Quantum Metrology beyond the Classical Limit under the Effect of Dephasing

Yuichiro Matsuzaki,[1,2,*] Simon Benjamin,[3] Shojun Nakayama,[4] Shiro Saito,[1] and William J. Munro[1,2,4]
[1]*NTT Basic Research Laboratories, NTT Corporation, 3-1 Morinosato-Wakamiya, Atsugi, Kanagawa 243-0198, Japan*
[2]*NTT Theoretical Quantum Physics Center, NTT Corporation, 3-1 Morinosato-Wakamiya, Atsugi, Kanagawa 243-0198, Japan*
[3]*Department of Materials, University of Oxford, Oxford OX1 3PH, United Kingdom*
[4]*National Institute of Informatics, 2-1-2 Hitotsubashi, Chiyoda-ku, Tokyo 101-8430, Japan*

(Received 4 August 2017; published 5 April 2018)

Quantum sensors have the potential to outperform their classical counterparts. For classical sensing, the uncertainty of the estimation of the target fields scales inversely with the square root of the measurement time T. On the other hand, by using quantum resources, we can reduce this scaling of the uncertainty with time to $1/T$. However, as quantum states are susceptible to dephasing, it has not been clear whether we can achieve sensitivities with a scaling of $1/T$ for a measurement time longer than the coherence time. Here, we propose a scheme that estimates the amplitude of globally applied fields with the uncertainty of $1/T$ for an arbitrary time scale under the effect of dephasing. We use one-way quantum-computing-based teleportation between qubits to prevent any increase in the correlation between the quantum state and its local environment from building up and have shown that such a teleportation protocol can suppress the local dephasing while the information from the target fields keeps growing. Our method has the potential to realize a quantum sensor with a sensitivity far beyond that of any classical sensor.

DOI: 10.1103/PhysRevLett.120.140501

It is well known that two-level systems are attractive candidates with which to realize ultrasensitive sensors, as the frequency of the qubit can be shifted by coupling it to a target field. Such a frequency shift induces a relative phase between the qubits basis states which can be simply measured in a Ramsey-type experiment. This method has been used to measure magnetic fields, electric fields, and temperature [1–4]. With the typical classical sensor measurement devices (including SQUIDs [5], Hall sensors [6], and force sensors [7]), the uncertainty in the estimation of the target fields scales as $1/\sqrt{T}$ with a total measurement time T. This scaling is considered classical [8]. With a qubit-based sensor using a Ramsey-type measurement, the readout signal is periodic against the amplitude of the target fields. So, unless the range of the target fields is known, the interaction time with the target fields should be limited, which reduces the sensitivity. In this case, the sensitivity decreases as $1/t\sqrt{N}$ by performing N repetitions with a short sensing time t. This sensitivity can be rewritten as $1/\sqrt{Tt}$ if fast qubit control is available. Although one could achieve the uncertainty with $1/T$ by setting $t = T$ with the knowledge of the target field range, a dynamic range, which allows us to estimate the fields unambiguously, becomes small due to the periodic structure of the readout signal. Fortunately, there is an ingenious way to improve the dynamic range by using a feedback control of the qubit [9,10]. Actually, several experimental demonstrations have shown a sensitivity that scales as $1/T$ with the high-dynamic range [10,11]. However, as quantum states are susceptible to decoherence, it has generally been considered that such a scaling $1/T$ can be realized only if the measurement time T is much shorter than the coherence time [9,12]. Recently, several approaches have been proposed and demonstrated that use quantum error correction [13] and dynamical decoupling [14–16] to circumvent this limitation. Using quantum error correction, we can measure the amplitude of the target field with an uncertainty scaling as $1/T$ under the effect of specific decoherence such as bit flip errors [17–23], while dynamical decoupling makes it possible to estimate the frequency of time-oscillating fields with a sensitivity beyond the classical limit on a time scale longer than the coherence time [24,25]. However, there is currently no known metrological scheme to achieve with an uncertainty of $1/T$ when measuring the amplitude of target fields with dephasing.

