Generation of a Nernst Current from the Conformal Anomaly in Dirac and Weyl Semimetals

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We show that a conformal anomaly in Weyl and Dirac semimetals generates a bulk electric current perpendicular to a temperature gradient and the direction of a background magnetic field. The associated conductivity of this novel contribution to the Nernst effect is fixed by a beta function associated with the electric charge renormalization in the material. We discuss the experimental feasibility of the proposed phenomenon.

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Dirac and Weyl semimetals are three-dimensional crystals whose low-energy excitations are solutions of the massless Dirac equation. The recent experimental realization in a large family of materials [1–5] has provided unexpected access to physical phenomena restricted so far to quite unreachable energy regions as the quark-gluon plasma [6]. Quantum anomalies [7] and anomaly-related transport [8] are at the center of interest of the actual research (an updated account is given in the reviews [9,10]).

After intense activity around the experimental consequences of the axial anomaly [11–14] including evidence for the chiral magnetic effect [15], thermal transport is now probing gravitational anomalies [16,17]. The main link that opened the door to study gravitational effects in condensed matter systems is provided by the Luttinger theory of thermal transport coefficients [18,19]. He proposes a gravitational potential as the local source of energy flows and temperature fluctuations. The basic idea is that the effect of a temperature gradient that drives a system out of equilibrium can be compensated by a gravitational potential [20]. This advance completed the condensed matter description of thermo-electric-magnetic transport phenomena.

A novel anomaly-induced transport phenomenon, the scale magnetic effect (SME) was described in a recent publication [21]. Using massless QED as an example, it was shown that, in the background of an external magnetic field, the conformal anomaly [22] induces an electric current perpendicular to the magnetic field and to the gradient of the conformal factor. The coefficient was fixed by the beta function of the charge. In this work, we show that a similar phenomenon will occur in Dirac and Weyl semimetals driven by a temperature gradient. The anomalous current

$$\boldsymbol{J} = \frac{e^2 v_F}{18\pi^2 T\hbar} \boldsymbol{B} \times \boldsymbol{\nabla} T \tag{1}$$

provides a novel contribution similar to the Nernst effect occurring at a zero chemical potential. Equation (1) comes from the original SME with two important additions: First, the original result, worked out in a conformally flat metric, has been extended to include smooth deformations from flat space that will allow us to include material lattice deformations. The technical details of the derivation are described in Supplemental Material [23]. Second, we use the Luttinger construction to trade the conformal factor to a temperature gradient. Finally, the Fermi velocity of the material v_F will substitute the speed of light c in the conductivity coefficient. In what follows, we will detail these steps.

The effective description of an interacting Dirac or Weyl semimetal around a single cone is given by the Lagrangian of massless QED in a flat Minkowski space-time:

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi} i \not{D} \psi, \tag{2}$$

where ψ is the Dirac four spinor, $\bar{\psi} = \psi^\dagger \gamma^0$, $\not{D} = \gamma^\mu D_\mu$ with the covariant derivative $D_\mu = \partial_\mu - ieA_\mu$ and the Dirac matrices γ^μ, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor of the gauge field A_μ. We notice that the electronic current $J^\mu = \bar{\psi}(\gamma^0, v_F \gamma^i)\psi$ is anisotropic. We will obviate this fact, which does not play a role in this part. The action $S = \int d^4x \mathcal{L}$ of Eq. (2) is invariant at a classical level under a simultaneous rescaling of all coordinates and fields according to their canonical dimensions:

$$x \to \lambda^{-1} x, \qquad A_\mu \to \lambda A_\mu, \qquad \psi \to \lambda^{3/2} \psi. \tag{3}$$

