Nonlocality, Correlations, and Magnetotransport in a Spatially Modulated Two-Dimensional Electron Gas

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It is shown that the classical commensurability phenomena in weakly modulated two-dimensional electron systems is a manifestation of the intrinsic properties of the correlation functions describing a homogeneous electron gas in a magnetic field. The theory demonstrates the importance for consideration of nonlocal response and removes the gap between classical and quantum approaches to magnetotransport in such systems.

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Magnetotransport properties of two-dimensional (2D) electrons in the presence of spatially varying weak electrostatic potential energy $U_{\mathbf{r}}$ or magnetic field $\delta B_{\mathbf{r}}$ have been extensively studied in connection with the problem of commensurability phenomena, in particular, Weiss oscillations, in periodically modulated systems [1–48]. The Weiss oscillations of the resistance of a unidirectionally modulated electron gas appear because of periodic dependence of the drift velocity, averaged over the path of cyclotron rotation, on the ratio of cyclotron radius R to modulation period a. Similar commensurability oscillations existing in the case of 2D (bidirectional) modulation have the same origin. Whereas the classical nature of Weiss oscillations was established [2] very soon after their discovery, the vast majority of theoretical works devoted to this phenomenon are based on application of the quantum linear response (Kubo) theory to calculation of conductivity. Within this approach, the resistance oscillations are explained in terms of modulation-induced transformation of Landau levels into one-dimensional subbands whose bandwidth oscillates as a function of the subband number. The classical analog of the Landau bandwidth is the average of the modulation energy over the path of cyclotron rotation [4,7]. However, the link between quantum and classical approaches to the problem is still incomplete. In the quantum linear response formalism, the oscillating dependence of conductivity appears as a result of direct influence of the modulation on the electron energy spectrum, so the classical origin of the commensurability phenomena is concealed. More important, the results obtained from the linear response theory deviate from the classical Boltzmann equation results [2,18,19] in the region $R \lesssim a$ corresponding to the high-field part of the oscillations and subsequent transition to the adiabatic regime.

In this Letter, the Kubo formalism is applied for calculation of the *nonlocal* conductivity $\sigma(\mathbf{r}, \mathbf{r}')$ of weakly modulated electron gas. It is shown that this approach is free from the difficulties mentioned above. In the regime of classically strong magnetic fields, relevant for observation of commensurability phenomena, the conductivity tensor is subdivided into the local part that describes the Drude response and the nonlocal one, entirely responsible for the effect of modulation. The nonlocal part is proportional to a product of the field of potential gradients, $\nabla_\gamma U_{\mathbf{r}} \nabla_{\gamma'} U_{\mathbf{r}'}$, or varying magnetic fields, $\delta B_{\mathbf{r}} \delta B_{\mathbf{r}'}$, by the spatial correlation functions of the *homogeneous* (unmodulated) 2D electron gas. Remarkably, the correlation functions already contain oscillating dependence on the magnetic field because they account for the cyclotron motion. This observation leads to a general point of view on the classical commensurability phenomena as manifestations of intrinsic properties of homogeneous 2D systems in the presence of modulation. The theory is valid for arbitrary weak and classically smooth $U_{\mathbf{r}}$ and $\delta B_{\mathbf{r}}$, and is applied as well for description of the magnetoresistance due to random modulation.

