Post-Newtonian Dynamics in Dense Star Clusters: Highly Eccentric, Highly Spinning, and Repeated Binary Black Hole Mergers

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We present models of realistic globular clusters with post-Newtonian dynamics for black holes. By modeling the relativistic accelerations and gravitational-wave emission in isolated binaries and during three- and four-body encounters, we find that nearly half of all binary black hole mergers occur inside the cluster, with about 10% of those mergers entering the LIGO/Virgo band with eccentricities greater than 0.1. In-cluster mergers lead to the birth of a second generation of black holes with larger masses and high spins, which, depending on the black hole natal spins, can sometimes be retained in the cluster and merge again. As a result, globular clusters can produce merging binaries with detectable spins *regardless* of the birth spins of black holes formed from massive stars. These second-generation black holes would also populate any upper mass gap created by pair-instability supernovae.

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Introduction.—With the recent detections of five binary black hole (BBH) mergers, and one binary neutron star merger, the era of gravitational wave (GW) astrophysics has arrived at last [1–5]. Despite significant theoretical work, the origins of these systems, particularly the heavier BBHs, remains an open question. Both stellar evolution in isolated massive binaries [6–10] and dynamical formation in dense star clusters [11–22] have been shown to produce merging BBHs similar to GW150914 [23,24]. Understanding which formation pathways are at play will be critical for the interpretation of GW data. While many signatures of dynamical assembly have been proposed, such as highly eccentric mergers occurring in strong chaotic encounters [25] or antialignment of the BH spins with the orbit [26], none of the BBH mergers detected so far by LIGO/Virgo have displayed any of those signatures clearly; see Ref. [27].

What *has* been displayed clearly in each BBH merger is the birth of a new rapidly spinning BH with a mass (almost) equal to the sum of its progenitor masses. Many of these new BHs, particularly the remnants of GW150914, GW170104, and GW170814, are significantly more massive than what is thought to form during the collapse of a single star, where the pair-instability mechanism limits the remnant BH mass to $\lesssim 50\,M_\odot$ [28]. Were one of these mergers to occur in a dense star cluster, however, the merger product could easily exchange into another BBH and merge again. Because of the distinct BH masses and spins in such second-generation (2G) mergers, it has been suggested that such a population could be easily identifiable with future LIGO/Virgo detections [29,30].

In this Letter, we present the first models of realistic globular clusters (GCs) with fully post-Newtonian (pN) stellar dynamics. While relativistic N-body dynamics has been studied previously for highly idealized systems, e.g., Refs. [31–40] or open clusters [22], we show here for the first time using self-consistent dynamical models of massive GCs that pN effects play a key role in assembling dynamically the merging BBHs detectable by LIGO/Virgo. In our new pN models, we observe that roughly half of all BBH mergers occur inside clusters, with a significant fraction of those (∼10%) merging with eccentricities greater than 0.1 following GW captures. In-cluster mergers produce a second generation of BHs that, if not ejected from the cluster through GW recoil, will dynamically exchange into new binaries only to merge again. These 2G BBH mergers have components with large spins and masses significantly beyond what is possible from the collapse of a single star; they may be quite common, with as many as ∼20% of BBH mergers from our models having components formed in a previous merger.

Throughout this Letter, we assume a flat ΛCDM cosmology with $h = 0.679$ and $\Omega_M = 0.3065$ [41].

Post-Newtonian dynamics.—We have computed the new GC models presented here using the cluster Monte Carlo (CMC) code. CMC has been developed over many years [42,43], and includes all the necessary physics for the long-term evolution of GCs, including two-body relaxation [44,45], single and binary stellar evolution [46–48], galactic tides, three-body binary formation [49], and three- and four-body gravitational encounters via the FEWBODY package [50,51]. We have shown in Ref. [52] that CMC can reproduce with a high degree of fidelity both the global cluster properties and BBH distributions computed with state-of-the-art direct N-body simulations [53], while at the same time being at least 2 orders of magnitude faster (essential for the sort of extensive parameter-space study presented here). Furthermore, CMC has been upgraded [21] to employ the most recent prescriptions for stellar-wind-driven mass loss [54,55] and compact-object formation [56], allowing us to compare our results directly to those of population synthesis studies for isolated binaries [23].