As a consequence of the scale invariance, the stress tensor of the model (2),

$$T^{\mu\nu} = -F^{\mu\alpha} F^{\nu}{}_{\alpha} + \frac{1}{4} \eta^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} + \frac{i}{2} \bar{\psi} (\gamma^\mu D^\nu + \gamma^\nu D^\mu) \psi - \eta^{\mu\nu} \bar{\psi} i \not{D} \psi, \tag{4}$$

is traceless: $(T^{\mu}_{\mu})_{\rm cl} \equiv 0$. The scale invariance (3) is broken by quantum corrections which make the electric charge $e = e(\mu)$ dependent on the renormalization energy scale μ. As a result, in the background of a classical electromagnetic field A_μ the expectation value of the trace of the stress-energy tensor (4) becomes [7]

$$\langle T^{\alpha}{}_{\alpha}(x)\rangle = \frac{\beta(e)}{2e} F^{\mu\nu}(x)F_{\mu\nu}(x), \tag{5}$$

where $\beta(e)$ is the beta function associated with the running coupling e: $\beta(e) = (de/d\ln\mu)$. Hereafter, we study quantum effects only in a classical electromagnetic background of the gauge fields $A_\mu \equiv A^{\rm cl}_\mu$.

The conformal anomaly (5) leads to anomalous transport effects which most straightforwardly reveal themselves in a conformally flat space-time metric:

$$g_{\mu\nu}(x) = e^{2\tau(x)}\eta_{\mu\nu}, \tag{6}$$

where $\tau(x)$ is a scalar conformal factor and $\eta_{\mu\nu}$ is the Minkowski metric tensor.

In a weakly curved ($|\tau| \ll 1$) gravitational background (6) and in the presence of background magnetic field $\boldsymbol{B}$, the conformal (scale) anomaly (5) generates an anomalous electric current via the SME [21]:

$$\boldsymbol{J} = -\frac{2\beta(e)}{e}\boldsymbol{\nabla}\tau(x) \times \boldsymbol{B}(x). \tag{7}$$

In the presence of the electric field background $\boldsymbol{E}$, the conformal anomaly leads to the scale electric effect

$$\boldsymbol{J} = \sigma(x)\boldsymbol{E}(x), \tag{8}$$

which has the form of the Ohm law with the metric-dependent anomalous electric conductivity [21]:

$$\sigma(t, \boldsymbol{x}) = -\frac{2\beta(e)}{e}\frac{\partial\tau(t,\boldsymbol{x})}{\partial t}. \tag{9}$$

Both anomalous currents (7) and (8) can be described by the same relativistically covariant expression:

$$J^\mu = \frac{2\beta(e)}{e} F^{\mu\nu}\partial_\nu \tau. \tag{10}$$

The anomalous currents are generated in a quantum vacuum so that they emerge at a zero chemical potential and in the absence of a classical current

$$J^\mu_{\rm cl} = -\partial_\nu F^{\mu\nu}, \tag{11}$$

in the space where the anomalous current is produced: $J^\mu_{\rm cl}(x) \equiv 0$.

Contrary to the axial anomaly, the scale anomaly is not exact in one loop. In particular, the beta function gets corrections at all orders in the perturbation theory. The leading contribution to the current is defined by the one-loop QED beta function:

$$\beta^{\rm 1\ loop}_{\rm QED} = \frac{e^3}{12\pi^2}. \tag{12}$$

In this Letter, we consider the anomalous transport effects for gapless fermionic quasiparticles, realized in Weyl and Dirac semimetals, for which the conformal invariance is unbroken in the infrared region. For massive Dirac fermions the SME is strongly suppressed [28].

Having in mind condensed matter applications of our study, in the rest of this Letter we will pay close attention only to the scale magnetic effect (7). However, we notice that its electric counterpart has certain interesting properties as well. For example, contrary to the usual Ohm conductivity, the anomalous conductivity (9) of the scale electric effect (8) may take negative values. The negative vacuum conductivity, which may play a role in the early Universe, has also been independently obtained in calculations for fermionic [29,30] and bosonic [31] electrically charged particles in expanding de Sitter space via the Schwinger pair-production mechanism.