General formalism.—Throughout the Letter, the Planck's constant $\hbar$ is set at unity. A parabolic spectrum of 2D electrons is assumed, and the Zeeman splitting is neglected. The Hamiltonian of noninteracting electrons in a perpendicular magnetic field $\mathbf{B}_{\mathbf{r}} = (0, 0, B + \delta B_{\mathbf{r}})$ has a standard form, $\hat{H} = \sum_j \hat{H}_{\mathbf{r}_j}$, $\hat{H}_{\mathbf{r}} = m\hat{\mathbf{v}}_{\mathbf{r}}^2/2 + V_{\mathbf{r}} + U_{\mathbf{r}}$, where $\hat{\mathbf{v}}_{\mathbf{r}} = [-i\boldsymbol{\nabla} - (e/c)(\mathbf{A}_{\mathbf{r}} + \delta\mathbf{A}_{\mathbf{r}})]/m$ is the velocity operator, $\mathbf{r}$ is the 2D coordinate, m is the effective mass of electron, $\mathbf{A}_{\mathbf{r}}$ and $\delta\mathbf{A}_{\mathbf{r}}$ are the vector potentials describing the uniform and the modulating magnetic fields, respectively. Next, $V_{\mathbf{r}}$ is a random impurity potential varying on a scale much smaller than the cyclotron radius $R = v_F/\omega_c$, where $v_F = \sqrt{2\varepsilon_F/m}$ is the Fermi velocity expressed through the chemical potential ε_F and $\omega_c = |e|B/mc$ is the cyclotron frequency. Finally, $U_{\mathbf{r}}$ is a potential varying on a scale much larger than the magnetic length $\ell = \sqrt{c/|e|B}$ with the amplitude much smaller than ε_F. Similar

conditions of smoothness and smallness apply for magnetic modulation. It is assumed that $U_{\mathbf{r}}$ and $\delta B_{\mathbf{r}}$ have zero average over the sample area.

The Kubo-Greenwood formula for the steady-state nonlocal conductivity tensor is written in the exact eigenstate representation as follows:

$$\sigma_{\alpha\beta}(\mathbf{r},\mathbf{r}') = \frac{i}{S^2}\sum_{\delta\delta'}\frac{\langle\delta'|\hat{I}^{\alpha}_{\mathbf{r}}|\delta\rangle\langle\delta|\hat{I}^{\beta}_{\mathbf{r}'}|\delta'\rangle(f_{\varepsilon_\delta}-f_{\varepsilon_{\delta'}})}{(\varepsilon_\delta-\varepsilon_{\delta'}-i\lambda)(\varepsilon_\delta-\varepsilon_{\delta'})}, \quad (1)$$

where $\hat{\mathbf{I}}_{\mathbf{r}} = e\sum_j\{\hat{\mathbf{v}}_{\mathbf{r}_j},\delta(\mathbf{r}_j-\mathbf{r})\}$ is the operator of current density, $\{,\}$ denotes a symmetrized product, $\lambda\to+0$, S is the normalization area, δ is the eigenstate index, and f_ε is the equilibrium Fermi distribution. It is convenient to transform Eq. (1) by using the operator identity

$$\hat{\mathbf{v}}_{\mathbf{r}} = \ell^2\hat{\epsilon}\boldsymbol{\nabla}\mathcal{U}_{\mathbf{r}} - \{\hat{\mathbf{v}}_{\mathbf{r}},\delta B_{\mathbf{r}}\}/B - i\omega_c^{-1}\hat{\epsilon}[\hat{\mathbf{v}}_{\mathbf{r}},\hat{H}_{\mathbf{r}}], \quad (2)$$

where $\mathcal{U}_{\mathbf{r}} = V_{\mathbf{r}} + U_{\mathbf{r}}$ is the total potential and $\hat{\epsilon}$ is the antisymmetric unit matrix in the Cartesian 2D coordinate space. After substituting Eq. (2) into Eq. (1), the last term in Eq. (2) gives the classical Hall conductivity, while the rest of the contributions come from the first two terms.

