Kern, Kadic, and Wegener Reply: In the preceding Comment [1] on our Letter [2], Oswald points out "that in Kern's structure the sign reversal happens already in the individual building blocks of their crystal, which makes its identification as a metamaterial effect questionable." He further argues that in photonic crystals "the interaction between the elementary building blocks is decisive, while this is not the case in Kern's structure." This appears to be a misunderstanding. Photonic crystals are distinct from metamaterials [3,4]. We will provide a clarifying discussion to the definition of a metamaterial below.

Apart from the question of what is a metamaterial and what is not, Oswald's Comment [1] points to the interesting connection between the "sign change" in the 3D chainmail-like metamaterial realized by us [2] and the individual "anti-Hall bars" introduced by Mani [5] years ago. This connection has given rise to a previous comment by Mani [6] and our response [7] and will not be repeated in full length here. In a nutshell, the sign change is connected to topology, which is the same for a Hall bar with a hole punched into it and for a single torus (also see Ref. [8]). Oswald states that "… the sign change happens already in the individual tori if the interconnections move gradually from the exterior to the interior and the design of the metamaterial provides a clever way to add up the individual interior Hall voltages of the tori and carry the total Hall voltage to the outside boundary of the complete structure." We agree, however, the credit for this clever arrangement leading to a 3D isotropic effective Hall coefficient should be given to previous mathematical [9] and numerical design work [10], which started from Bergman's homogenization formula [11,12]. Our Letter [2] merely reported the experimental realization of a chainmail-like metamaterial with a sign-inverted 3D isotropic effective Hall coefficient.

This brings us back to the definition of a metamaterial. Historically, in the pioneering work on negative-index metamaterials by Pendry [3] and Smith and co-workers [4], the split-ring resonator (SRR) [3] was the building block providing the mechanism for the underlying sign change of the magnetic permeability. An important aspect was that the size of the SRR, and hence the size of the crystal unit cell from which the metamaterial is built, can be much smaller than the electromagnetic wavelength. In the stationary regime relevant for us [2], the wavelength is mathematically infinitely large. For a photonic crystal, unit cell size and wavelength are generally comparable, leading to collective modes via Bragg reflection. In sharp contrast, for metamaterials in the spirit of [3,4], the effective properties are determined by the properties of the individual building blocks or unit cells. Interactions between different building blocks in one unit cell and between different unit cells have mostly been considered as a nuisance. This understanding is the polar opposite of Oswald's opening paragraph. Following Oswald, the identification of these pioneering structures [3,4] as metamaterials would be "questionable," too.

The discussion can be concretized by using the notion of "effective media," which was established much earlier [12]. If a rationally designed artificial structure, which can be periodic but need not be periodic, can adequately be described by effective-medium material parameters, it qualifies as a metamaterial in the strict sense. The material parameters can refer to optics, electrostatics, magnetostatics, mechanics, thermodynamics, transport, etc. The mapping of our chainmail microstructure [2] onto an effective scalar Hall coefficient has a sound mathematical basis [9–12]. It is therefore a metamaterial in the strict sense. In sharp contrast, the mapping of a photonic crystal band structure onto effective material parameters is problematic in general—with the long wavelength limit being a notable exception.

As a side remark, referring to his own work from 2005–2006, Oswald points to a possible mathematical description by discrete electrical network models. Indeed, such models [11,13] are an interesting avenue not only for describing anti-Hall bars and existing metamaterials but also for designing novel metamaterials with targeted effective parameters. Along these lines, it might be possible to construct anisotropic metamaterials with any wanted effective Hall-coefficient tensor. We have already taken steps in this direction [14].

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