Now let us consider possible thermal effects which may play a role here. The basic idea is that the effect of a temperature gradient that drives a system out of equilibrium can be compensated by a gravitational potential Φ [18,19]:

$$\frac{1}{T}\boldsymbol{\nabla}T = -\frac{1}{c^2}\boldsymbol{\nabla}\Phi, \tag{13}$$

where c is the speed of light. For weak gravitational fields, the gravitational potential Φ, to leading order, is related to the metric as follows:

$$g_{00} = 1 + \frac{2\Phi}{c^2}, \tag{14}$$

while other components of the metric tensor are unmodified. This metric is not conformally flat. To get our result, Eq. (10) needs to be generalized to an arbitrary background metric. The technical derivation is given in Supplemental Material [23]. We can see that the electric current induced by the conformal effects with the metric (14) is determined by Eq. (7) with the factor $\tau(x) \equiv \varphi(x)$ given by the last three equations in Supplemental Material [23]:

$$\varphi(x) = -\frac{\Phi(x)}{3c^2}. \tag{15}$$

In particular, for a time-independent gravitational potential Φ the scale electric effect (8) is absent. Thus, the current density given by the conformal anomaly is

$$\boldsymbol{J} = C_{\rm conf}\boldsymbol{B} \times \boldsymbol{\nabla}T. \tag{16}$$

The conformal anomaly leads to a Nernst effect (16) with the coefficient described by the QED beta function (12):

$$C_{\rm conf} = \frac{2\beta(e)}{3e} \equiv \frac{e^2 c}{18\pi^2 T\hbar}, \tag{17}$$

where we have restored the powers of $\hbar$ and c. A similar strategy has been used in Ref. [32] to derive a new correction to the chiral vortical effect that arises in the presence of a temperature gradient.

To take into account the Fermi velocity, we will now restore all $\hbar$ and c in the fermionic part of Lagrangian (2) in the SI system of units. The result (16) and (17) corresponds to the fermionic Lagrangian

$$\mathcal{L} = \bar{\psi}\left(\gamma^0 i\hbar\frac{\partial}{\partial t} + c\boldsymbol{\gamma}(i\hbar\boldsymbol{\nabla} - e\boldsymbol{A})\right)\psi, \tag{18}$$

where we identified $A^\mu = (0, \boldsymbol{A})$ and set $A^0 = 0$, as it does not affect the scale magnetic effect. Therefore, we conclude that the c in the numerator of the anomalous current (16) and (17) is the c which appears in the spatial derivative term of the Lagrangian (18).

Taking into account that the fermionic Lagrangian of Dirac and Weyl semimetals is given by Eq. (18) with the substitution $c \to v_F$,

$$\mathcal{L} = \bar{\psi}\left[i\gamma^0\hbar\frac{\partial}{\partial t} + v_F\boldsymbol{\gamma}(i\hbar\boldsymbol{\nabla} - e\boldsymbol{A})\right]\psi, \tag{19}$$

we get our main result (1) by making the appropriate change in Eqs. (16) and (17).

To estimate the order of magnitude of the proposed effect, we have to remember that the Nernst effect is defined in open-circuit conditions, $\boldsymbol{J} = 0$, thus appearing a voltage drop across the sample:

$$J_i = \sigma_{ij}E_j + \mathcal{L}^{12}_{ir}(-\nabla_r T) = 0 \tag{20}$$

(the notation $\mathcal{L}^{ab}_{ir}$ for transport coefficients is standard, and we have, for instance, $\mathcal{L}^{11}_{ir} = \sigma_{ir}$. See, e.g., [33] for a modern reference). The induced electric field is, thus,

$$E_j = \rho_{ji}\mathcal{L}^{12}_{ir}(-\nabla_r T), \tag{21}$$

where $\rho_{ji} = (\sigma^{-1})_{ji}$ is the resistivity tensor. For definitiveness, let us choose the gradient of temperature to point, say, along the x direction, $\nabla_1 T$, and the magnetic field $\boldsymbol{B}$ to point along z as it is shown in Fig. 1. Then from Eq. (1) the only component of the tensor $\mathcal{L}^{12}_{ir}$ is

$$\mathcal{L}^{12}_{21} = \frac{e^2 v_F B_3}{18\pi^2\hbar T}. \tag{22}$$

Under these conditions, two coefficients are usually defined. The Ettingshausen-Nernst coefficient is defined as

$$S_{11} \equiv \frac{E_1}{B_3\nabla_1 T} = \frac{\rho_{12}\mathcal{L}^{12}_{21}}{B_3}, \tag{23}$$

