Charge Fractionalization in the Two-Channel Kondo Effect

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The phenomenon of charge fractionalization describes the emergence of novel excitations with fractional quantum numbers, as predicted in strongly correlated systems such as spin liquids. We elucidate that precisely such an unusual effect may occur in the simplest possible non-Fermi liquid, the two-channel Kondo effect. To bring this concept down to experimental test, we study nonequilibrium transport through a device realizing the charge two-channel Kondo critical point in a recent experiment by Iftikhar *et al.* [Nature (London) **526**, 233 (2015)]. The shot noise at low voltages is predicted to result in a universal Fano factor $e^*/e = 1/2$. This allows us to experimentally identify elementary transport processes of emergent fermions carrying half-integer charge.

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Introduction and results.—Deconfinement and fractionalization are fascinating phenomena in which particles that are initially found as bound states become independent of each other. Such phenomena emerge in strongly interacting systems, e.g., in the form of spin-charge separation in Luttinger liquids [1], the possible emergence of a spinon Fermi sea in spin liquids [2], or the appearance of magnetic monopoles in spin ice [3]. In this Letter we argue that a similar phenomenon may be observed in ongoing experiments on a charge-Kondo device [4].

As introduced in 1980, the two-channel Kondo (2CK) model [5] describes a single impurity spin-1/2 \vec{S} coupled to two electronic channels $\alpha = 1$, 2 via

$$H_{K} = J \sum_{\alpha=1,2} \psi^{\dagger}_{\alpha\uparrow}(0) \psi_{\alpha\downarrow}(0) S^{+} + \text{H.c.}$$
(1)

While a *single* channel of electrons can completely screen the impurity spin, the presence of two (or more) competing channels turns this model into a paradigmatic example of frustration and non-Fermi liquid (NFL) behavior, with possible broader significance in bulk systems such as heavy fermion materials. Because of the spin-flip process H_K , the number of electrons from each channel $\alpha = 1$, 2 and each spin $\sigma = \uparrow, \downarrow, N_{\alpha\sigma} = \int dx \psi^{\dagger}_{\alpha\sigma}(x) \psi_{\alpha\sigma}(x)$, may change by integer amounts. As a precursor to fractionalization in this model, through the Emery-Kivelson (EK) solution [6], one introduces charge, spin, flavor, and spin-flavor quantum numbers,

$$\mathcal{N}_{c,s} = \frac{1}{2} (N_{1\uparrow} \pm N_{1\downarrow} + N_{2\uparrow} \pm N_{2\downarrow}),$$

$$\mathcal{N}_{f,sf} = \frac{1}{2} (N_{1\uparrow} \pm N_{1\downarrow} - N_{2\uparrow} \mp N_{2\downarrow}), \qquad (2)$$

with associated new fermions, $\psi^{\dagger}_{\mu}(x)$ (here, $\mu = c, s, f, sf$), that change only the corresponding \mathcal{N}_{μ} quantum numbers by ± 1 . An exact rewriting of the Kondo interaction is [6] $H_K = J\{[\psi_s(0)\psi_{sf}^{\dagger}(0)] + [\psi_s(0)\psi_{sf}(0)]\}S^+ + \text{H.c.}$ Crucially, at weak coupling, physical operators such as H_K involve the new fermions in pairs [7,8]. This is a necessary constraint to describe FLs. Otherwise, upon inverting Eq. (2), each individual new fermionic particle ψ^{\dagger}_{μ} changes electronic numbers $N_{\alpha\sigma}$ by half integers [7,8]; for example, $\psi_{\rm sf}^\dagger$ takes $\mathcal{N}_{\rm sf}
ightarrow \mathcal{N}_{\rm sf} + 1$ or, equivalently, $\delta(N_{1\uparrow}, N_{1\downarrow}, N_{2\uparrow}, N_{2\downarrow}) = (\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{2})$. In this sense, the new fermions are "confined" to occur in pairs in physical processes in FLs. However, the nonperturbative Kondo interaction leads to NFL behavior [5]. In light of this, one may wonder: is the unpairing of these fermions possible and can it be manifest in a physical system?