In the case of purely potential modulation, $\delta B = 0$, the dissipative part of the conductivity is

$$\sigma^d_{\alpha\beta}(\mathbf{r},\mathbf{r}') = 2\pi e^2\ell^4\epsilon_{\alpha\gamma}\epsilon_{\beta\gamma'}\int d\varepsilon\left(-\frac{\partial f_\varepsilon}{\partial\varepsilon}\right) \times\langle(\nabla_\gamma\mathcal{U}_{\mathbf{r}})(\nabla_{\gamma'}\mathcal{U}_{\mathbf{r}'})\mathcal{A}_\varepsilon(\mathbf{r},\mathbf{r}')\mathcal{A}_\varepsilon(\mathbf{r}',\mathbf{r})\rangle, \quad (3)$$

where the angular brackets define the average over the random potential, and $\mathcal{A}_\varepsilon(\mathbf{r},\mathbf{r}') = (2\pi i)^{-1}[\mathcal{G}^A_\varepsilon(\mathbf{r},\mathbf{r}') - \mathcal{G}^R_\varepsilon(\mathbf{r},\mathbf{r}')]$ is the spectral function in the coordinate representation, expressed through the nonaveraged Green's functions $\mathcal{G}^s$ ($s = R,\ A$ denotes the retarded and the advanced ones). Since the case of degenerate electron gas is assumed, the energy ε stands in a narrow interval around the Fermi level and can be replaced by ε_F if the correlation function in Eq. (3) slowly varies with energy, in particular, in the classical transport regime. Evaluating Eq. (3) within the accuracy up to the first power in the random potential correlator $w(q)$ defined as a Fourier transform of the correlation function $\langle V_0 V_{\mathbf{r}}\rangle$ leads to two contributions: $\sigma^d_{\alpha\beta}\simeq\sigma^{(1)}_{\alpha\beta}+\sigma^{(2)}_{\alpha\beta}$,

$$\sigma^{(1)}_{\alpha\beta}(\mathbf{r},\mathbf{r}') = 2\pi e^2\ell^4\epsilon_{\alpha\gamma}\epsilon_{\beta\gamma'}(\nabla_\gamma U_{\mathbf{r}})(\nabla_{\gamma'}U_{\mathbf{r}'}) \times\int d\varepsilon\left(-\frac{\partial f_\varepsilon}{\partial\varepsilon}\right)\langle\mathcal{A}_\varepsilon(\mathbf{r},\mathbf{r}')\mathcal{A}_\varepsilon(\mathbf{r}',\mathbf{r})\rangle, \quad (4)$$

$$\sigma^{(2)}_{\alpha\beta}(\mathbf{r},\mathbf{r}') = 2\pi e^2\ell^4\epsilon_{\alpha\gamma}\epsilon_{\beta\gamma'}\int d\varepsilon\left(-\frac{\partial f_\varepsilon}{\partial\varepsilon}\right) \times\int\frac{d\mathbf{q}}{(2\pi)^2}q_\gamma q_{\gamma'}w(q)e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')}A_\varepsilon(\mathbf{r},\mathbf{r}')A_\varepsilon(\mathbf{r}',\mathbf{r}), \quad (5)$$

where $A_\varepsilon(\mathbf{r},\mathbf{r}') = \langle\mathcal{A}_\varepsilon(\mathbf{r},\mathbf{r}')\rangle$ is the averaged spectral function. The first contribution describes the conductivity due to the presence of smooth potential gradients. The second one is the leading term in the expansion of the conductivity in powers of the ratio of the scattering rate to cyclotron frequency. Keeping only these contributions is sufficient in the case of classically strong magnetic fields, $(\omega_c\tau_{\mathrm{tr}})^2\gg1$, where τ_{tr} is the transport time.

The difference between the present technique and previous applications of the Kubo formalism to the problem is a consideration of nonlocal response instead of the local one, which is necessary for correct evaluation of the conductivity, and the application of the identity Eq. (2), which separates the drift-induced $\sigma^{(1)}$ and diffusion-induced $\sigma^{(2)}$ contributions and removes the necessity to specify eigenstates and Green's functions at the early stage of calculations.