In this Letter, we propose a scheme for measuring the amplitude of target fields with an uncertainty of $1/T$ under the effect of dephasing. We will use a similar concept to the quantum Zeno effect (QZE) [26–28]. For shorter time scales than the correlation time of the environment τ_c, the interaction with the environment induces a quadratic decay rate that is much slower than the typical exponential decay [29]. Frequent measurements can be used to reset the correlation with the environment and so keep this state in the initial quadratic decay region, which suppresses the decoherence [26–28]. However, if we naively apply the QZE to quantum metrology, the frequent measurements freeze all the dynamics so that the quantum states cannot acquire any information from the target fields. Instead, we use quantum teleportation (QT) based on concepts taken from one-way quantum computation [30–34] to reset the correlation between the system and the environment [35]. If we transfer the quantum states to a new site, we can prevent any increase in the correlation between the system and environment in the previous site, and the quantum states are affected then only by a slow quadratic decay due to the local environment in the new site. This noise suppression with a qubit motion using a concept drawn from QT has been proposed and demonstrated by using superconducting qubits [35]. The crucial idea in this Letter is to use this one-qubit teleportation-based noise suppression for quantum metrology. Interestingly, although the QT protocol eliminates the deterioration effect caused by the dephasing from the local environment, we can accumulate the phase information from the global target fields during this protocol. We have shown that, as long as nearly perfect QT is available, we can achieve a sensor with the uncertainty scaling $1/T$ with dephasing. Moreover, we have found that, even when the QT is moderately noisy, the sensitivity of our protocol is superior to that of the standard Ramsey measurement.

Noise and its suppression.— Our system and the environment in this situation can be described by a Hamiltonian of the form $H = H_S + H_I + H_E$ [36], where $H_S = \sum_{j=1}^{L} (\omega/2)\sigma_z^{(j)} \otimes \mathbb{1}_E^{(j)}$ ($H_E = \sum_{j=1}^{L} \mathbb{1}_S^{(j)} \otimes C_j$) denotes the system (environmental) Hamiltonian while $H_I = \sum_{j=1}^{L} \lambda\sigma_z^{(j)} \otimes B_j$ denotes the interaction between the system and the environment. Here $\sigma_z^{(j)}$ is the usual Pauli Z operator of the jth qubit with frequency ω, while B_j and C_j denote the environmental operator at that jth site. $\hat{\mathbb{1}}_S^{(j)}$ ($\hat{\mathbb{1}}_E^{(j)}$) denotes an identity operator for the system (environment). Furthermore, we set $\hbar = 1$. In an interaction picture, we have $H_I(t) = \lambda \sum_{j=1}^{L} \sigma_z^{(j)} \otimes \tilde{B}_j(t)$, where $\tilde{B}_j(t) = e^{iH_E t} B_j e^{-iH_E t}$. The separable initial state is given as $\rho(0) = \otimes_{j=1}^{L} [\rho_S^{(j)}(0) \otimes \rho_E^{(j)}]$, where we have assumed $\rho_E^{(j)}$ is in thermal equilibrium ($[\rho_E^{(j)}, H_E] = 0$) and our noise is nonbiased ($\mathrm{Tr}[\rho_E^{(j)} B_j] = 0$) for all j. If the initial state is separable, we consider the first site by tracing out the others. Solving Schrodinger's equation gives

$$\rho_I^{(1)}(\tau) \simeq \rho^{(1)}(0) - i\lambda \int_0^{\tau} dt' [\sigma_z^{(1)} \otimes \tilde{B}_1(t'), \rho^{(1)}(0)] - \lambda^2 \int_0^{\tau} \int_0^{t'} dt' dt'' [\sigma_z^{(1)} \otimes \tilde{B}_1(t'), [\sigma_z^{(1)} \otimes \tilde{B}_1(t''), \rho^{(1)}(0)]]$$

using a second-order perturbation expansion in λ [36]. Tracing out the environment, we have

$$\rho_S^{(1)}(\tau) \simeq \rho_S^{(1)}(0) - \lambda^2 \int_0^{\tau} \int_0^{t'} dt' dt'' C_{t'-t''}^{(1)} [\hat{\sigma}_z^{(1)}, [\hat{\sigma}_z^{(1)}, \rho_S^{(1)}(0)]],$$

