Parametrization and Optimization of Gaussian Non-Markovian Unravelings for Open Quantum Dynamics

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We derive a family of Gaussian non-Markovian stochastic Schrödinger equations for the dynamics of open quantum systems. The different unravelings correspond to different choices of squeezed coherent states, reflecting different measurement schemes on the environment. Consequently, we are able to give a single shot measurement interpretation for the stochastic states and microscopic expressions for the noise correlations of the Gaussian process. By construction, the reduced dynamics of the open system does not depend on the squeezing parameters. They determine the non-Hermitian Gaussian correlation, a wide range of which are compatible with the Markov limit. We demonstrate the versatility of our results for quantum information tasks in the non-Markovian regime. In particular, by optimizing the squeezing parameters, we can tailor unravelings for improving entanglement bounds or for environment-assisted entanglement protection.

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Introduction.—Perhaps the most dramatic effect of the coupling of a quantum system to an environment is the loss of quantum properties of its state [1]. Yet, decoherence seldom occurs in a simple manner. In the last decade, advances in experimental techniques made it possible to observe non-Markovian dynamics in open quantum systems as, for example, micromechanical [2] and optical [3] systems, highlighting the central part it plays in preserving the coherent features of the system [4]. Non-Markovian dynamics has been proven to be essential for improvements in quantum metrology [5,6], advances in quantum thermodynamics [7], and optimal control scenarios [8], in which the persistence of correlations such as entanglement is crucial. The interplay of (non-Markovian) open system dynamics and time evolution of quantum correlations is an active field of research [9].

General open quantum system dynamics can be approached from various perspectives. One can, as is most commonly done, use the projection operator formalism [10,11], time local master equations [12], or hierarchical equations of motion [13,14], all of which describe the dynamics on the level of the density matrix. Alternatively, a stochastic description in terms of pure state unravelings [stochastic Schrödinger equations (SSEs)] is possible. Quantum jumps and quantum state diffusion are then suitable methods both in the Markovian [15,16] and non-Markovian [17,18] regimes. In particular, a complete parametrization of diffusive SSEs in the Markovian regime is known [19,20]. Changing these parameters allows control over the noise correlations driving the stochastic dynamics, which can be used to optimize the trajectories, e.g., for entanglement detection [21,22]. Moreover, in the Markov case, a physical interpretation for the stochastic states can be given in terms of continuous monitoring of the environment of the open system [23].

Recently, there have been similar efforts in the non-Markovian regime. Diósi and Ferialdi [24,25] have studied the structure of general Gaussian non-Markovian SSE going beyond the standard non-Markovian quantum state diffusion (NMQSD) [18,26–28]. The same class of SSEs was then re-examined by Budini from the perspective of its symmetries [29]. However, a microscopic justification and derivation of general Gaussian non-Markovian SSEs is still lacking.

In this Letter we aim to fill this gap by providing a novel parametrization of the Gaussian noise correlations using squeezed states. We offer a single shot measurement interpretation for our family of Gaussian non-Markovian unravelings. Moreover, we demonstrate their potential: due to the explicit parametrization and physical interpretation we are able to significantly improve entanglement bounds and perform environment-assisted entanglement protection in the non-Markovian regime.

Open system model and general Gaussian unravelings.—We investigate the dynamics of a system linearly coupled to a bosonic bath. The Hamiltonian of the total system is given by $H = H_S + H_B + H_{SB}$, where H_S is the Hamiltonian of the system, $H_B = \sum_\lambda \omega_\lambda b_\lambda^\dagger b_\lambda$ is the bath Hamiltonian, and $H_{SB} = \sum_\lambda g_\lambda (L b_\lambda^\dagger + L^\dagger b_\lambda)$ describes their interaction. Here, b_λ and $b_\lambda^\dagger$ are bosonic annihilation and creation operators of the bath mode λ, with frequency ω_λ, satisfying $[b_\lambda, b_{\lambda'}^\dagger] = \delta_{\lambda\lambda'} \mathbb{1}$. Furthermore L is an arbitrary coupling operator acting on the system and accounting for its interaction with all modes of the bath through coupling amplitudes g_λ, which, without loss of generality, are chosen to be real. We switch to the interaction picture with respect to the bath, in which the transformed Hamiltonian reads $H_I(t) = H_S + \sum_\lambda g_\lambda (L b_\lambda^\dagger e^{i\omega_\lambda t} + L^\dagger b_\lambda e^{-i\omega_\lambda t})$. We assume, for simplicity, that the bath is initially at zero temperature, $\rho_B(0) = \otimes_\lambda |0_\lambda\rangle\langle 0_\lambda|$.