To incorporate pN effects into CMC, we make the following modifications. We account for relativistic effects during three- and four-body encounters by adopting a modified version of the FEWBODY code with pN accelerations up to and including the 2.5 pN order. This code has been described in detail in Refs. [27,57] and has been shown to conserve energy to 2 pN order and to reproduce the inspiral times for compact binaries [58]. For BBHs which merge during an encounter, we perform a standard sticky-sphere merger, using detailed, spin-dependent fitting formulas from analytic and numerical relativity calculations [59–71]. The new masses, spins, and recoil kicks are applied immediately during any merger, allowing us to model the retention of BHs by the cluster self-consistently. See Supplemental Material A [72] for details, which includes Refs. [73–78]. We initially assume all BHs from stellar collapse have no spins at birth ($\chi_b = 0$, where χ is the dimensionless Kerr spin parameter), though we relax this assumption in Sec. V. For BBHs that do not merge during a FEWBODY encounter, we directly integrate the orbit-averaged Peters equations [58] for the change in semimajor axis and eccentricity due to GW emission. This represents a departure from our previous work where we relied on the binary stellar evolution module (BSE) [47]. By default, BSE only applies GW energy loss to binaries with $a < 10R_\odot$. This assumption leads BSE to *significantly* underestimate the number of GW-driven mergers for binaries in a typical cluster, which can be highly eccentric and very massive. When accounting for GW energy loss in eccentric binaries, the number of in-cluster mergers becomes comparable to the number of merging binaries that are ejected from the cluster. This is a significant improvement over previous results in the literature [15,20,21,24,79], where ejected BBHs dominated the merger rate in the local universe. Semianalytic approaches to cluster dynamics [80,81] have reported significantly higher fractions of in-cluster mergers, similar to those presented here, and have noted the possibility of multiple mergers in galactic nuclei [80].

We generate 24 GC models covering a range of masses, metallicities, galactocentric distances, and virial radii, similar to those observed in the Milky Way and beyond. These initial conditions are identical to those from Ref. [21], allowing us to explicitly compare our pN results to those in the literature. Our physics for single and binary stellar evolution is nearly identical to Ref. [21]. We have added a prescription for stellar mass loss via pulsational-pair-instability supernovae and stellar destruction via pair-instability supernovae. This physics, powered by the rapid production of electron-positron pairs in the stellar core [28], places a well-understood upper limit on the masses of BHs that can form from the collapse of a single star. We take the limit from Ref. [82] of $\sim 45\ M_\odot$, which is reduced to $\sim 40\ M_\odot$ via neutrino emission. See Supplemental Material B [72] for details, which includes Refs. [83–85]. This limit is in tentative agreement with the BH mass distribution measured by LIGO/Virgo [86]. In our simulations, no BH can be born with a mass above $40\ M_\odot$ unless the BH or its stellar progenitor has undergone a dynamical merger or mass transfer. Finally, unlike previous studies [20,21,24], we have not weighted our models according to the distribution of observed GCs. We will explore more realistic sets of models in future work focusing specifically on LIGO/Virgo detection rates. In practice a more realistic weighting should make little difference, as our previously adopted weighting scheme primarily selected BBHs from the most massive clusters, which also contribute the majority of sources in our current grid.

In-cluster mergers.—With the addition of the pN physics, we see a significant increase in the number of in-cluster mergers. Whereas before the number of in-cluster mergers was a minor correction to the BBH mergers in the local universe (0.06% of mergers at $z < 1$, see Ref. [21]), we now find that nearly half of mergers now occur inside the cluster. For the 24 models considered here we find a total of 2819 mergers, 55% of which occur in the cluster. At low redshifts ($z < 1$), this number decreases to 45%, as the primordial binaries that merged at early times after a common-envelope phase had merged many Gyr ago. Compared to similar models without pN physics [21], the number of ejected BBH mergers at $z < 1$ decreases by $\sim$20% (496 versus 410). However, the number of in-cluster mergers has jumped significantly, from 1 to 338. This increases the total number of mergers (in-cluster and ejected) by $\sim$50%.