where we define the correlation function of the environment as $C_{t'-t''}^{(1)} \equiv \frac{1}{2} \mathrm{Tr}[(\tilde{B}_1(t')\tilde{B}_1(t'') + \tilde{B}_1(t'')\tilde{B}_1(t'))\rho_E^{(1)}]$. If we are interested in a time scale much shorter than the correlation time of the environment, we can approximate the correlation function as $C_{t'-t''}^{(1)} \simeq C_0^{(1)}$. For most solid state systems, this is readily satisfied, as the environment correlation time is much longer than the coherence time of the qubit [37–40], and so this condition is readily satisfied for many systems. In such a case, $\rho_S^{(1)}(\tau) \simeq (1 - \epsilon_\tau) U_{1,\tau} \rho_S^{(1)}(0) U_{1,\tau}^{\dagger} + \epsilon_\tau \hat{\sigma}_z^{(1)} \times U_{1,\tau} \rho_S^{(1)}(0) U_{1,\tau}^{\dagger} \sigma_z^{(1)}$ with $U_{j,\tau} = e^{-i\omega\tau\hat{\sigma}_z^{(j)}/2}$ specifying the unitary operator for a site j and $\epsilon_\tau = \lambda^2 C_0 \tau^2$ denoting the error rate for $\lambda^2 C_0 \tau^2 \ll 1$. Since the error rate has a quadratic form in time t, the decoherence effect is negligible for short time scales $t \ll 1/\lambda\sqrt{C_0}$. This has been discussed in the field of the QZE [26–28]. On the other hand, if we consider longer time scales of $t > 1/\lambda\sqrt{C_0}$ with the same environment, error accumulation will destroy the quantum coherence of the qubit.

Let us now describe the noise suppression technique using QT. It begins with the free evolution of the qubit for a time $\tau = t/n$ (where t is the total time and n is the number of times QT is to be performed). After this, QT transports $\rho_S^{(1)}$ to site 2. The quantum state starts interacting with a new local environment described by a density matrix $\rho_E^{(2)}$. The error rate will be suppressed due to the quadratic decay [35]. Performing QT n times (each time to a fresh qubit) yields $\rho_S^{(n)}(t) \simeq [1 - (\lambda^2 C_0 t^2/n)] U_{n,t} \rho_S(0) U_{n,t}^{\dagger} + (\lambda^2 C_0 t^2/n) \hat{\sigma}_z \times U_{n,t}^{\dagger} \rho_S(0) U_{n,t}^{\dagger} \sigma_z$ at site n. For a large n, this approaches the pure state $\rho_S(t) \simeq U_{n,t} \rho_S(0) U_{n,t}^{\dagger}$, and so our approach can suppress dephasing.

Definition of parameters.—Here, we discuss the key parameters that we will use in our scheme. We define L and M as the total number of the probe qubits and the size of the entangled state, respectively. In our scheme, there are three time scales: $\tau \ll t \ll T$. The interaction time between teleportations is denoted τ. This interaction is repeated n times between the state preparation and measurement, giving a total time denoted $t = n\tau$. The whole procedure including preparation and measurement is repeated N times, giving a total interaction time T. As regards the dephasing model, although our general approach described above uses a perturbative analysis typically valid for a short time scale, we need to examine the dynamics of our system for arbitrary time scales. So we will consider a more specific noise model that is given by $\hat{\mathcal{E}}_j[\rho_S^{(j)}(\tau)] = [(1 + e^{-\gamma^2\tau^2})/2] U_{j,\tau} \rho_S^{(j)}(0) \times U_{j,\tau}^{\dagger} + [(1 - e^{-\gamma^2\tau^2})/2] \hat{\sigma}_z^{(j)} U_{j,\tau} \rho_S^{(j)}(0) U_{j,\tau}^{\dagger} \sigma_z^{(j)}$ at the site j during the evolution for a time $\tau = t/n$ (with γ representing the dephasing rate). This model is consistent with the general results described above when we choose $\gamma = \sqrt{2\lambda^2 C_0}$ [41]. Typical dephasing models [37–39,54] show this behavior if the correlation time of the environment is much longer than the dephasing time. If we consider a state ρ composed of M qubits, the noise channel during the time evolution of τ is described as $\hat{\mathcal{E}}_1 \hat{\mathcal{E}}_2 \ldots \hat{\mathcal{E}}_M(\rho)$. We consider