The key ingredient for our derivation of the generalized Gaussian SSE are *Bargmann squeezed states*. For mode λ, these states are defined as $\|z_\lambda, \xi_\lambda\rangle \equiv R(z_\lambda, \xi_\lambda)|0_\lambda\rangle$, where $R(z_\lambda, \xi_\lambda) = \exp[z_\lambda b_\lambda^\dagger - (\xi_\lambda/2) b_\lambda^{\dagger 2}]$, with $z_\lambda,\ \xi_\lambda \in \mathbb{C}$, and $|\xi_\lambda| < 1$ [30]. Since $R(z_\lambda, \xi_\lambda)$ is not unitary, the $\|z_\lambda, \xi_\lambda\rangle$ are not normalized, yet the condition $|\xi_\lambda| < 1$ guarantees that they are normalizable; see the Supplemental Material [31]. Most notably, the states $\|z_\lambda, \xi_\lambda\rangle$ are analytic both in z_λ and ξ_λ, and overcomplete. In the multimode bosonic environment, one can then write

$$\mathbb{1} = \int d^2\mathbf{z}\, p_{\boldsymbol{\xi}}(\mathbf{z}) \|\mathbf{z}, \boldsymbol{\xi}\rangle\langle \mathbf{z}, \boldsymbol{\xi}\|, \tag{1}$$

with $\|\mathbf{z}, \boldsymbol{\xi}\rangle = \otimes_\lambda \|z_\lambda, \xi_\lambda\rangle$, and measure $d^2\mathbf{z}\, p_{\boldsymbol{\xi}}(\mathbf{z}) = \prod_\lambda [(d\mathrm{Re} z_\lambda d\mathrm{Im} z_\lambda)/(\pi\sqrt{1-|\xi_\lambda|^2})] \exp(-\{[|z_\lambda|^2 - \frac{1}{2}(\xi_\lambda^* z_\lambda^2 + \xi_\lambda z_\lambda^{*2})]/(1-|\xi_\lambda|^2)\})$.

With the above completeness relation for squeezed coherent states at hand, we now turn to the system dynamics. Using relation (1), a pure state $|\Psi_t\rangle$ of the composite system, evolving according to Schrödinger equation

$$\frac{d}{dt}|\Psi_t\rangle = -iH_I(t)|\Psi_t\rangle, \tag{2}$$

can, at all times t, be expanded as

$$|\Psi_t\rangle = \int d^2\mathbf{z}\, p_{\boldsymbol{\xi}}(\mathbf{z}) |\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*, t)\rangle \|\mathbf{z}, \boldsymbol{\xi}\rangle, \tag{3}$$

where $|\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*, t)\rangle \equiv \langle \mathbf{z}, \boldsymbol{\xi}\|\Psi_t\rangle$ is the *unnormalized* state vector of the system *relative* to the environment squeezed coherent state $\|\mathbf{z}, \boldsymbol{\xi}\rangle$. This approach is a generalization of non-Markovian quantum state diffusion with an additional freedom through squeezing parameters $\boldsymbol{\xi}$ [18,27,28]. We emphasize that $|\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*, t)\rangle$ is an analytical function of both $\mathbf{z}^*$ and $\boldsymbol{\xi}^*$. Tracing over the bath, we find

$$\begin{aligned}\rho_S(t) &= \mathrm{tr}_B\{|\Psi_t\rangle\langle\Psi_t|\} \\ &= \int d^2\mathbf{z}\, p_{\boldsymbol{\xi}}(\mathbf{z}) |\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*, t)\rangle\langle\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*, t)| \\ &\equiv \mathcal{M}[|\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*, t)\rangle\langle\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*, t)|].\end{aligned} \tag{4}$$