In recent years the multichannel Kondo effect was experimentally studied in highly tunable semiconductor quantum dot systems [4,9–12]. Our work is primarily motivated by charge 2CK setups [13] recently realized in the quantum Hall regime [9]. The impurity "spin" is encoded by two nearly degenerate macroscopic charge states of a large quantum dot (see Fig. 1), which is coupled to normal leads via quantum point contacts (QPCs), allowing us to flip the "spin" via single electron tunneling. Upon decreasing temperature below the Kondo temperature T_K , the conductance reaches half the quantum conductance $G \rightarrow G_0 = \frac{1}{2}(e^2/h)$, corresponding to two perfectly transmitting quantum resistors in series.

We study nonequilibrium transport through such devices and analyze the nonlinear current I(V) and shot noise S(V), focusing on the vicinity of the 2CK critical point. Shot noise provides information on the charge of the current carrying particles, with examples ranging from the fractional quantum Hall effect [14,15] to superconductor



FIG. 1. Schematics of the device: two QPCs coupled to a large quantum dot, with coupling constants J_1 and J_2 . The overall charge of the dot is controlled by a gate voltage V_q .

junctions exhibiting Cooper pair tunneling [16]. Applying full counting statistics (FCS) [17] methods borrowed from Gogolin and Komnik [18] and Schiller and Hershfield [19], we find interesting universal properties in the nonequilibrium Kondo regime $T \ll eV \ll T_K$,

$$I = \frac{e^2 V}{2h} \left[1 - \frac{\pi^2}{8} \frac{e|V|}{T_K} + O\left(\frac{eV}{T_K}\right)^2 \right],$$

$$S = \frac{e^3 V}{2h} \left[\frac{\pi^2}{8} \frac{e|V|}{T_K} + O\left(\frac{eV}{T_K}\right)^2 \right]$$
(3)

(for simplicity, we set T = 0). The current contains a nonlinear correction that corresponds to a backscattering current $I_b = [(e^2V)/(2h)] - I$. While the first term describes noiseless current through two perfectly transmitting QPCs, the backscattering current produces shot noise S = $2e^*I_b + O(V^3)$, with a Fano factor $F = S/(2eI) \equiv e^*/e$, with $e^* = e/2$.

This fractional Fano factor can be precisely interpreted in terms of the unpairing of a spin-flavor fermion ψ_{sf}^{\dagger} in physical processes at the NFL state. Below, we identify the elementary backscattering processes consisting of annihilation of this individual fermion, which means half-integer changes in electronic occupation numbers. Finally, we suggest an alternative three-lead setup [4] to probe this fractionalization.

Model.—Our system in Fig. 1 consists of a large metallic quantum dot in the quantum Hall regime hosting spinless electrons coupled to two normal leads via QPCs. It is described by the standard 2CK Hamiltonian [13,20]

$$H_{\rm K} = \sum_{\alpha=1,2} \left(i\hbar v_F \sum_{\sigma} \int dx \psi^{\dagger}_{\alpha\sigma}(x) \partial_x \psi_{\alpha\sigma}(x) + J_{\alpha} [\psi^{\dagger}_{\alpha\uparrow}(0)\psi_{\alpha\downarrow}(0)\hat{S}^- + {\rm H.c.}] \right) + \Delta E \hat{S}^z. \quad (4)$$

Here, $\sigma = \uparrow$ describes states in the leads and $\sigma = \downarrow$ in the dot, the index $\alpha = 1$, 2 labels the two QPCs, and v_F is the Fermi velocity. We assume [9,13] that the dot is sufficiently large that its level spacing is small compared to the temperature, as a result of which edge states in the dot

near different QPCs are incoherently coupled. In the opposite regime we expect 1CK behavior. We specialize to the large charging energy limit $E_c \gg eV$, T, such that only two macroscopic charge states, with $N = N_0$ or $N_0 + 1$ electrons in the dot, are relevant and play the role of the impurity spin \hat{S} . Thus, $\hat{S}^z = \frac{1}{2}|N_0+1\rangle\langle N_0+1|-\frac{1}{2}|N_0\rangle\langle N_0|$, and $\hat{S}^- = |N_0\rangle\langle N_0 + 1|$. Upon detuning the gate voltage from the degeneracy point, an energy splitting ΔE is formed between these macroscopic charge states.