To find $\sigma^{(1)}$, one needs to calculate the pair correlation function in Eq. (4), which is determined, in the Born approximation, by the particle-hole ladder series. In the case of arbitrary $w(q)$, the problem cannot be solved analytically even in the classical limit. Therefore, the case of white noise random potential is assumed when $w(q)$ is replaced by a constant. Introducing the correlator $C^{ss'}_\varepsilon(\mathbf{r},\mathbf{r}') = w\langle\mathcal{G}^s_\varepsilon(\mathbf{r},\mathbf{r}')\mathcal{G}^{s'}_\varepsilon(\mathbf{r}',\mathbf{r})\rangle$ and applying a standard technique of summation leads to the integral equation $C^{ss'}_\varepsilon(\mathbf{r},\mathbf{r}') = K^{ss'}_\varepsilon(\mathbf{r},\mathbf{r}') + \int d\mathbf{r}_1 K^{ss'}_\varepsilon(\mathbf{r},\mathbf{r}_1)C^{ss'}_\varepsilon(\mathbf{r}_1,\mathbf{r}')$, where $K^{ss'}_\varepsilon(\mathbf{r},\mathbf{r}') = wG^s_\varepsilon(\mathbf{r},\mathbf{r}')G^{s'}_\varepsilon(\mathbf{r}',\mathbf{r})$ is the "bare" correlator expressed through the averaged Green's functions. It is convenient to rewrite this equation for the double Fourier transforms of C and K:

$$C_\varepsilon(\mathbf{q},\mathbf{q}') = K_\varepsilon(\mathbf{q},\mathbf{q}') + \int\frac{d\mathbf{q}_1}{(2\pi)^2}K_\varepsilon(\mathbf{q},\mathbf{q}_1)C_\varepsilon(\mathbf{q}_1,\mathbf{q}'). \quad (6)$$

Since only the terms with $s\neq s'$ are important, the repeating s indices are omitted here and below. The correlators C and K are essentially different. While $K_\varepsilon(\mathbf{r},\mathbf{r}')$ describes correlations on the $2R$ scale, $C_\varepsilon(\mathbf{r},\mathbf{r}')$ has no definite correlation length and logarithmically depends on $|\mathbf{r}-\mathbf{r}'|$. This is a consequence of the diffusion-pole divergence of $C_\varepsilon(\mathbf{q},\mathbf{q}')$, as in the limit of small q Eq. (6) can be reduced to a diffusion equation. The long-range behavior of correlations is a general property topologically dictated by the dimensionality 2 [49,50].

In contrast to $\sigma^{(1)}$, the contribution $\sigma^{(2)}$ can be treated locally, because it contains the exponential factor $e^{i\mathbf{q}\cdot(\mathbf{r}-\mathbf{r}')}$, where $\mathbf{q}$ has meaning of the momentum transferred in the scattering of electrons by the potential V. Since q is comparable to the Fermi momentum (except for the scattering on very small angles), the correlation length is much smaller than both R and the modulation length, and it is sufficient to consider the local conductivity,

$$\sigma^{(2)}_{\alpha\beta}(\mathbf{r}) = \int d\Delta\mathbf{r}\sigma^{(2)}_{\alpha\beta}(\mathbf{r}+\Delta\mathbf{r}/2,\mathbf{r}-\Delta\mathbf{r}/2). \quad (7)$$

Classical conductivity.—The contribution $\sigma^{(1)}$ is already proportional to the squared gradient of the smooth potential $U_{\mathbf{r}}$. In the classical case, when the Landau quantization is neglected, accounting for $U_{\mathbf{r}}$ in the Green's functions entering C_ε leads to an expansion in powers of small parameters $U_{\mathbf{r}}/\varepsilon_F$ and $\nabla U_{\mathbf{r}} R/\varepsilon_F$. Therefore, to calculate $\sigma^{(1)}$ in the classical limit, it is sufficient to employ the Green's functions of a homogeneous system:

$$G^{R,A}_\varepsilon(\mathbf{r},\mathbf{r}') = \frac{e^{i\theta(\mathbf{r},\mathbf{r}')}}{2\pi\ell^2}\sum_{N=0}^{\infty}\frac{L_N^0(|\Delta\mathbf{r}|^2/2\ell^2)e^{-|\Delta\mathbf{r}|^2/4\ell^2}}{\varepsilon-\varepsilon_N\pm i/2\tau}, \tag{8}$$

