Nonclassicality of the Harmonic-Oscillator Coherent State Persisting up to the Macroscopic Domain

S. Bose,[1] D. Home,[2] and S. Mal[3]

[1]*Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom*
[2]*CAPSS, Physics Department, Bose Institute, Salt Lake, Sector V, Kolkata 700097, India*
[3]*S.N. Bose National Center for Basic Sciences, Block JD, Sector III, Salt Lake, Kolkata 700098, India*

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Can the most "classical-like" of all quantum states, namely the Schrödinger coherent state of a harmonic oscillator, exhibit nonclassical behavior? We find that for an oscillating object initially in a coherent state, merely by observing at various instants which spatial region the object is in, the Leggett-Garg inequality (LGI) can be violated through a genuine negative result measurement, thereby repudiating the everyday notion of macrorealism. This violation thus reveals an unnoticed nonclassicality of the very state which epitomizes classicality within the quantum description. It is found that for any given mass and oscillator frequency, a significant quantum violation of LGI can be obtained by suitably choosing the initial peak momentum of the coherent state wave packet. It thus opens up potentially the simplest way (without coupling with any ancillary quantum system or using nonlinearity) for testing whether various recently engineered and sought after macroscopic oscillators, such as feedback cooled thermal trapped nanocrystals of $\sim 10^6$–10^9 amu mass, are indeed bona fide nonclassical objects.

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Introduction.—Ever since the advent of quantum mechanics (QM), a perennial intriguing question has been how to reconcile classical features of the macroworld with the description of the microworld provided by QM, in other words, to what extent the principles of QM can be extrapolated to the macrodomain. In this context, the coherent states of a linear harmonic oscillator (LHO) introduced by Schrödinger [1] have a unique significance in embodying classical behavior within the quantum description: the peak of such a coherent state nonspreading wave packet (satisfying the position-momentum minimum uncertainty product) follows classical dynamics of an isolated LHO, while such states also emerge as the stable states of a LHO under decoherence [2]. On the other hand, central to the classical world view is the notion of "realism," viz. that at any instant, irrespective of measurement, a system is in a definite one of the available states for which all its observable properties have definite values. It should therefore be of interest to investigate whether the Schrödinger coherent state gives rise to a testable incompatibility with this notion of realism even in the macroscopic domain, thereby displaying nonclassicality of a system that has a well-defined classical description and is prepared to be in a most classical-like of all quantum states.

For the study of the aforementioned possibility, we invoke the Leggett-Garg inequality (LGI) [3–5] that can be regarded as a temporal analogue of Bell's inequality (BI) in terms of the time-separated correlation functions corresponding to successive measurement outcomes for a system. Apart from realism, an additional ingredient for obtaining LGI, replacing the locality condition underlying BI, is the notion of noninvasive measurability (NIM), which means assuming that it is possible to determine which of the states the system is in, without affecting the system's subsequent behavior. Thus, if the condition of NIM is satisfied by adopting a suitable measurement procedure (this is explained later), the testing of LGI would enable a scrutiny of the notion of realism itself in the form defined above. The finding of this paper is that even a macroscopic LHO in a coherent state can be made to repudiate the everyday notion of realism by violating the LGI based on suitable spatial measurements that satisfy NIM.

While the original motivation that led to LGI was for testing the possible limits of QM in the macroscopic regime, e.g., in terms of suitable experiments involving the rf-SQUID device [6], in recent years, a primary aim has been the certification of nonclassical behavior [7,8] in varied discrete variable microscopic quantum systems: solid-state qubits [9,10], nuclear spins [11], photons [12], electrons [13], oscillating kaons and neutrinos [14], qubit-oscillator hybrid systems [15], and atoms hopping in a lattice [16]. However, none of these systems have a well-defined classical description. There has not yet been any study for a system that possesses both quantum and classical descriptions, such as an isolated LHO which is uncoupled to qubits or nonlinearities.