The 2CK state is a critical point occurring at charge degeneracy $\Delta E = 0$ and for left-right symmetry $J_1 = J_2 = J$. Below, we will comment on deviations from these conditions which eventually lead to a crossover at low energies to a FL state [13,20]. The parameters of the model include the density of states $\nu = [1/(2\pi\hbar v_F)]$ and a high-energy cutoff *D*, set by the minimum of the bandwidth and the charging energy, defining through the tunneling amplitude *J* the Kondo temperature $T_K \sim De^{-(1/\nu J)}$.

The model equation (4) is an anisotropic *XY* Kondo Hamiltonian. In our calculations below we will add a term $H_z = J_z \sum_{\alpha,\sigma,\sigma'} \psi^{\dagger}_{\alpha\sigma}(0) (\vec{\sigma}_{\sigma\sigma'}/2) \psi_{\alpha\sigma'}(0) S^z$, keeping in mind that spin anisotropy is an irrelevant perturbation [21].

Mapping to the Toulouse Hamiltonian.—Following the standard EK transformation [6,19], we (i) bosonize the fermionic fields $\psi_{\alpha\sigma}(x) \sim (1/\sqrt{2\pi a})e^{i\Phi_{\alpha\sigma}(x)}$, with a short distance cutoff *a*, (ii) perform the EK transformation [Eq. (2)] to define charge, spin, flavor, and spin-flavor bosons, $\Phi_{\alpha\sigma} \rightarrow \Phi_{\mu}$ ($\mu = c, s, f, sf$), and (iii) refermionize these bosons into new fermion operators $\psi_{\mu} \sim (1/\sqrt{2\pi a})e^{i\Phi_{\mu}}$. After a unitary transformation the Hamiltonian becomes

$$H_{K} = i\hbar v_{F} \sum_{\mu} \int dx \psi_{\mu}^{\dagger}(x) \partial_{x} \psi_{\mu}(x) + i\mathcal{J}\chi_{\rm sf}(0)\hat{b}$$
$$-\frac{\mathrm{eV}}{2} \int dx [\psi_{\rm sf}^{\dagger}(x)\psi_{\rm sf}(x) + \psi_{f}^{\dagger}(x)\psi_{f}(x)]$$
$$+ i(J_{z} - 2\pi\hbar v_{F})\psi_{s}^{\dagger}(0)\psi_{s}(0)\hat{a}\,\hat{b}, \qquad (5)$$

where \hat{a} and \hat{b} are local Majorana operators associated with the impurity degrees of freedom, $i\hat{a} \hat{b} = \hat{S}^z$, satisfying $\hat{a}^2 = \hat{b}^2 = \frac{1}{2}$, $\chi_{sf}(x) = \{[\psi_{sf}^{\dagger}(x) + \psi_{sf}(x)]/\sqrt{2}\}$, and $\mathcal{J} = [(J_1 + J_2)/\sqrt{2\pi a}]$. We included the source-drain voltage eV, setting a chemical potential difference in the leads $eV[(N_{1\uparrow} - N_{2\uparrow})/2]$, which after the transformation equation (2) becomes a chemical potential of the sf and ffermions [19]. The last term accounts for deviations from the Toulouse point $J_z = 2\pi\hbar v_F$. The current operator is given by $\hat{l} = [(ie)/\hbar]\{[(N_{1\uparrow}N_{2\uparrow})/2], H_K\}$.

At the Toulouse point the Hamiltonian has a free fermion form and one obtains the current [19,22] $I(V) = [(e^2V)/(2h)][1+O(V^2/T_K^2)]$ and noise $S(V)=O(V^3/T_K^2)$, where $T_K = \pi \nu \mathcal{J}^2$, valid for $eV \ll T_K$. However, the

quadratic voltage corrections in I(V) and the cubic term in S(V) are artifacts of the free fermion structure at the Toulouse point. In order to obtain the leading universal corrections in eV/T_K to the current and noise, we allow finite deviations from the free fermion Toulouse point. Computing the current as well as the shot noise using nonequilibrium Keldysh Green function methods in the framework of Eq. (5), to infinite order in \mathcal{J} and to the leading second order in $v_1 = J_z - 2\pi\hbar v_F$, we obtain [22] Eq. (3). Notably, we can define the backscattering current $I_b \equiv G_0 V - I = [(e^2 V)/h][(\pi^2 e|V|)/(16T_K)]$ and write the shot noise as $S = 2e^*I_b$, with $e^* = e/2$. In fact, one may conjecture, as we have actually verified [22], that near the 2CK fixed point the FCS of charge transfer is described by half-integer charges.