where $\Delta\mathbf{r}=\mathbf{r}-\mathbf{r}'$, the sum is taken over the Landau level numbers, L_N^M is the Laguerre polynomial, $\varepsilon_N=\omega_c(N+1/2)$ is the Landau level spectrum, $\tau=1/mw$ is the scattering time, and $\theta(\mathbf{r},\mathbf{r}')=(e/c)\int_{\mathbf{r}'}^{\mathbf{r}} d\mathbf{r}_1\cdot\mathbf{A}_{\mathbf{r}_1}$. Because of the homogeneity, Eq. (6) is solved analytically:

$$C_\varepsilon(\mathbf{q},\mathbf{q}')=C_{\varepsilon q}(2\pi)^2\delta(\mathbf{q}-\mathbf{q}'), C_{\varepsilon q}=K_{\varepsilon q}/(1-K_{\varepsilon q}), \tag{9}$$

where

$$K_{\varepsilon q}=\frac{w}{2\pi\ell^2}\sum_{N,N'}\frac{(-1)^{N+N'}e^{-\beta}L_N^{N-N'}(\beta)L_{N'}^{N'-N}(\beta)}{(\varepsilon-\varepsilon_N+i/2\tau)(\varepsilon-\varepsilon_{N'}-i/2\tau)} \tag{10}$$

and $\beta=q^2\ell^2/2$. The classical limit corresponds to treatment of the Landau level numbers as continuous variables and to application of the asymptotic form of $L_N^M(\beta)$ at large N. With $\varepsilon=\varepsilon_F$ and $q\ll mv_F$, this leads to

$$K_{\varepsilon q}\simeq K_q=\sum_{n=-\infty}^{\infty}\frac{J_n^2(qR)}{1+(n\omega_c\tau)^2}, \tag{11}$$

where J_n is the Bessel function. If $(\omega_c\tau)^2\gg 1$, it is sufficient to retain a term with $n=0$. As a result,

$$C_{\varepsilon q}\simeq C_q=J_0^2(qR)/[1-J_0^2(qR)] \tag{12}$$

and

$$\sigma^{(1)}_{\alpha\beta}(\mathbf{r},\mathbf{r}')=\frac{e^2\tau}{\pi m\omega_c^2}\epsilon_{\alpha\gamma}\epsilon_{\beta\gamma'}\int\frac{d\mathbf{q}_1}{(2\pi)^2}\int\frac{d\mathbf{q}_2}{(2\pi)^2}\int\frac{d\mathbf{q}}{(2\pi)^2} \times e^{i(\mathbf{q}-\mathbf{q}_1)\cdot\mathbf{r}}e^{i(\mathbf{q}_2-\mathbf{q})\cdot\mathbf{r}'}q_{1\gamma}q_{2\gamma'}\frac{U_{-\mathbf{q}_1}U_{\mathbf{q}_2}J_0^2(qR)}{1-J_0^2(qR)}, \tag{13}$$

where $U_{\mathbf{q}}$ is the Fourier transform of $U_{\mathbf{r}}$.

Using the Green's functions (8) for calculations of the local contribution $\sigma^{(2)}_{\alpha\beta}(\mathbf{r})$ in the classical limit gives the isotropic Drude conductivity at $(\omega_c\tau)^2\gg 1$:

$$\sigma^{(2)}_{\alpha\beta}=\delta_{\alpha\beta}\frac{e^2 n_s}{m\omega_c^2\tau}, \tag{14}$$

where n_s is the electron density. Consideration of higher-order terms (not included in $\sigma^{(2)}$) leads to an additional contribution $-\sigma^{(2)}_{\alpha\beta}/[1+(\omega_c\tau)^2]$ that complements the conductivity to the full Drude form. A generalization to the case of arbitrary $w(q)$ is straightforward and results in a substitution of the transport time τ_{tr} in place of τ. The effect of $U_{\mathbf{r}}$ on $\sigma^{(2)}$ leads to contributions of the order $(\omega_c\tau)^{-2}\sigma^{(1)}$ and, therefore, is neglected.