In order to apply LGI to a LHO, we consider a spatial dichotomization of the harmonic well: coarse-grained

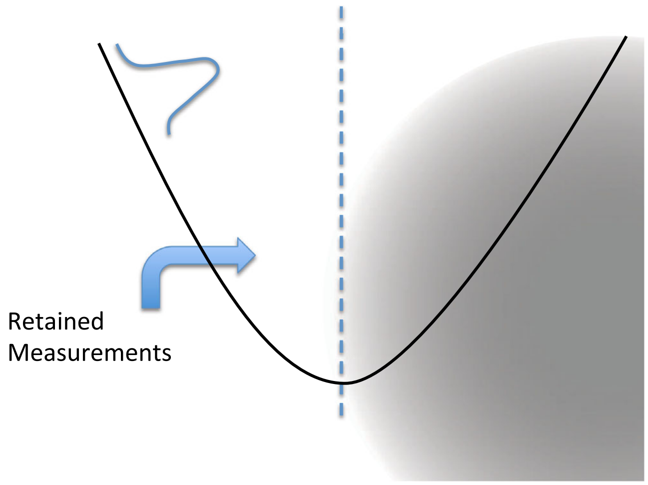

FIG. 1. Setup for testing the violation of macrorealism for an object in a coherent state in a harmonic well. A probe beam, shown as applied to the greyed circular region, illuminates one half of a harmonic well. If no light is scattered using this probe beam, it is concluded that the object must be on the unilluminated half of the well. These outcomes constitute an ideal negative result measurement as the probe has not interacted with the object corresponding to these outcomes. Two-time correlation functions computed using such outcomes then provide a loophole-free violation of macrorealism.

measurement of a type that would determine which one of the halves of the well the oscillating particle is in at any given instant, without providing any further information about the position of the particle, as shown in Fig. 1. This type of measurement is similar to the kind of spatial measurement used in a recent realization of the violation of LGI in a multiple coupled well structure [16] and likewise, satisfies NIM. Although a number of experiments have tested LGI, only two to date [10,16] have claimed to have satisfied the condition of NIM by implementing what is called the negative result measurement (NRM) procedure (to be explained later in the context of our example), and thereby can claim to have scrutinized the notion of realism. However, whether such violation of realism in the macroscopic domain is possible for a system with a classical description is yet to be investigated.

LGI and the notion of NRM.—In the one-dimensional LHO example considered in this paper, the temporal evolution involves oscillation between two states, one of which corresponds to the particle being found within, say, the negative half of the region ($x = 0$ to $x \to -\infty$) which we call state 1, while state 2 pertains to the particle being found within the positive half ($x = 0$ to $x \to +\infty$). Let $Q(t)$ be an observable quantity such that at any instant, it takes a value $+1(-1)$ depending on whether the system is in the state 1(2). Now, consider a set of runs starting from the identical initial state such that on the first subset Q is measured at times t_1 and t_2, on the second at t_2 and t_3, on the third at t_3 and t_4, and on the fourth at t_1 and t_4 (here $t_1 < t_2 < t_3 < t_4$). From such measurements, one can obtain the temporal correlations $C_{ij} \equiv \langle Q(t_i)Q(t_j)\rangle$. Then, adapting in this context, the standard argument leading to a Bell-type inequality with the measurement times t_i playing the role of apparatus settings, the following consequence of the assumptions of realism and NIM is invoked. For sets of runs corresponding to the same initial state, an individual $Q(t_i)$ is taken to have the same definite value ($+1$ or -1), irrespective of the pair $Q(t_i)Q(t_j)$ in which it occurs; i.e., the value of $Q(t_i)$ in any pair does not depend on whether any prior measurement has been made on the system. Consequently, the combination $[Q(t_1)Q(t_2) + Q(t_2)Q(t_3) + Q(t_3)Q(t_4) - Q(t_1)Q(t_4)]$ is always $+2$ or -2. If all these product terms are replaced by their respective averages over the entire ensemble, assuming the principle of induction [5], the following LGI is obtained:

$$C \equiv C_{12} + C_{23} + C_{34} - C_{14} \leq 2. \tag{1}$$

The above is, thus, a testable inequality imposing realist constraints on the time-separated correlation functions. Now, to explain how the notion of NIM can be satisfied by invoking NRM, let us consider the case in which Q is measured at t_1, followed by at t_2, corresponding to the determination of the correlation function $C_{12} = P_{++}(t_1, t_2) - P_{+-}(t_1, t_2) + P_{--}(t_1, t_2) - P_{-+}(t_1, t_2)$ where $P_{++}(t_1, t_2)$ is the joint probability of finding the particle in the state 1 at both the instants t_1 and t_2, similarly, for $P_{+-}(t_1, t_2), P_{--}(t_1, t_2), P_{-+}(t_1, t_2)$. Note that the derivation of LGI requires essentially the first measurement of each such pair to satisfy NIM. This can be ensured through the NRM procedure by arranging the measuring setup so that if, say, the probe is triggered, one can infer $Q(t_1) = +1$, while if it is not, $Q(t_1) = -1$; thereby the probe being untriggered provides information about the value of $Q = -1$, although there is no interaction occurring between the probe and the measured particle; NIM is, thus, satisfied. Now, if the results of those runs are only used for which $Q(t_1) = -1$, followed by the measurement of Q at t_2, discarding the results of the rest runs, these results can be used for determining the joint probabilities $P_{-+}(t_1, t_2)$ and $P_{--}(t_1, t_2)$. Similarly, for determining the other two joint probabilities $P_{+-}(t_1, t_2)$ and $P_{++}(t_1, t_2)$, the measuring setup can be inverted. In this way, one can determine all the two-time correlation functions occurring in the LGI by ensuring NIM (using NRM) for the first measurement of any pair. Thus the experiment takes place as independent sets of runs on identically prepared systems with two measurements during each run whose outcomes are correlated. Each of these set of runs is chosen at random from the entire ensemble and if such a set is sufficiently large, by invoking the fair-sampling assumption, the measurement results obtained for each set can be taken as the fair representative of the entire ensemble, thereby enabling use of the joint probabilities thus measured for determining each correlation function occurring in Eq. (1). The violation of LGI thus obtained would then repudiate the notion of realism in the sense defined earlier because, as Leggett [3–5] has argued, the realist statement that the particle has a definite one of the available states at any instant is hard to justify if the state's evolution can be affected by the NRM procedure. It is, therefore, necessary to invoke the NRM procedure in order to ensure NIM for achieving loophole-free verification of LGI that can be regarded as a clear test of realism. Next, we proceed to discuss the specifics of our example.

LGI using the LHO Schrödinger coherent state.—Let us consider the following initial Gaussian wave function peaked at $x = 0$,

$$\psi(x, t = 0) = \sqrt{\frac{1}{\sqrt{2\pi}\sigma_0}} \exp\left(-\frac{x^2}{4\sigma_0^2} + \frac{ip_0 x}{\hbar}\right), \quad (2)$$

with the initial momentum expectation value p_0, and the width $\sigma_0 = \sqrt{[\hbar/(2m\omega)]}$ where ω is the angular frequency of oscillation. It is well known that under the LHO potential, the above $\psi(x, 0)$ evolves into $\psi(x, t)$ (whose detailed expression is given in Supplemental Material [17]), whence the probability density is given by

$$|\psi(x, t)|^2 = \sqrt{\frac{m\omega}{\hbar\pi}} \exp\left(-m\omega \frac{(x - \frac{p_0}{m\omega}\sin\omega t)^2}{\hbar}\right), \quad (3)$$