Unpairing of EK fermions and the leading irrelevant operator.—In order to better understand the result [Eq. (3)], we express the 2CK fixed point Hamiltonian and its leading irrelevant correction, of known dimension 3/2 [23], in the language of EK fermions ψ_{μ}^{\dagger} .

In the framework of Eq. (5), at the 2CK fixed point dominated by \mathcal{J} , the local \hat{b} fermion hybridizes with the spin-flavor fermion $\chi_{sf}(x)$, responsible for fluctuations of spin between the channels. Solving this quadratic Majorana fermion model, one obtains the operator relation [24,25]

$$\hat{b} = \frac{1}{\sqrt{\pi\nu T_K}} \tilde{\chi}_{\rm sf}(0), \tag{6}$$

which becomes exact at energies $\ll T_K$. Here, $\tilde{\chi}_{sf}(x) = \chi_{sf}(x) \operatorname{sgn}(x)$ is a boundary condition modified spin-flavor Majorana fermion [22], reflecting the absorption of the local Majorana fermion \hat{b} . The impurity spin \hat{S} is only partially screened at the 2CK fixed point and has residual entropy of $\frac{1}{2}\log(2)$ [6]. Using Eq. (6), its dynamics is described in terms of the unpaired spin-flavor fermion, for example, $\hat{S}^z = (i/\sqrt{\pi\nu T_K})\hat{a}\tilde{\chi}_{sf}(0)$. This unpairing of the EK fermion can be manifest via any coupling to the impurity spin, e.g., applying a gate detuning $\Delta E \hat{S}^z$, as discussed later. Even at $\Delta E = 0$, however, any finite deviation from the Toulouse point $v_1 = J_z - 2\pi\hbar v_F$ in Eq. (5) results in the leading irrelevant operator [22,26]

$$H_{\rm irr} = \frac{1}{\nu^{3/2} \sqrt{T_K}} i \psi_s^{\dagger}(0) \psi_s(0) \tilde{\chi}_{\rm sf}(0) \hat{a}, \tag{7}$$

containing an odd number of EK fermions.

Our results rely on the 2CK theory at the vicinity of the anisotropic Toulouse point. One may question this approach, given that it actually fails [19,27] to yield correctly the known [27–31] $e^*/e = 5/3$ Fano factor in the 1CK case. The failure of the Toulouse point description in this case traces back to the fact that it produces an anisotropic version of the leading irrelevant operator

: $\vec{J}_s(0)^2$:, where $\vec{J}_s(x)$ is the SU(2) spin current [23]. Expanding this dimension 2 operator in electron processes, one finds [27] both 1*e* and 2*e* backscattering amplitudes whose weighted average gives the correct $e^*/e = 5/3$ Fano factor [27]. However, their ratio, and hence the Fano factor, becomes different in the anisotropic Toulouse point which yields an anisotropic operator of the form $:J_s^z(0)^2:$. The state of affairs is very different in the 2CK case. The dimension 3/2 operator [Eq. (7)] is identified [25,26] with the SU(2) symmetric operator $J_{-1}^s \vec{\phi}$ in conformal field theory, where $\vec{\phi}$ is the vector field of the spin SU(2) sector. This observation provides full confidence that our results are not artifacts of the anisotropic Toulouse point.