Magnetic modulation.—If the modulation $\delta B_{\mathbf{r}}$ instead of $U_{\mathbf{r}}$ is present, $\sigma^{(1)}$ of Eq. (4) is replaced by

$$\sigma^{(1)}_{\alpha\beta}(\mathbf{r},\mathbf{r}')=2\pi e^2\frac{\delta B_{\mathbf{r}}\delta B_{\mathbf{r}'}}{B^2}\int d\varepsilon\left(-\frac{\partial f_\varepsilon}{\partial\varepsilon}\right) \times\langle\tilde{v}_{\mathbf{r}\alpha}\tilde{v}_{\mathbf{r}'\beta}\mathcal{A}_\varepsilon(\mathbf{r},\mathbf{r}')\mathcal{A}_\varepsilon(\mathbf{r}',\mathbf{r})\rangle, \tag{15}$$

where $\tilde{\mathbf{v}}_{\mathbf{r}}=[-i\nu\partial/\partial\mathbf{r}-(e/c)\mathbf{A}_{\mathbf{r}}]/m$ is a differential operator with $\nu=1/2$ ($\nu=-1/2$) when acting on the first (second) coordinate variable of the Green's functions. The response is determined by the correlator $M^{\alpha\beta}_\varepsilon(\mathbf{r},\mathbf{r}')=w\langle\tilde{v}_{\mathbf{r}\alpha}\tilde{v}_{\mathbf{r}'\beta}\mathcal{G}^s_\varepsilon(\mathbf{r},\mathbf{r}')\mathcal{G}^{s'}_\varepsilon(\mathbf{r}',\mathbf{r})\rangle$ with $s\neq s'$:

$$M^{\alpha\beta}_\varepsilon(\mathbf{r},\mathbf{r}')=\mathcal{M}^{\alpha\beta}_\varepsilon(\mathbf{r},\mathbf{r}')+\int d\mathbf{r}_1\int d\mathbf{r}_2\tilde{v}_{\mathbf{r}\alpha}K_\varepsilon(\mathbf{r},\mathbf{r}_1) \times[\delta(\mathbf{r}_1-\mathbf{r}_2)+C_\varepsilon(\mathbf{r}_1,\mathbf{r}_2)]\tilde{v}_{\mathbf{r}'\beta}K_\varepsilon(\mathbf{r}_2,\mathbf{r}'), \tag{16}$$

where $\mathcal{M}^{\alpha\beta}_\varepsilon(\mathbf{r},\mathbf{r}')=w\tilde{v}_{\mathbf{r}\alpha}\tilde{v}_{\mathbf{r}'\beta}G^s_\varepsilon(\mathbf{r},\mathbf{r}')G^{s'}_\varepsilon(\mathbf{r}',\mathbf{r})$. In the classical case, using Green's functions of Eq. (8) and $C_{\varepsilon q}$ of Eq. (12), one gets the expression for Fourier transform of $M^{\alpha\beta}_\varepsilon(\mathbf{r},\mathbf{r}')$ at $\varepsilon=\varepsilon_F$ and $q\ll mv_F$:

$$M^{\alpha\beta}_{\mathbf{q}}\simeq\epsilon_{\alpha\gamma}\epsilon_{\beta\gamma'}\frac{q_\gamma q_{\gamma'}}{q^2}v_F^2\frac{J_1^2(qR)}{1-J_0^2(qR)}. \tag{17}$$

Therefore, $\sigma^{(1)}_{\alpha\beta}(\mathbf{r},\mathbf{r}')$ of Eq. (15) can be written in the form of Eq. (13), when the latter is modified by the substitution $q_{1\gamma}q_{2\gamma'}U_{-\mathbf{q}_1}U_{\mathbf{q}_2}J_0^2(qR)\to q_\gamma q_{\gamma'}\delta B_{-\mathbf{q}_1}\delta B_{\mathbf{q}_2}(\varepsilon_F/B)^2J_1^2(qR)/(qR/2)^2$.