This increase in the number of BBH mergers occurring in the cluster primarily arises from properly accounting for GW emission for binaries regardless of their semimajor

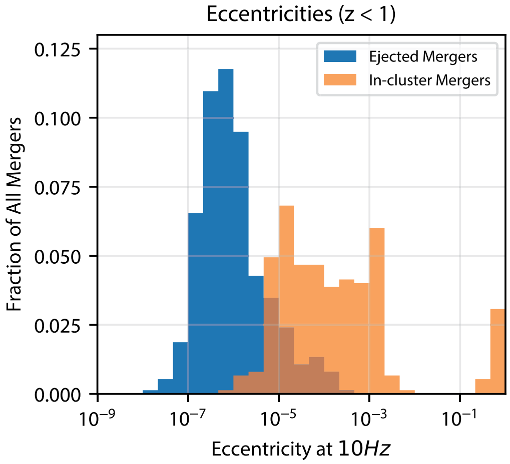

FIG. 1. The eccentricities of BBHs from the 24 GC models that merge at low redshifts. We calculate the eccentricity when the BBH enters the LIGO/Virgo detection band at a (circular) GW frequency of 10 Hz. The distribution is clearly trimodal: the first peak corresponds to BBHs which merger after ejection from the cluster (similar to Ref. [21], Fig. 10). The second peak corresponds to BBH mergers which occur in the cluster. The final peak, at $e > 0.1$, corresponds to in-cluster mergers which occur during a strong encounter, when the BBH enters the LIGO/Virgo band during a GW capture. Note that the two distributions are normalized to the total number of mergers (in-cluster and ejected).

axis. For example, a typical $30\ M_\odot + 30\ M_\odot$ BBH is ejected from a GC with $a \sim 0.4$ AU (roughly 10 times greater than the $a < 10R_\odot$ cutoff in BSE) after undergoing $\mathcal{O}(10)$ dynamical encounters [24]. During a typical encounter, the BBH semimajor axis will characteristically shrink while the orbital eccentricity randomly drawn from the thermal distribution, $p(e)de = 2ede$ [87]. These "hardening" encounters continue, shrinking the binary's semimajor axis until either the BBH is ejected from the cluster by the third body or until GWs drive the binary to merger. The timescale for each BBH to merge can be roughly approximated by [58]

$$t_{\rm GW} \sim \frac{e}{|de/dt|} \sim 400\ {\rm Gyr}\left(\frac{a}{0.4\ {\rm AU}}\right)^{4}\left(\frac{m_{\rm BH}}{30\ M_\odot}\right)^{-3}(1-e^2)^{7/2}. \qquad (1)$$

As Eq. (1) makes clear, a large eccentricity can significantly decrease the merger timescale. For $e \gtrsim 0.95$ (roughly %10 postencounter binaries) $t_{\rm GW}$ will decrease by more than 10^3, leading the BBH to promptly merge in the cluster. On the other hand, for BBHs that never reach a high eccentricity, these encounters will continue to harden the binary until it is ejected from the cluster (where its eccentricity at ejection is set by a single draw from the thermal distribution). Because the $(1-e^2)^{7/2}$ dependence in Eq. (1) preferentially selects in-cluster mergers from a super-thermal distribution, we expect these mergers to have larger eccentricities than their ejected counterparts by the time they reach the LIGO/Virgo band.