which oscillates without spreading or changing shape, while its peak follows classical motion, and $\Delta x \Delta p = \hbar/2$ at all instants. Such a wave packet is known as the Schrödinger coherent state—a much-discussed remarkable example of a quasiclassical state in quantum mechanics. In order to apply LGI in this context, we consider coarse-grained measurement of a type that determines at any instant whether the oscillating particle is in the region between $x \to -\infty$ and $x = 0$ (yielding the measurement outcome $+1$) or is in the region between $x = 0$ and $x \to +\infty$ (yielding the measurement outcome -1). Such a measurement can be represented by the localization operator $\hat{O} = \int_{-\infty}^{0} |x\rangle\langle x| dx - \int_{0}^{\infty} |x\rangle\langle x| dx$, which has two eigenstates $\int_{-\infty}^{0} \langle x|\psi\rangle |x\rangle dx$ and $\int_{0}^{\infty} \langle x|\psi\rangle |x\rangle dx$ corresponding to the eigenvalues $+1$, -1, respectively. We later comment on the feasibility of measuring an operator close to $\hat{O}$. Now, note that the probability of obtaining the outcome $+1(-1)$ for such a measurement at the instant, say, t_1, is given by

$$P_{\pm}(t_1) = \frac{1}{2}\left(1 \mp \mathrm{erf}\left(\frac{\langle x(t_1)\rangle}{\sqrt{2}|\sigma_t|}\right)\right), \quad (4)$$

where the error function $\mathrm{erf}(t_1) = (2/\sqrt{\pi}) \int_0^{t_1} \exp(-z^2) dz$ and $\sigma_{t_1} = (i\hbar \sin\omega t_1 + 2m\omega\sigma_0^2 \cos\omega t_1)/2m\omega\sigma_0$.

TABLE I. Taking the angular frequency of oscillation $\omega = 2 \times 10^6$ Hz, for various values of mass m, different choices of the initial peak momentum p_0 (initial peak velocity v_0) of the coherent state wave packet are indicated for which the respective QM values of the lhs (C) of the LGI inequality (1) are computed. The corresponding values of the constant width (σ_0) of the coherent state wave packet and the classical amplitude (A_{Cl}) of oscillation are given.

m(amu)	$\sigma_0(m)$	p_0(kg m/s)	v_0(m/s)	$A_{Cl}(m)$	C
10^3	3.9×10^{-9}	3.3×10^{-23}	2×10	10^{-5}	2.58
10^6	1.2×10^{-10}	3.3×10^{-21}	2.0	10^{-6}	2.5
10^{10}	1.2×10^{-12}	3.3×10^{-21}	2×10^{-4}	10^{-10}	2.7
10^{20}	1.2×10^{-17}	3.3×10^{-15}	2×10^{-8}	10^{-14}	2.65

Next, given the result of the above measurement at the instant t_1 to be $+1(-1)$, obtained using the NRM procedure (its suggested empirical implementation in this case is discussed later), the subsequent time evolution of the postmeasurement state is subjected to a measurement at an instant, say, t_2. For this latter measurement, the conditional probability of obtaining the outcome $+1$, contingent upon the outcome $+1(-1)$ obtained for the measurement at the earlier instant t_1, is

$$P_{\pm/+}(t_1, t_2) = \int_{-\infty}^{0} |\psi_{\pm}^{PM}(x, t_2)|^2 dx, \quad (5)$$

while such a conditional probability for the outcome -1 at the instant t_2 is of the form

$$P_{\pm/-}(t_1, t_2) = \int_{0}^{\infty} |\psi_{\pm}^{PM}(x, t_2)|^2 dx, \quad (6)$$

where $\psi_{\pm}^{PM}(x, t_2)$ is the time-evolved form of the postmeasurement state that has evolved up to the instant t_2 (whose expression is given in Supplemental Material [17]).