Periodic modulation.—In the case of a periodic $U_{\mathbf{r}}$ or $\delta B_{\mathbf{r}}$, the problem becomes macroscopically homogeneous and described by the conductivity tensor

$$\sigma_{\alpha\beta}=\frac{1}{S}\int d\mathbf{r}\int d\mathbf{r}'\sigma_{\alpha\beta}(\mathbf{r},\mathbf{r}'), \tag{18}$$

which can be also viewed as the average of the local conductivity over the elementary cell of modulation lattice. Application of Eq. (18) to Eq. (13) gives, for potential and magnetic modulation, respectively,

$$\sigma^{(1)}_{\alpha\beta} = \frac{e^2 n_s \tau}{m} \int d\mathbf{q} \frac{\Omega_{\mathbf{q}}}{2q^2} \frac{\epsilon_{\alpha\gamma}\epsilon_{\beta\gamma'} q_\gamma q_{\gamma'}}{1 - J_0^2(qR)} \begin{cases} (qR)^2 J_0^2(qR) \\ 4J_1^2(qR) \end{cases}, \quad (19)$$

with $\Omega_{\mathbf{q}} = \sum_{k_1,k_2} |u_{k_1,k_2}|^2 \delta(\mathbf{q} - k_1\mathbf{Q}_1 - k_2\mathbf{Q}_2)$, where k_1 and k_2 are integers, $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are the Bravais vectors of the reciprocal lattice, and u_{k_1,k_2} are the Fourier coefficients of the relative modulation strength, $u(\mathbf{r}) = U_{\mathbf{r}}/\varepsilon_F$ for the potential modulation and $u(\mathbf{r}) = \delta B_{\mathbf{r}}/B$ for the magnetic one. For harmonic unidirectional modulation, $u(\mathbf{r}) = \eta\cos(Qx)$, the vectors are $\mathbf{Q}_1 = (Q, 0)$ and $\mathbf{Q}_2 = (0, 0)$, while nonzero coefficients are $u_{1,0} = u_{-1,0} = \eta/2$. Thus, only the component $\sigma^{(1)}_{yy}$ survives, leading to the resistivity $\rho^{(1)}_{xx} \simeq \sigma^{(1)}_{yy}/\sigma_H^2$, where σ_H is the classical Hall conductivity. This contribution is identified with the Weiss oscillations term, in full agreement with the results of theories based on the Boltzmann equation [2,18,19]. Previous theories based on the Kubo formula for local conductivity miss the term J_0^2 in the denominator. This would occur if the correlators C_q and $M^{\alpha\beta}_{\mathbf{q}}$ were replaced by the bare correlators K_q and $\mathcal{M}^{\alpha\beta}_{\mathbf{q}}$. Such an approximation is justified at $qR \gg 1$, when $J_l^2(qR) \simeq (2/\pi qR) \times \cos^2(qR - l\pi/2 - \pi/4)$. In the general case of anharmonic 2D modulation, Eq. (19) gives a superposition of Weiss oscillations with different $\mathbf{q}$ in both ρ_{xx} and ρ_{yy} [9]. In the adiabatic limit, $qR \ll 1$, $\rho^{(1)} \propto B^2$ in agreement with the experiment [10].