Fermi's golden rule.—Our result Eq. (3), supplemented by an intelligible physical picture, can be reobtained by a simple calculation based on Fermi's golden rule applied directly with respect to the irrelevant operator [Eq. (7)]. Decomposing the operator $\tilde{\chi}_{\rm sf}(0) = (1/\sqrt{2L}) \sum_{k_{\rm sf}} (\tilde{c}_{k_{\rm sf}}^{\dagger} +$ $\tilde{c}_{k,c}$) into normal fermionic modes [8], we see that it creates either a particle or a hole in the spin-flavor Fermi sea. Equation (5) shows that the source-drain voltage sets an enhanced chemical potential eV/2 of the spin-flavor Fermi sea. Thus, annihilation of one spin-flavor particle at k_{sf} above the equilibrium Fermi level $0 < \varepsilon_{k_{sf}} < eV/2$ lowers the energy. Energy conservation is attained via a creation of a particle-hole excitation in the spin sector via the operator $\psi_s^{\dagger}(0)\psi_s(0) = (1/L)\sum_{k_+}\sum_{k_-} c_{s,k_+}^{\dagger}c_{s,k_-}$ in Eq. (7). This process depicted in Fig. 2 creates a unit change in the quantum numbers in Eq. (2), $\tilde{\mathcal{N}}_{sf} \rightarrow \tilde{\mathcal{N}}_{sf} - 1$; i.e., it annihilates an unpaired spin-flavor fermion. Evaluating the total rate for this process via Fermi's golden rule gives, using $\hat{a}^2 = \frac{1}{2}$,

$$-\frac{d\langle \mathcal{N}_{\rm sf}\rangle}{dt} = \frac{2\pi}{\hbar L^3} \sum_{k_{\pm}, k_{\rm sf}} \frac{1}{\pi \nu^3 T_K} \theta(\epsilon_{k_+}) \theta(\epsilon_{k_-}) \theta(eV/2 - \epsilon_{k_{\rm sf}}) \times \delta(\epsilon_{k_+} - \epsilon_{k_-} - \epsilon_{k_{\rm sf}}) = \frac{\pi}{\hbar} \frac{(eV)^2}{16T_K}.$$
(8)

Crucially, the unit change in \tilde{N}_{sf} , modifies *electronic* occupations by half integers. Thus, this Poissonian process actually describes backscattering of charge $e^* = e/2$, and the backcattering current is

$$I_b = -e^* \frac{d \langle \tilde{\mathcal{N}}_{\rm sf} \rangle}{dt} = \frac{e^2 V}{h} \frac{\pi^2 e|V|}{16T_K},$$

with an associated noise $S = 2e^*I_b$, in agreement with Eq. (3).

While the process displayed in Fig. 2 looks simple in terms of the EK fermions, it cannot be described by a finite number of electron tunneling events. This reflects the NFL nature of the 2CK fixed point [5], lacking a notion of a local



FIG. 2. Energy diagram of EK fermions. Elementary backscattering processes consist of annihilation of a single spin-flavor fermion accompanied by a creation of a particle-hole excitation in the spin Fermi sea.

Fermi surface [32]. Nevertheless, an effective FL picture holds in terms of EK fermions [7,8], allowing us to draw the simple process in Fig. 2.

Finite temperature effects.—The universality of our results [Eq. (3)] is revealed by the ratio between the coefficients of eV/T_K in the current and noise. Another experimentally testable ratio can be obtained from the leading *T* dependence of the current, which we find to be $I = G_0 V [1 - (\pi^2/8) \{ [(e|V|)/T_K] + \pi^2(T/T_K) \}]$. Similarly, the noise S(V, T) has a temperature dependence where, at $T \gg eV$, it must cross from the shot noise limit [Eq. (3)] to thermal noise $S = 4k_BTG$, with $G = dI/dV|_{V=0}$.

High temperature Coulomb blockade regime.-At energies that are high compared to the Kondo scale $\max(T, eV) \gg T_K$ but still lower than the charging energy, the system crosses over to the Coulomb blockade classical regime. The Fano factor $S/(2eI) = (R_1^2 + R_2^2)/(R_1 + R_2)^2$ shows a dependence on the asymmetry of the QPC resistances $R_{1,2}$, and it approaches 1/2 in the symmetric case, $R_1 = R_2$ [33]. In that case, two consecutive tunneling events, through the left and then the right QPC, are required to transfer a single electron across the device. Thus, the effective charge defined from noise may be misleading. To distinguish this mathematical peculiarity from our physical mechanism, one may look at the FCS. The FCS, as calculated in Ref. [34], reflects [22] the fact that the actual tunneling particles in the Coulomb blockade regime are electrons and can be contrasted with our result of a FCS described fully in terms of half-integer charges. Furthermore, as we subsequently show, this fractional FCS has a distinctive finite stability against channel asymmetry.