Results.—Using Eqs. (4)–(6), for suitable choices of the relevant parameters, one can compute the QM values of the joint probabilities $P_{++}(t_1, t_2)$, $P_{+-}(t_1, t_2)$, $P_{--}(t_1, t_2)$, $P_{-+}(t_1, t_2)$ and evaluate the temporal correlation function C_{12}. Similarly, the other temporal correlation functions C_{23},C_{34},C_{14} occurring in LGI of the form (1) can be calculated. In our setup, the key parameters are m, p_0, and ω. Suitably choosing the values of m, p_0, ω while taking the temporal intervals to be the same, i.e., $t_2 - t_1 = t_3 - t_2 = t_4 - t_3 = \Delta t$, and by numerically integrating the relevant integrals occurring in Eqs. (4)–(6), the key results of the quantitative studies are presented in Table I. Here it needs to be mentioned that for given values of m, p_0, and ω, by varying the choices of the time interval Δt and the first instant of measurement t_1, it is found that the maximum value of C on the lhs of the inequality (1) is attained when Δt is chosen within the neighborhood of $T/4$ or $3T/4$, and t_1 is slightly larger than 0 or is within the neighborhood of $T/2$, where T is the time period of oscillation. Note that for computing all the results we have chosen the same values of $\Delta t = 2.4 \times 10^{-6}$ s (close to $3T/4$) and $t_1 = 1.5 \times 10^{-6}$ s (close to $T/2$) with $\omega = 2 \times 10^6$ Hz (this value of ω is close to the value 100 kHz—1 MHz for optically levitated oscillating masses). We now proceed to summarize the results.

(a) It is found that while for the initial peak momentum $p_0 = 0$, LGI is always satisfied, by appropriately choosing p_0, it is possible to obtain a significant amount of QM violation of LGI for any m corresponding to a given ω, and this violation can be maximized over Δt and t_1 (as per the choices of Δt and t_1 mentioned above). This is illustrated in Table I where the maximum obtained values of C are given for different sets of values of the relevant parameters while the m is varied from 10 to 10^{20} amu. Note that appreciable QM violations of LGI ($C > 2$) are found by suitable choices of p_0 given in Table I for, say, masses up to 10^{10} amu, such that the respective values of the classical amplitude of oscillation $A_{Cl} = p_0/m\omega$ range from 10^{-4} to 10^{-10} m. If the mass is further increased to, say, 10^{20} amu, it is found that in order to obtain significant QM violation of LGI, p_0 needs to be chosen such that the corresponding A_{Cl} becomes much smaller, and the required value of v_0 (initial peak velocity of the wave packet) for showing the QM violation of LGI also becomes increasingly smaller. Thus, although theoretically one can obtain the QM violation of LGI for any given m and ω by suitably choosing p_0, actual testability of this violation becomes gradually impracticable for sufficiently large mass as the requirement to controllably impart exactly the appropriate momentum becomes more stringent.

(b) If by keeping the parameters p_0, ω fixed, one increases the mass m, the QM violation of LGI is found to gradually diminish, and eventually for sufficiently large mass, LGI is satisfied; i.e., $C < 2$.

(c) For given values of m and ω, if p_0 is increased, the corresponding A_{Cl} is also increased, the QM value of C is found to be gradually decreasing, and eventually $C < 2$ for appropriately large p_0.

The results discussed above, therefore, serve to highlight the efficacy of LGI in not only revealing an earlier unnoticed nonclassicality of the oscillator coherent state, but also in exploring the extent to which such a nonclassical feature persists for masses larger than the typical microsopic masses.

