler shifted $\sim m/\eta(\theta)$ as opposed to the bosonic case where the population at low momenta can be captured by a Doppler-shifted effective temperature [38,50].

Realistic models: Combination of adiabatic and superluminal cooling.—For $\tau = 0$, cooling occurs due to spatial separation of cold and hot regions—one can verify that $\int dx \epsilon_{\text{th}}(x, t_q) = 1$, and thus independent of v_s. Thus, one anticipates that interactions, which lead to a homogenization of the energy density, spoil the cooling effect. We now provide arguments showing how the introduction of finite τ resolves this issue. First, we note that the superluminal quench can be analyzed in a Lorentz-boosted frame moving at speed $u_s = 1/v_s < 1$. In this frame, the quench occurs uniformly in space. This analogy is clearly useful for $\tau = 0$: one may recover the result of the superluminal quench, Eq. (2), using the uniform, adiabatic quench result in Eq. (4) and Doppler shifting the momenta to obtain population of modes in the laboratory frame.

For large momenta $k \gg 1/L$, we can ignore the breaking of Lorentz symmetry by the walls, and use the above intuition to find the population of modes at finite τ and v_s. In the boosted frame, the mass term transforms as $f[(x - v_s t)/(v_s\tau)] \to f[-t'/\tau']$; thus, $\tau' = \gamma_s\tau$ emerges as the effective timescale for the quench in the boosted frame. Doppler shifting back into the laboratory frame, we find that the population of modes begins to decay rapidly for $\eta(\theta)\omega_k \ll \tau^{-1}/\gamma_s$. Thus, the cutoff determining the direction-dependent energy density ϵ_θ is now set by m/γ_s for $\tau^{-1} \approx m$ instead of m [see Eq. (2)]. This implies that the average energy density $\epsilon_{\text{ss+ad}} = \epsilon_{\text{ss}}/\gamma_s^d$ goes to zero in the limit $v_s \to c^+$ for finite τ, suggesting that the protocol can be useful for preparing the ground state of interacting models.

UV effects.—The linear dispersion $\omega_k = ck$ is crucial to the Doppler physics we rely on for our diabatic protocol. Beyond a certain energy scale (for instance, set by the lattice), this assumption breaks down and UV modes have a k-dependent group velocity $v_g(k) < c$. The effective cooling or heating factor for these modes can be estimated [51] as before with $\gamma_s(k) = 1/\sqrt{1 - (v_g(k)/v_s)^2}$—this is going to deviate minimally from 1 when $v_g(k)$ is much smaller than c. UV modes are thus excited in a nonchiral way. In the event that the UV scale $\Lambda < m/\eta(\theta)$ and $\tau'^{-1}/\eta(\theta)$, we expect the energy density $\epsilon_\theta \sim 1/\eta(\theta)$, which is yet different from previous cases. In $d = 1$, this predicts a cold region with energy density $m/4\eta_0$ separated from a hot region with energy density $\gamma_s m/4$; the average energy density is $m/4\gamma_s$.

We next describe simulations of the proposed quench on a classical model of noninteracting phonons to study UV physics, and subsequently study its efficacy on an interacting model—the Heisenberg spin chain.

Classical phonons.—We study the classical system with Hamiltonian $H = \frac{1}{2}\sum_i (x_i - x_{i+1})^2 + v_i^2 + m^2 x_i^2$, where m is quenched as per Eq. (1). Each mode is given an initial energy $\epsilon_k \sim \sqrt{m^2 + 4\sin^2(k/2)}$ akin to the vacuum point energy in an equivalent quantum model. The simulations allow us to verify some expectations for the role of τ and v_s in a noninteracting setting; see Fig. 3(a).