Deviations from the critical point.—The intricate properties of the critical point are destabilized by left-right asymmetry $\Delta J = J_1 - J_2$ or by gate voltage deviations from the charge degeneracy point ΔE . These relevant perturbations create an energy scale, $T^* = c_1 T_K (\nu \Delta J)^2 + c_2 (\Delta E)^2 / T_K$, with $c_{1,2}$ coefficients of order unity. Below this energy scale the system crosses over to a FL state, whereby the nonlinear conductance gradually decreases below $G_0 = e^2/2h$ [13,20] till it vanishes at $eV \ll T^*$. Since T_K may become high and approach the charging energy (~290 mK) in charge-Kondo devices [4,9], one may



FIG. 3. Charge fractionalization using a weak probe: one electron tunnels from the weakly coupled lead no. 3 and is equally and simultaneously partitioned into the two leads.

realistically assume $T^* \ll T_K$. This gives a finite voltage window $T^* \ll eV \ll T_K$ within which our shot noise predictions, dominated by the leading irrelevant operator, hold. Nevertheless, what is the leading influence of finite T^* ? Extending the calculations of Ref. [20], we find that, to leading order in T^* , the effective charge remains unchanged. This is in accordance with the general argument above, stating that any coupling to the impurity spin in the 2CK fixed point involves the unpaired spin-flavor fermion. Thus, although relevant operators eventually destabilize the critical point, the $e^*/e = 1/2$ Fano factor is remarkably stable and includes the leading effects of relevant—as well marginal—operators [22].

Three-lead setup.—We briefly present an alternative setup that displays charge fractionalization. Consider attaching a third lead [4] at voltage V and weakly coupling it to the large dot with the two other leads held at V = 0; see Fig. 3. The model Hamiltonian is Eq. (4), where now $\alpha = 1, 2, 3$ and $J_1 = J_2 > J_3$. We analyzed the current and noise at the 2CK fixed point for this device. The injected current through the third QPC is given by $I_3(V) = [(e^2V)/\hbar](\pi^2/2)(\nu J_3)^2[1 + O(eV/T_K)]$. In the two strongly coupled leads $\alpha, \beta = 1, 2$, where, by channel symmetry, the average ejected current is $I \equiv I_{\alpha} = I_3/2$, we find, based on the linear relation between the currents at the fixed point [35], the current-current correlation $S_{\alpha\beta}(\omega) \simeq 2e^*I$, with $e^* = e/2$.

Interpreting this result, each individual charge *e* tunneling through the third QPC into the dot, is *equally and simultaneously* partitioned into both leads; see Fig. 3. This fractionalization can also be understood along the above Fermi golden rule picture: the tunneling process from the weakly coupled lead increases the dot's charge, $N_0 \rightarrow N_0 + 1$; i.e., it involves the operator $\hat{S}^+ = (\hat{a} - i\hat{b})/\sqrt{2}$. Using Eq. (6), this operator becomes the unpaired fermion $\tilde{\chi}_{sf}$, changing electronic numbers in the leads by half integers.

This setup has the advantage that charge fractionalization can be seen in the shot noise of the full current as measured in one of the leads, rather than in the backscattering current.

Before concluding, it is interesting for us to mention other mechanisms for fractional charge transfer in related devices, e.g., in a QPC coupled to a two-level system [36], or near a Mott transition described by the Luther-Emery point [37], which has a resemblance to Coulomb drag experiments [38]. Summary.—We analyzed nonequilibrium transport properties of charge 2CK devices and found that shot noise encodes fractional charges signaling key NFL features. One may compare this to the single-channel spin-Kondo effect in quantum dots, where a Fano factor $e^*/e = 5/3$ has been predicted based on a FL theory [27–31] and tested [39–43]. This is a weighted average of 1e and 2e processes, while our current result $e^* = e/2$ cannot be accommodated within such a Fermi liquid picture.

In contrast to charge-Kondo devices [4,9], calculations of nonequilibrium transport in spin-multichannel Kondo devices [10,12] remain challenging. Also, nonequilibrium noise in N > 2 multichannel charge-Kondo devices [4], whose linear transport properties were addressed recently [44], remains an interesting question for future work. One may exploit connections, e.g., to topological Kondo devices [45–49] and their nonequilibrium properties [50,51].

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