That is, the reduced density operator of the system is obtained by averaging over the unnormalized relative states with the Gaussian probability density $p_{\boldsymbol{\xi}}(\mathbf{z})$; we denote this weighted integral over $d^2\mathbf{z}$ by $\mathcal{M}[\cdot]$. Equations (3) and (4) follow directly from the resolution of the identity in the form of Eq. (1). In order to obtain Eq. (4), one has to insert that resolution of the identity under the partial trace. Note, that both the relative states $|\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*, t)\rangle$ and the probability density $p_{\boldsymbol{\xi}}(\mathbf{z})$ depend parametrically on $\boldsymbol{\xi}$ in such a way that $\rho_S(t)$ is independent of $\boldsymbol{\xi}$; this reflects the basis independence of the partial trace over the environmental degrees of freedom. Equation (4) represents a family of unravelings of the open system dynamics parametrized by the squeezing parameters $\boldsymbol{\xi}$. Moreover, decomposition (3) allows for a single-shot measurement interpretation of the unraveling; a topic to which we come back later in the Letter.

We are now in a position to state the first main result of this Letter. Starting from the Bargmann squeezed state representation of the total state, Eq. (3), we derive an SSE for the time evolution of the relative states $|\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*, t)\rangle$. Combining Eqs. (3) and (2), and using the relations $b_\lambda^\dagger \|z_\lambda, \xi_\lambda\rangle = (\partial/\partial z_\lambda)\|z_\lambda, \xi_\lambda\rangle$ and $b_\lambda \|z_\lambda, \xi_\lambda\rangle = [z_\lambda - \xi_\lambda(\partial/\partial z_\lambda)]\|z_\lambda, \xi_\lambda\rangle$ [30,31,40], we are able to derive a closed linear non-Markovian SSE for the open system state $|\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*, t)\rangle$,

$$\begin{aligned}\frac{d}{dt}|\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*, t)\rangle &= -iH_S|\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*, t)\rangle + L z_t^* |\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*, t)\rangle \\ &\quad - \int_0^t ds[\alpha(t,s)L^\dagger + \eta(t,s)L] \\ &\quad \times \frac{\delta}{\delta z_s^*}|\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*, t)\rangle.\end{aligned} \tag{5}$$

Here we use the chain rule $(\partial/\partial z_\lambda^*)(\cdot) = \int ds (\partial z_s^*/\partial z_\lambda^*)(\delta/\delta z_s^*)(\cdot)$, and introduce the quantities

$$z_t^* \equiv -i\sum_\lambda g_\lambda e^{i\omega_\lambda t} z_\lambda^*, \tag{6a}$$

$$\alpha(t,s) \equiv \sum_\lambda g_\lambda^2 e^{-i\omega_\lambda(t-s)}, \tag{6b}$$

$$\eta(t,s) \equiv -\sum_\lambda \xi_\lambda^* g_\lambda^2 e^{i\omega_\lambda(t+s)}. \tag{6c}$$

Equation (4) shows that the reduced state dynamics is obtained by a Gaussian average over the solutions of Eq. (5). This amounts to regarding z_t^* to be a Gaussian stochastic process; see Eq. (6a). A simple calculation gives $\mathcal{M}[z_t^*] = \mathcal{M}[z_t] = 0$, and the second order correlations

$$\mathcal{M}[z_t z_s^*] = \alpha(t,s), \qquad \mathcal{M}[z_t^* z_s^*] = \eta(t,s), \tag{7}$$

completely specifying the Gaussian process z_t^*.