In Fig. 1, we show the eccentricity distribution of merging binaries as they enter the LIGO/Virgo band (which we define as a circular GW frequency of 10 Hz). We see the expected separation in eccentricity between BBHs which merge in the cluster and those that merge after being ejected from the cluster. For the in-cluster mergers, we also find a clear bimodality, with the lower peak corresponding to isolated binaries that merge after a dynamical encounter and the higher peak ($e > 0.1$) corresponding to sources which merge *during* the encounter via GW capture. Although previous work [25,27,88] has shown through scattering experiments that such mergers are to be expected at the 1% level, this is the first work to show that these mergers occur in realistic GC environments. From our combined 24 models, we find that about 10% of the in-cluster mergers (~3% of all mergers) at $z < 1$ occur during these GW captures, in good agreement with analytic work [89].

Mergers over cosmic time.—In Fig. 2, we show the mergers of BBHs as a function of cosmological redshift. What is immediately striking is that the mass distributions for in-cluster and ejected binaries are significantly different at low redshifts. This arises from the delay times between formation and mergers for ejected BBHs. When a BBH is ejected from the cluster, it may still take several Gyr to merge in the field; see e.g., Ref. [90] and references therein. Even for the most massive clusters, the median inspiral time for ejected binaries is ~10 Gyr, see Fig. 1 in Ref. [21]. In effect, the ejected BBHs which merge today drew their components from the *initial* distribution of BH masses in the cluster, where the masses varied from $5\ M_\odot$ to $40\ M_\odot$. On the other hand, the in-cluster mergers have effectively no delay time, and their components are drawn from the *present-day* distribution of BH masses in the cluster. Because old GCs have ejected their most-massive BHs many Gyr ago [91], the BBHs merging in the cluster today are typically lower-mass than those that were ejected many Gyr ago.

Another interesting feature of Fig. 2 is the presence of BBH mergers in the upper-mass gap, beyond the mass limit imposed by pair-instability supernovae. The increased number of in-cluster mergers allows the GCs to produce significant numbers of 2G BBH mergers, some of which will have components above the maximum mass for BHs born from a single stellar collapse. As these systems can only be produced through multiple mergers, they will immediately be identifiable as having arisen from a dynamical environment. The rate of such mergers is small, but LIGO/Virgo is more sensitive to mergers with more massive components (the detection horizon scales with the mass of the more massive component as $m^{2.2}$ [86]). At the expected sensitivity for Advanced LIGO's third observing