Experimental tests of nonclassicality of macro-objects.— In recent years, an active experimental field has opened up in the trapping and cooling nanoscale objects of masses 10^6 amu and above, much larger than the mass of any object for which nonclassicality has been demonstrated to date. However, all the schemes suggested in this area for ascertaining whether such objects are nonclassical involve the initial preparation of non-Gaussian states such as Schrödinger cat states or require coupling the object to ancillary quantum systems [15,18,19]. Can our scheme open up a much simpler way of testing their nonclassicality? Remarkable technical developments reported in some very recent experiments [20–23] have now opened up the way for the implementation of our scheme. For preparation of the initial state, the center of the trap can be suddenly displaced (as demonstrated in ion traps [24]) so that the wave packet is centered at $A_{Cl} \sim \mu$m. Then, after a quarter oscillation, at $x = 0$, the peak of the wave packet can gain the appropriate momentum p_0 required for the LGI violation. As regards the required dichotomic measurement, the mirror-based levitation procedure reported in Refs. [20,21] should enable us to focus two optical beams of different frequency simultaneously in the trapping region. One will create the harmonic well and the other, slightly off-axis, and thereby traversing only one side of the well, will scatter off the levitated object and measure its position. It is this measuring beam that is shown in Fig. 1 as applied to the greyed circular region. When no scattered light from the object is detected, we conclude that the object is on the half where there is no beam to scatter light from and only those outcomes are retained—this is a loophole-free implementation of NRM as no probe interacts with the object for the retained outcomes. We consider two illustrative cases: (i) a nano-object of 10^6 amu trapped by laser fields that generate a harmonic well of $\omega \sim$ MHz [25] (from Table I, it is seen that in this case, for example, $C = 2.5$ with $A_{Cl} \sim \mu$m) and (ii) an ionized nano-object of 10^9 amu trapped in an ion trap of $\omega \sim 100$ Hz [22,26] (for this case we have estimated that $C = 2.7$ for $A_{Cl} \sim \mu$m). Damping and decoherence in both the cases are negligible in the experimental time scale of $1/\omega$ so that the time evolution is well approximated by the unitary dynamics as used in our treatment [18,27]. A primary criterion is to be able to differentiate reasonably sharply the presence of mass on the left or on the right half of the harmonic well in implementing the measurement of the operator $\hat{O}$ in a detection window of duration $\ll 1/\omega$, so that the measurements can essentially be regarded as instantaneous. This detection time window corresponds to the duration for which the measurement beam, illuminating one half of the well, is on—such control of pulse durations has been demonstrated in recent squeezing experiments [20]. The positions can be detected with extremely high spatial resolution by means of photodiodes using the interferometric (phase sensitive) detection of light scattered from the objects [20,23,28]. Within the detection time window of duration $\ll 1/\omega$, this technique enables the detection of positions of the aforementioned masses in their corresponding traps with sub-Angstrom resolutions (much sharper than the spread σ_0). Note that it has been shown [29] that the QM violation of LGI is retained for a significant unsharpness of the observable measured, where the idea of unsharpness is considered in conformity with the definition of unsharp measurement given in terms of positive-operator valued measure (POVM) elements comprising projection operators mixed with white noise [30]. On the other hand, the numerical results presented in Supplemental Material note III [17] serve the purpose of showing that the QM violation of LGI is extermely robust to the left and right spatial halves not being precisely distinguished; in particular, the QM violation of LGI persists as long as the blurring of the line separating the left-right spatial half measurements is less than the classical amplitude of oscillation A_{Cl}. Moreover, as is clear from the numerical results presented in Supplemental Material note IV [17], the predicted LGI violation persists even if the initial state is a mixed thermal state of the center of mass of the trapped object with lower than 10^4 phonons (i.e., as hot as 0.1 K for a MHz trap), which is quite standard to prepare by cooling [20,22,23,31–36]. Strikingly, the better the fine control one can acquire on trap displacements or momenta, the larger the mass for which LGI violation can be observed, thus offering an avenue for extending the test of the limits of quantum behavior to the macroscopic domain. In practice, larger m requires lower ω in order to have σ_0 larger than the feasible precision of position measurements, and this in turn extends the time scale of the experiment. When this would exceed the typical decoherence time (1–10 ms for levitated objects), our calculations would need to be modified to include the decoherence effects.

Concluding remarks.—In a nutshell, our work provides a means for testing the everyday notion of macrorealism for the first time in a loophole-free way for a macroscopic system having a classical analogue. This provides an alternative means for probing the macrolimit of the quantum world that differs from current methods in that it does not require prior preparation of any highly nonclassical state. We have shown how a system such as the quantum harmonic oscillator, which has a well-defined classical analogue, can be made to violate LGI through suitable spatial measurements even when the initial state is the most classical of all states—namely, the Schrödinger coherent state. Moreover, the violation is quite robust—it can be observed for mixed thermal states as well as for significant blurring of the distinction between left-right measurements of the spatial observable. Further, since the LGI violation for an isolated oscillator is in itself yet unexplored, this should also be worth testing even in the microdomain with trapped ions, using electromagnetic fields in cavity and circuit QED.

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