Our representation in terms of squeezed Bargmann states allows for nonzero $\eta(t,s)$ correlation, determined by the squeezing parameter $\boldsymbol{\xi}$. The choice $\boldsymbol{\xi} = 0$ leads to $\eta(t,s) = 0$, which is the standard NMQSD [18,27,28]. We are thus able to give a microscopic derivation of the generalized Gaussian non-Markovian SSE.

Families of Gaussian unravelings, Markov limits, and single shot measurements.—Since the partial trace over the environment is basis independent, it is clear that the dynamics of the reduced state $\rho_S(t)$ cannot depend on the squeezing parameter $\boldsymbol{\xi}$ and therefore must be independent of $\eta(t,s)$. However, different choices of $\boldsymbol{\xi}$, and thus $\eta(t,s)$, define different unravelings, Eq. (5), with corresponding correlations, Eq. (7).

General Gaussian non-Markovian SSEs, similar to Eq. (5), have been recently postulated based on the properties of Gaussian processes and symmetry properties of the system-environment interaction [24,29]. There, arbitrary $\alpha(t,s)$ and $\eta(t,s)$ are considered, satisfying a general positivity condition. The physics of our model [Eqs. (6b) and (6c)] determines $\alpha(t,s) = \alpha(t-s)$ to be a stationary correlation while $\eta(t,s) = \eta(t+s)$ is a function of $t+s$. The general positivity condition on the correlations, when applied here, corresponds to the normalizability condition $|\xi_\lambda| < 1$ mentioned earlier.

Note that in the Markov limit, when $\alpha(t,s) \to \gamma\delta(t-s)$, the average dynamics follows the Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) master equation for any choice of $\boldsymbol{\xi}$ in Eq. (6c). We stress that this can be shown starting from Eq. (5) without having to resort to the microscopic origin. We can conclude that the Markov limit fixes solely the form of the correlation $\alpha(t,s)$ leaving a wide range of choices for $\eta(t,s)$.

Let us now return to state decomposition (3) and consider the connection it provides between our formalism and measurement theory. A *single shot* measurement interpretation of the state $|\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*,t)\rangle$ in Eq. (5) can be offered: It is the state of the system after a generalized measurement of the bath with an outcome labeled by $\mathbf{z}$ has been performed at time t. This can be made evident by noticing that the set of operators $\mathsf{E}_{\boldsymbol{\xi}}(\mathbf{z}) = d^2\mathbf{z}\, p_{\boldsymbol{\xi}}(\mathbf{z})\|\mathbf{z},\boldsymbol{\xi}\rangle\langle\mathbf{z},\boldsymbol{\xi}\|$ is a positive-operator valued measure (POVM), see Supplemental Material [31]. Then, from representation (3) of state $|\Psi_t\rangle$, the probability of obtaining a measurement outcome in the vicinity of $\mathbf{z}$, when at a time $t > 0$ a measurement of the observable $\mathsf{E}_{\boldsymbol{\xi}}$ is done on the bath, is

$$P_{\boldsymbol{\xi}}(\mathbf{z},t)d^2\mathbf{z} = p_{\boldsymbol{\xi}}(\mathbf{z})\|\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*,t)\|^2 d^2\mathbf{z}. \tag{8}$$

Freedom to choose $\boldsymbol{\xi}$ allows us to optimize the measurement on the environment for certain tasks. Next we will discuss two of them: $\boldsymbol{\xi}$-optimal bounds on entanglement dynamics [21,41] and environment-assisted entanglement protection [42,43].

SL-invariant entanglement measures.—We now address the problem of entanglement evolution in open multipartite systems. First steps using a diffusive unraveling for the quantification of entanglement dynamics in non-Markovian open systems were taken in Ref. [41]. With the new family of unravelings at hand, Eq. (5), we are now in a position to tackle challenging tasks in quantum information dynamics in the non-Markovian regime.

We consider the entanglement evolution in multipartite open systems in which the subsystems do not interact among themselves, but one or more of them may be coupled to its own local bosonic environment. The system consists of N subsystems, described by a Hilbert space $\mathcal{H}_S = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \cdots \otimes \mathcal{H}_N$, each with arbitrary finite dimension. The system Hamiltonian H_S is a sum of local Hamiltonians. Subsystem k couples to its local bath through traceless operators L_k with real coupling amplitudes $g_{k,\lambda}$.