Heisenberg chain.—We perform time-dependent DMRG simulations (using iTensor [52]) of the antiferromagnetic

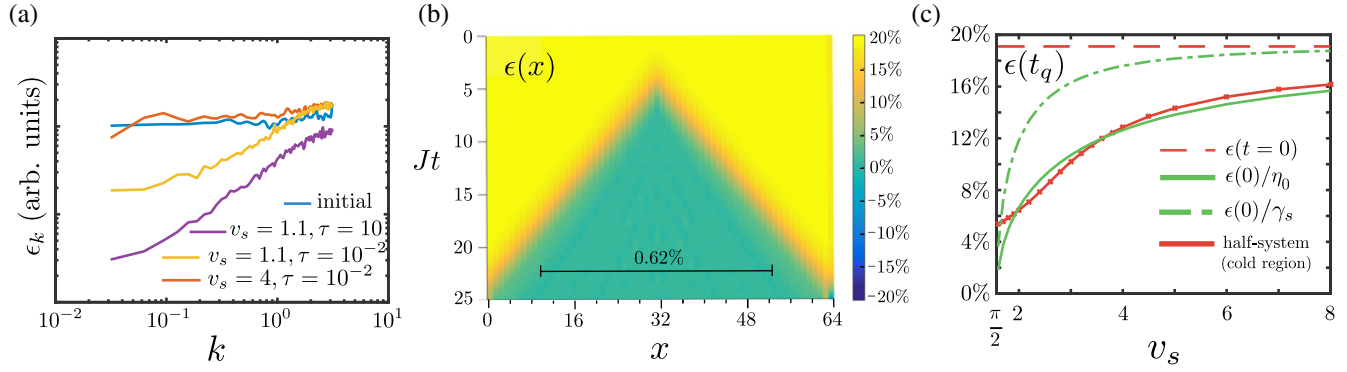

FIG. 3. (a) Energy per phonon mode at fixed momentum k. At large v_s, all modes are uniformly excited. As $v_s \to 1$, low energy "critical" modes are cooled, but UV modes remain unaffected. UV mode population is controlled by increasing τ. (b) TDMRG simulation of the quench in a Heisenberg chain for $h_z = 3$, $\tau = 2$, and $v_s/c = 3.2/\pi$. The initial state has an energy ~20% of bandwidth above ground state while the final energy density is ~0.62%. A linear ramp yields a state that is 3 times hotter in the same time. (c) Energy density in the middle half of the chain at t_q is seen to lie below $\epsilon(0)/\gamma_s$ and follows $\epsilon(0)/\eta_0$ for a large range of v_s. Here, $h_z = 3$, $\tau = 0.1$, $c = \pi/2$.

Heisenberg chain with alternating local fields setting the mass. A two-dimensional version of this model applies to the low-energy physics of the half-filled Fermi-Hubbard model at large Hubbard U, currently of interest in several ultracold atom experiments. The Hamiltonian reads $H = J\sum_x \boldsymbol{S}_x \cdot \boldsymbol{S}_{x+1} + \sum_x (-1)^x h(x,t)$, where the magnetic field $h(x,t)$ is eliminated from the center outward according to the functional form set by $h_z f(x)$ as in Eq. (1). In the noninteracting assumption, the middle region $x \in [L/2 - ct, L/2 + ct]$ is illuminated by only cold excitations moving against the front, while both hot (moving with the front) and cold excitations inundate space elsewhere; these "heat waves" are also observed in the Heisenberg chain (see Ref. [44]) although boundaries are blurred due to interactions. The cooling efficacy of the protocol is illustrated in Fig. 3(b) for $v_s/c \approx 1$ and $\tau = 2$; in Fig. 3(c) we examine the energy density in the middle half of the chain at the end of the quench and quantitatively verify it lies below the expected energy density $\sim\epsilon(0)/\gamma_s$ (either at small or large τ, as per the above arguments) for the complete chain.

Summary and conclusions.—In this work, we provided a nonadiabatic method for preparing the ground states of models with Lorentz symmetry. An exact analysis was presented for free relativistic bosons and fermions while analytical arguments and numerical simulations were used to examine the usefulness of our approach for realistic systems with UV effects and interactions. Our protocol should be accessible in experiments in a wide range of setups hosting artificial quantum matter [14,53], particularly ultracold atoms [1,2,54,55], where an adapted version may serve as an alternate route to preparation of the ground state of the Hubbard model at half-filling in the strong-coupling limit.

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