Random modulation.—In the case of weak modulation by random potential or magnetic field, the problem is again macroscopically homogeneous. The current density averaged over a large area is approximately related to the averaged driving electric field by the local Ohm's law with the conductivity tensor of Eq. (18), averaged over the random modulation distribution. This approximation is valid because of the assumed weakness of modulation, while in the general case the problem of linear response in inhomogeneous media remains very complicated even in the local formulation [51]. The averaging of $\sigma^{(1)}_{\alpha\beta}$ written in the form of Eq. (13) is equivalent to a substitution $u_{-\mathbf{q}_1} u_{\mathbf{q}_2} \to S\delta_{\mathbf{q}_1,\mathbf{q}_2} W(q_1)$, where $W(q)$ is the Fourier transform of the correlator $\langle u(0)u(\mathbf{r})\rangle$. This leads to the isotropic conductivity

$$\sigma^{(1)} = \frac{e^2 n_s \tau}{m} \int_0^\infty \frac{dq}{8\pi} \frac{qW(q)}{1 - J_0^2(qR)} \begin{cases} (qR)^2 J_0^2(qR) \\ 4J_1^2(qR) \end{cases}. \quad (20)$$

The function $W(q)$ is expected to decrease with q on the scale of inverse mean modulation length r_0^{-1}. For example, $W(q) \propto e^{-r_0 q}$ in the case of remote ionized impurity potential relevant for 2D electrons in high-mobility heterostructures. According to Eq. (20), in the adiabatic limit $R \ll r_0$ one has $\rho^{(1)} \propto B^2$ for both types of modulation, while at $R \gg r_0$ $\rho^{(1)} \propto B$ for the potential modulation and $\rho^{(1)} \propto B^3$ for the magnetic one. Though both $V_{\mathbf{r}}$ and $u(\mathbf{r})$ are random, the problem studied here is not equivalent to the problem of electron motion in the presence of two kinds of scatterers, the short-ranged and the long-ranged ones. Indeed, the effect of modulation accounted in $\sigma^{(1)}$ is electron drift rather than scattering-assisted diffusion, while the diffusion occurs due to the potential $V_{\mathbf{r}}$. The positive magnetoresistance described above is a consequence of the drift motion (although the drift along closed contours is also known to be a cause of localization, which cannot be accounted within the Born approximation). A different model of two-component disorder [52] can lead to a negative magnetoresistance.

Finally, one should discuss possible effects of electron-electron (Coulomb) interaction on the magnetoresistance of modulated 2D electron gas. Although this interaction conserves the total momentum of electrons, it does contribute to the Green's functions, modifying the energy spectrum and, consequently, the conductivity. The combined effect of the periodic modulation and the Coulomb interaction is essential in strong magnetic fields, when the interaction changes the shape of the Shubnikov–de Haas oscillations [53,54]. Next, the interaction-induced correction to conductivity [55] generates oscillations in ρ_{yy} [56], which are not related to the Landau quantization and, therefore, are important as well in the classical region of fields studied in this Letter. Apart from that, the interaction-induced imaginary part of self-energy in the Green's functions, which can be described by the temperature-dependent inelastic scattering time τ_{in}, leads to a cutoff of the diffusion pole in the correlator C_q. As a result, one should expect a suppression of the conductivity $\sigma^{(1)}$ when the modulation length (period) increases and becomes comparable to the diffusion length $l_D = \sqrt{\tau_{\text{in}} D}$, where $D = R^2/2\tau$ is the diffusion coefficient. Since $l_D \gg R$, owing to the assumed $\tau_{\text{in}} \gg \tau$ at low temperatures, this effect may influence the resistance in the adiabatic limit only.

In summary, the problem of magnetotransport in modulated 2D electron systems requires consideration of nonlocal response. The classical commensurability phenomena are described as a result of mapping of the modulation structure onto the spatial correlation pattern of a homogeneous electron system. The correlation functions responsible for potential and magnetic modulation in the regime of classically strong magnetic fields [Eqs. (12) and (17)] depend only on the cyclotron radius. A random modulation leads to a positive magnetoresistance that is sensitive to the modulation type until the adiabatic limit is reached. It remains a question whether similar conclusions apply to 2D systems with Dirac band spectrum such as graphene and related materials.

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