In order to quantify entanglement in this system we use special linear (SL)-invariant multipartite measures of entanglement μ_{inv} [44–46]. These are polynomial measures defined by the following two properties: (i) They are invariant under local linear transformations $G = G_1 \otimes G_2 \otimes \cdots \otimes G_N$, where G_i acts on subsystem i and $\det G_i = 1$, that is, $\mu_{\mathrm{inv}}(G\psi) = \mu_{\mathrm{inv}}(\psi)$. (ii) They are homogeneous functions of degree two for all $u \in \mathbb{C}$, i.e., $\mu_{\mathrm{inv}}(u\psi) = |u|^2\mu_{\mathrm{inv}}(\psi)$. SL-invariant multipartite measures can be used on mixed states by means of their convex roof extension

$$\mu_{\mathrm{inv}}(\rho_S) = \min_{\{p_k,\psi_k\}} \sum_k p_k \mu_{\mathrm{inv}}(|\psi_k\rangle), \tag{9}$$

where the minimum is taken over all possible pure state decompositions of ρ_S, i.e., $\rho_S = \sum_k p_k|\psi_k\rangle\langle\psi_k|$ [47]. The well known concurrence is the prime example of such an SL-invariant measure [48]. Note that there exists also a generalized multipartite concurrence for mixed states [49,50], which however, is not SL invariant [51].

Following Ref. [41], for a system satisfying the above conditions, given an initial normalized state $|\tilde{\psi}(0)\rangle$, a scaling relation between the entanglement $\mu_{\mathrm{inv}}(|\tilde{\psi}_{\boldsymbol{\xi}}(\mathbf{z},t)\rangle)$ of the normalized relative state $|\tilde{\psi}_{\boldsymbol{\xi}}(\mathbf{z},t)\rangle = [(|\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*,t)\rangle)/(\|\psi_{\boldsymbol{\xi}^*}(\mathbf{z}^*,t)\|)]$ and the initial entanglement in the system $\mu_{\mathrm{inv}}(|\tilde{\psi}(0)\rangle)$ can be established:

$$x_{\boldsymbol{\xi}}(\mathbf{z},t) \equiv \frac{\mu_{\mathrm{inv}}(|\tilde{\psi}_{\boldsymbol{\xi}}(\mathbf{z},t)\rangle)}{\mu_{\mathrm{inv}}(|\tilde{\psi}(0)\rangle)} = f_{\boldsymbol{\xi}}(\mathbf{z},t)\frac{P_{\boldsymbol{\xi}}(\mathbf{z},0)}{P_{\boldsymbol{\xi}}(\mathbf{z},t)}. \tag{10}$$

The second equality follows from measurement outcome probabilities (8), and the details of the scaling function $f_{\boldsymbol{\xi}}(\mathbf{z},t)$ can be worked out similar to Ref. [41]; see Supplemental Material [31]. Crucially, the new scaling relation now depends on the squeezing parameter $\boldsymbol{\xi}$.

$\boldsymbol{\xi}$-optimal bounds on entanglement dynamics.—Estimating and finding bounds on multipartite entanglement is a long-standing problem in entanglement theory [9]. Based on Eq. (10), the freedom provided by the dependence of the scaling function $f_{\boldsymbol{\xi}}(\mathbf{z},t)$ on $\boldsymbol{\xi}$ allows us to look for the tightest possible upper bound on the entanglement $\mu_{\mathrm{inv}}(\rho_S(t))$ of the reduced state of the system