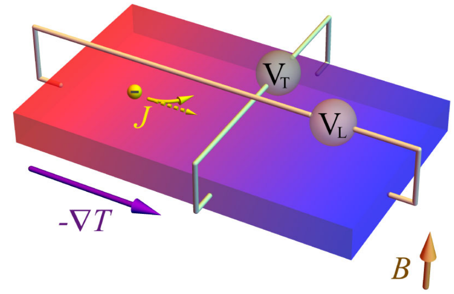

FIG. 1. The setup of the Nernst-Ettingshausen effect in open-circuit conditions. Voltage drops are induced by simultaneously applying an external magnetic field and a temperature gradient. Depending on whether the measured voltage is perpendicular (V_T) or parallel to the gradient of T (V_L), we speak about Nernst or Nernst-Ettingshausen effects (see the details in the main text).

and the Nernst coefficient is as follows:

$$S_{12} \equiv \frac{E_2}{B_3\nabla_1 T} = \frac{\rho_{22}\mathcal{L}^{12}_{21}}{B_3}. \tag{24}$$

In general, for three-dimensional (isotropic) metals, we have

$$\rho_{22} = \frac{\sigma_0}{\sigma_0^2 + \sigma_H^2}, \qquad \rho_{12} = \frac{\sigma_H}{\sigma_0^2 + \sigma_H^2}, \tag{25}$$

where σ_0 is the longitudinal conductivity and σ_H is the transverse (Hall) conductivity. The longitudinal transport in undoped Weyl semimetals is strongly suppressed due to the absence of free carriers (transport coefficients are proportional to the chemical potential [33]), and the current is carried by counterpropagating electrons and holes [34]. However, at a zero chemical potential, Weyl semimetals have a finite topological anomalous Hall current:

$$\boldsymbol{J} = \frac{e^2}{2\pi^2\hbar}\boldsymbol{b}\times\boldsymbol{E}, \tag{26}$$

where $\boldsymbol{b}$ is the separation between Weyl nodes. Choosing $\boldsymbol{b}$ to point along the z direction, we have

$$\sigma_0 \ll \sigma_H = \frac{e^2}{2\pi^2\hbar}|\boldsymbol{b}|, \tag{27}$$

so $\rho_{22} \sim (\sigma_0/\sigma_H^2)$ and $\rho_{12} \sim (1/\sigma_H)$.

The Ettingshausen-Nernst coefficient is then, approximately,

$$S_{11} \equiv \frac{E_1}{B_3\nabla_1 T} = \frac{\rho_{12}\mathcal{L}^{12}_{21}}{B_3} \sim \frac{v_F}{9|\boldsymbol{b}|T}. \tag{28}$$

The Nernst coefficient S_{12} appears to be strongly suppressed due to $\sigma_0 \ll \sigma_H$. For this reason, we propose to measure S_{11}. A small comment is in order here: It might be surprising that a transverse current as (1) leads to a longitudinal measurable quantity as it is S_{11}. The reason is that, due to the way the thermoelectric transports presented here are measured, the current in (1) is entangled to the resistivity tensor, which is dominated by the transverse Hall component, leading to a large coefficient S_{11} compared with S_{12}.

For typical Fermi velocities in Weyl semimetals, $v_F \sim 10^5$ m/s, $T \sim 10$ K, and separation of Weyl nodes $|2\boldsymbol{b}| \sim 0.3$ Å^{-1}, the Nernst coefficient divided by T is of the order of $S_{11}/T \sim 0.6\ \mu\mathrm{V/T\,K^{-2}}$, which is within the range of current Nernst measurements [35].

The importance of the Nernst and other thermomagnetic effects for thermoelectric power generation justifies the interest of the analysis of new sources even if small in magnitude. The Nernst effect was explored in the early stages of novel Dirac materials [33,36–39], and some experimental results are already available in the literature [35,40]. In most of the theoretical works, the main ingredient are the magnetization of the materials or the Berry curvature acting as an effective magnetization in a semiclassical analysis. The Nernst effect described in the present Letter appears in the *linearized* model at a zero chemical potential, and it is attached to the scale anomaly and not to the chiral anomaly. This observation makes the presence of a nontrivial Berry curvature and the described effect not obvious from the effective action formalism that we use in the present Letter. A calculation of the same response by using the Kubo formula and the Landau levels formalism is in progress [41].

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