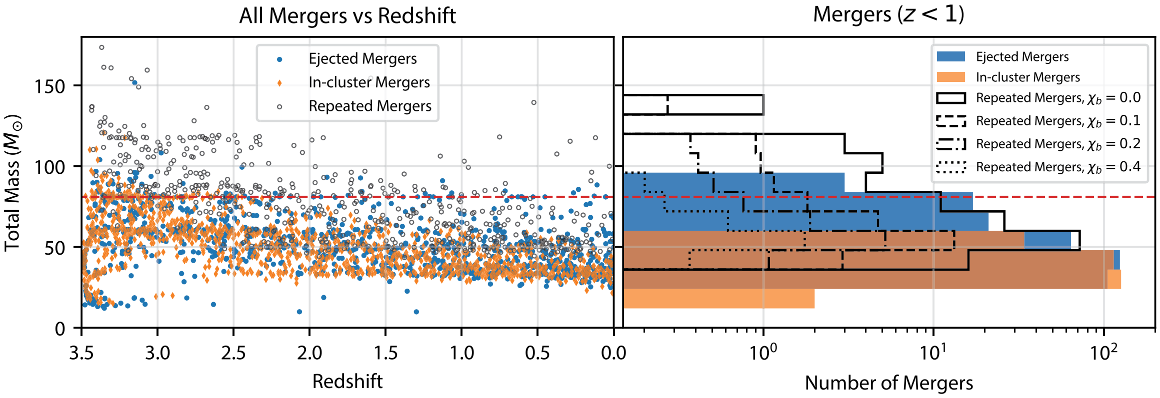

FIG. 2. The total mass of merging BBHs from all 24 GC models. On the left, we show all mergers as a function of redshift, with the orange diamonds and blue points showing in-cluster and ejected mergers, respectively. The black circles show 2G mergers (both in-cluster and ejected) which have at least one component that was formed from a previous BBH merger. The right panel shows the mass distribution of these mergers at low redshifts ($z < 1$). As the spins are increased from $\chi_b = 0$ to $\chi_b = 0.4$, the number of 2G mergers decreases significantly, as their progenitors were less likely to be retained in the cluster. See the discussion in Sec. V. The red-dashed line indicates the maximum mass of first-generation BBHs ($\sim 81\ M_\odot$) with our assumed pair-instability supernova limit. The handful of first-generation BBHs that merge above this are the result of either stable mass transfer or stellar collisions prior to BH formation.

run [92], a BBH with component masses of $40\ M_\odot + 80\ M_\odot$ could be detected out to $z \sim 1$, encompassing a comoving volume of space 3 times larger than was observed during LIGO's second science run [93].

Black hole spin and recoil kicks.—As a conservative assumption, we have assumed that all BHs in the cluster are born with no intrinsic spin. This is consistent with all but one (GW151226 [5]) of the BBHs detected by LIGO/Virgo so far. However, the presence of high BH spins, suggested by observations of BH x-ray binaries—see Ref. [94] for a review—can radically change the results presented here: depending on the spin magnitudes and orientations, merging BBHs can get kicks as high as 5000 km/s [63,70,95], significantly larger than the escape speed of a typical GC. As a result, the 2G mergers shown in the left-hand panel of Fig. 2 would not have formed if BHs are born with large spins, since their components would not have been retained in the cluster [14].

We can estimate how the numbers in Fig. 2 would have changed under different assumptions for BH birth spins. For each repeated merger, we calculate the probability that each of the components would have been retained in the cluster given different birth spins. This is done by computing the recoil kicks over 1000 realizations of the spin orientations at merger. The probability of retaining each progenitor is simply the fraction of mergers for which the recoil speed is smaller than the cluster escape speed where the merger occurred. For each 2G BBH merger, we take the product of the retention probabilities for each component as the probability of that 2G merger occurring. We show the retention of these BBHs in the right panel of Fig. 2 by weighting each 2G BBH merger by its retention probability. As expected, the number of 2G BBH mergers decreases as the birth spins of the BHs are increased. When $\chi_b = 0$, we find that $\sim 20\%$ of mergers at $z < 1$ are 2G mergers. As the spins are increased, this number decreases, and once $\chi_b = 0.4$, we observe $\mathcal{O}(1)$ 2G mergers, compared to the 672 first-generation mergers which occur at $z < 1$.

These assumption have significant implications for the measurable spins of BBH mergers. As shown by numerical relativity [64,96,97] and idealized pN N-body simulations with spins [40], repeated mergers of BBHs in clusters with near-equal masses will tend to produce BHs with $\chi \sim 0.7$, (assuming the initial spins are isotropically distributed [98]). But what LIGO/Virgo is most sensitive to is not the spin magnitudes of the BBHs components [99,100], but the effective spin of the BBH, defined as the mass-weighted projection of the two spins onto the orbital angular momentum:

$$\chi_{\rm eff} \equiv \left(\frac{m_1\vec{\chi}_1 + m_2\vec{\chi}_2}{m_1 + m_2}\right) \cdot \hat{L}, \tag{2}$$

where $\hat{L}$ is the direction of the orbital angular momentum and $\vec{\chi}_{1,2}$ are the dimensionless-spin vectors for the BHs. For dynamically formed binaries, the isotropic distribution of the orbit and spin vectors means that Eq. (2) will be peaked at $\chi_{\rm eff} = 0$ with symmetric tails whose extent depends on the BH spin magnitudes. We show the distributions of $\chi_{\rm eff}$ in Fig. 3. When the initial BH spins are low, the 2G systems are the only BBHs that merge with observably large spins. The fraction of systems with large spins increases as a function of total mass, since these larger systems (particularly those beyond the pulsational-pair instability limit) are predominantly formed through repeated mergers. As the birth spins are increased, the number of 2G mergers (with their characteristically large spins) decreases as their components are more likely to be ejected from the cluster during their first merger. But the total number of BBH