$\rho_S(t)$, within the family of unravelings Eq. (5). In the framework of Markovian open quantum system dynamics diffusive equations have been used to achieve this goal [21,22]. The new family of non-Markovian unravelings permits us to generalize these results to the non-Markovian regime. The pure state decomposition (4) provides an upper bound for the entanglement of the open system state, we find

$$\frac{\mu_{\text{inv}}(\rho_S(t))}{\mu_{\text{inv}}(|\tilde{\psi}(0)\rangle)} \leq \bar{x}_{\boldsymbol{\xi}}(t), \tag{11}$$

where $\bar{x}_{\boldsymbol{\xi}}(t)$ is the mean entanglement in the multipartite open system; see the Supplemental Material [31]. Here we use Eq. (10), which also leads to the expression $\bar{x}_{\boldsymbol{\xi}}(t) = \int d^2\mathbf{z} P_{\boldsymbol{\xi}}(\mathbf{z}, 0) f_{\boldsymbol{\xi}}(\mathbf{z}, t)$.

Remarkably, both the scaling relation (10) and the upper bound (11) are independent of the initial state as well as of the specific entanglement measure used, as long as it is SL invariant.

We demonstrate the significance of our findings with an example of non-Markovian multipartite open quantum system dynamics. For concreteness, let us assume that M $(M \leq N)$ of the subsystems are qubits (two-level systems), each one of them coupled to its own local dephasing bath via $L_k = L = \sigma_z$, $(k = 1\ldots M)$, while the rest of the $N - M$ subsystems remain isolated [52]. For dephasing environments the scaling function becomes independent of $\mathbf{z}$ (see Supplemental Material [31]) and the mean entanglement in the system reduces to

$$\bar{x}_{\boldsymbol{\xi}}(t) = f_{\boldsymbol{\xi}}(t) = \prod_k^M \exp\left(-\frac{1}{2}\int_0^t ds \gamma_k(s)\right), \tag{12}$$

with time-dependent dephasing rates

$$\gamma_k(s) = 4\text{Re}\int_0^s ds'[\alpha_k(s,s') + \eta_k(s,s')]. \tag{13}$$

Clearly, any choice of η_k provides, via Eq. (12), an upper bound on the entanglement of the state of the system $\rho_S(t)$. One can now ask for the optimal choice η_k^{opt} (and therefore $\boldsymbol{\xi}_k^{\text{opt}}$), which would yield the tightest possible bound. Before going into the search for this optimal unraveling a reminder on the meaning of our theory is due here. As a result of the single shot measurement interpretation of the system state, an optimization of the mean entanglement $\bar{x}_{\boldsymbol{\xi}}$ at time $t = T$ must target that specific time and may not be the optimal choice for a different time $t \neq T$. With this in mind, we may now return to the task of minimizing $\bar{x}_{\boldsymbol{\xi}}(T)$ in Eq. (12). We assume for simplicity that all local dephasing channels are identical so that $\gamma_k(t) = \gamma(t)$. For a given time T, $\bar{x}_{\boldsymbol{\xi}}(T)$ is minimal if the integral in Eq. (12) is maximized for each bath mode λ. A simple calculation shows that the optimal value for $\bar{x}_{\boldsymbol{\xi}}(T)$ is obtained by setting the squeezing parameter $\xi_\lambda^{\text{opt}} = -e^{i\omega_\lambda T}$ [53], and yields the upper bound

$$\bar{x}_{\boldsymbol{\xi}^{\text{opt}}}(t) = \exp\left(-\frac{M}{2}\int_0^t ds \gamma^{\text{opt}}(s)\right). \tag{14}$$

Exact results on entanglement evolution in multipartite open systems with non-Markovian dynamics are scarce [9,54], making it difficult to assess how tight our bound really is. Yet, for the case of two qubits with only one of them coupled to a dephasing channel, the exact entanglement dynamics is given in Ref. [22] for Markovian and in Ref. [55] for non-Markovian dynamics. Our bound exactly reproduces this entanglement evolution in both cases. Indeed, our $\boldsymbol{\xi}$-optimal bound is also exact for any N-partite open system, where only one subsystem of dimension two is exposed to an arbitrary dephasing channel [45]. In Fig. 1 we show the entanglement dynamics, the $\boldsymbol{\xi}$-optimal entanglement bound (indistinguishable), and the previously obtained $\boldsymbol{\xi} = 0$ bound for Markov-, Ohmic-, and super-Ohmic dephasing environments.

The multichannel result, Eq. (14), coincides with the upper bound of the entanglement dynamics for a multipartite mixed state given in Corollary 4 of Ref. [45].

Environment-assisted entanglement protection.—It has been shown that if and only if the dynamics of the open system is given by a random unitary channel, then there exists a protocol perfectly restoring the lost quantum information [42]. The error correction scheme is conditional on the measurement performed on the system's quantum environment. Such a procedure has been explicitly constructed for two qubit [43] and for N-qubit [56] pure dephasing dynamics.

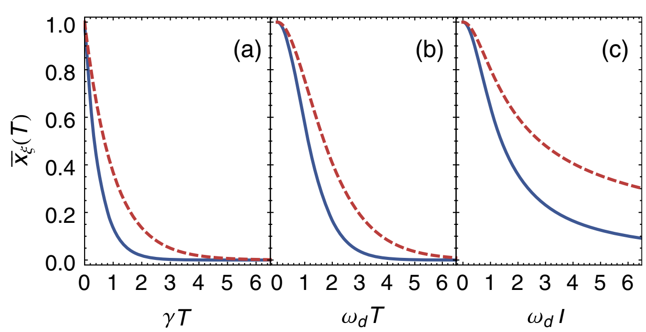

FIG. 1. Mean entanglement evolution in a N-partite open system, where only one subsystem of dimension two is exposed to a dephasing channel. (a) Markov, (b) Ohmic, and (c) super-Ohmic environment [41], where γ is the Markov decay rate and ω_d is the cutoff frequency. In all cases the upper bounds given by $\bar{x}_{\boldsymbol{\xi}^{\text{opt}}}(T)$ (blue continuous line) coincide with the exact dynamics of the reduced state entanglement (cf. Refs. [22,55]). For comparison we show the bound obtained for $\bar{x}_{\boldsymbol{\xi}=0}(T)$ (red dashed line), corresponding to the bound using the standard NMQSD $(\eta = 0)$.

Our dephasing channel is of random unitary type. However, having restricted the measurement to Bargmann squeezed state POVMs we do not expect to recover the initial state but instead we aim to restore the initial entanglement.

Indeed, by choosing $\xi_\lambda^{\rm prot} = e^{i\omega_\lambda T}$, Eq. (12) gives the bound $\bar{x}_{\xi^{\rm prot}}(T) = 1$. This means that for any outcome $\mathbf{z}$ of this optimal measurement at time T the conditional state of the open system contains the initial amount of entanglement.

Remarkably, we are able to construct explicitly a measurement on a realistic quantum environment that realizes an environment-assisted entanglement protection scenario, generalizing earlier considerations on Markov open quantum systems [22,57,58].

Let us finally remark that with our SSE we are able to assess the dynamics of entanglement without first solving the reduced state dynamics [22].

Conclusions.—In the present Letter we derived a generalized Gaussian non-Markovian SSE by expanding the environment in a Bargmann squeezed state basis. Each choice of the squeezing parameter $\boldsymbol{\xi}$ corresponds to a different unraveling and reflects a different measurement done on the environment of the open quantum system. Thus our results also add to the discussion on the objectivity of collapse models in the non-Markovian regime [59,60]. Our microscopic approach leads to a stationary Hermitian correlation $\alpha(t,s)$ and to a nonstationary non-Hermitian correlation $\eta(t,s)$ for the Gaussian noise z_t^*. By construction, the reduced dynamics is independent of $\eta(t,s)$. In the Markov limit we see that a wide range of different $\eta(t,s)$ are allowed, being compatible with the GKSL dynamics. We demonstrated the power of our family of unravelings for quantum information tasks in the non-Markovian regime. In particular, for local quantum channels, by optimizing over the squeezing parameter $\boldsymbol{\xi}$, we can tailor the ensemble of relative states for $\boldsymbol{\xi}$-optimal entanglement bounds or for environment-assisted entanglement protection.

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