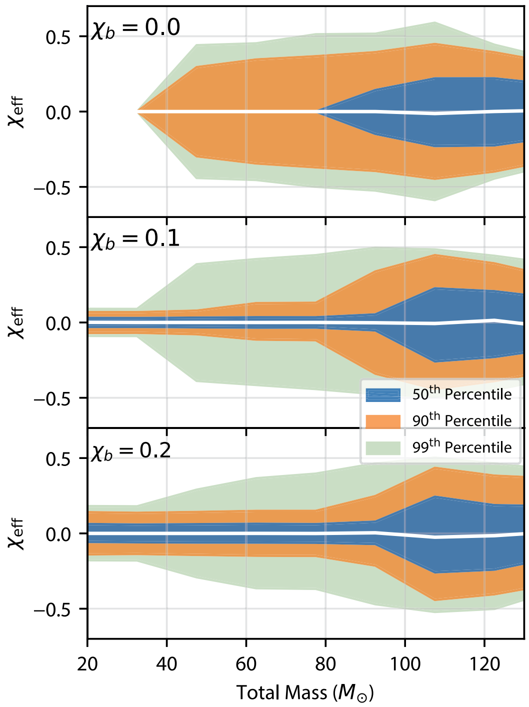

FIG. 3. The distributions of $\chi_{\rm eff}$ from BBHs that merge at $z < 1$, divided into bins of 15 $M_\odot$. Each bin shows the median (white line), 50th, 90th, and 99th percentiles of $\chi_{\rm eff}$ for all BBH mergers with that mass. For each binary, we average over $N = 10^3$ random spin orientations. For the 2G mergers, we use $N = 10^3$ times the probability of each component having been retained in the cluster following its earlier mergers (see discussion in Sec. V). As the birth spins (χ_b) of the BHs are increased, the fraction of 2G BBHs retained in the cluster decreases; however, the overall magnitudes of $\chi_{\rm eff}$ increases, as the first generation of BBHs begin to produce mergers with measurable spins. Note that while the large spin magnitudes for BBHs with total masses above 80 $M_\odot$ does not depend on the birth spins, the number of mergers in that mass range decreases sharply with increasing χ_b (see Fig. 2).

systems with nonzero $\chi_{\rm eff}$ increases, as the first generation of BHs will now form mergers with observable spins. This result is key: one of the most promising ways for identifying a dynamically formed BBH merger is by the alignment of the spins, with antialigned systems ($\chi_{\rm eff} < 0$) being a clear indicator of dynamical formation [26]. These results indicate that dynamical assembly in dense star clusters will inevitability produce a merger with $\chi_{\rm eff} < 0$, regardless of the BH birth spins.

Conclusion.—We have shown that the inclusion of pN effects can have significant implications for BBH mergers from dense star clusters detectable by LIGO/Virgo. By accounting for GW emission from isolated binaries and during three- and four-body dynamical encounters, we find that a significant number of mergers occur in the cluster, and that about 3% of all mergers (and ~10% of in-cluster mergers) in our models will enter the LIGO/Virgo detection band with high residual eccentricity ($e > 0.1$). Because of this, GCs can potentially produce a significant number of 2G BBH mergers with detectable spins and with masses larger than those produced through the collapse of single stars. Dynamics in dense star clusters can therefore produce BBH mergers with antialigned spins (a clear indicator of a dynamical origin) regardless of the initial spins of first-generation BHs: if natal BH spins are large, then GCs can produce BBH mergers with $\chi_{\rm eff} < 0$ from first-generation systems. If the spins are initially small (as predicted by, e.g., Ref. [27]), then the BBH merger products can often be retained in the cluster, forming a second generation of BBHs with large spins ($\chi \sim 0.7$).

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