Greenberger-Horne-Zeilinger (GHZ) states $|\psi^{(\mathrm{GHZ})}\rangle = (1/\sqrt{2})[\otimes_{j=1}^{M}|0\rangle_j + \otimes_{j=1}^{M}|1\rangle_j]$ as a metrological resource [8,55,56]. For a given L qubits, we create GHZ states with a size of M qubits, and the number of the GHZ states is L/M. In realistic situations, there will be errors caused during the QT operations, and so we consider an imperfect QT. If we teleport a state ρ_1 from $j = 1, 2, \ldots, M$ sites to $j' = 1 + M, 2 + M, \ldots, 2M$ sites, we obtain a state of $\rho_2' = (1-p)^M \rho_2 + [1-(1-p)^M]\rho_2^{(\mathrm{error})}$, where p is the error rate on a single qubit, ρ_2 is the ideal state (that we could obtain by a perfect QT), and $\rho_2^{(\mathrm{error})} = \frac{1}{2}(\otimes_{j=1+M}^{2M}|0\rangle_j\langle 0| + \otimes_{j=1+M}^{2M}|1\rangle_j\langle 1|)$ is the dephased state.

Quantum metrology with QT.—Here, we focus on using the QT scheme to enable quantum metrology with an uncertainty scaling as $1/T$. Consider the situation in which the qubit frequency ω is shifted depending on the amplitude of the target fields, and so the measurement of the qubit's frequency shift allows us to infer the amplitude of the target field. Such a qubit frequency shift is estimated from the relative phase between quantum states. The key idea is to use the QT in a ring arrangement with $2L$ qubits where each qubit has a tunable interaction with another qubit. Half of the qubits are used to probe the target fields, while the remaining qubits are used as an ancilla for QT. The QT is accomplished by implementing a control-phase gate between a probe qubit and an ancilla qubit, followed by a $\hat{\sigma}_x$ measurement on the probe qubit (and single qubit corrections depending on the measurement result). This QT approach has been widely used in one-way quantum computation [30,57].

Scheme with entanglement.—Our scheme for measuring the amplitude of the target fields is as follows: First, we prepare GHZ states of $\otimes_{k=0}^{(L/M)-1}|\psi_k^{(\mathrm{GHZ})}\rangle$ between the probe qubits where $|\psi_k^{(\mathrm{GHZ})}\rangle = (1/\sqrt{2})[\otimes_{j=1+2kM}^{M+2kM}|0\rangle_j + \otimes_{j=1+2kM}^{M+2kM}|1\rangle_j]$ for $k = 0, 1, \ldots, [(L/M)-1]$, while the other qubits (which we call ancillary qubits) are prepared in $|0\rangle$. Second, we let the state evolve for time $\tau = t/n$ and then teleport the state of the probe qubit at the site j to another site $j + M$. We assume that our gate operations are much faster than τ. Third, we repeat the second step $(n-1)$ times, while in the fourth step we let this state evolve for time $\tau = (t/n)$ and read out the states. Finally, we repeat these steps N times during time T, where $N \simeq T/t$ is the repetition number.

We derive the sensitivity using imperfect QT and entanglement with general conditions and subsequently discuss special cases. By letting the GHZ states $|\psi_k^{(\mathrm{GHZ})}\rangle$ evolve with low-frequency dephasing for time τ, we have

$$\rho_k(\tau) = \frac{1}{2}\Big(\bigotimes_{j=1+2kM}^{M+2kM}|0\rangle_j\langle 0| + \otimes_{j=1+2kM}^{M+2kM}|1\rangle_j\langle 0|e^{-iM\omega\tau - M\gamma^2 t^2} + \bigotimes_{j=1+2kM}^{M+2kM}|0\rangle_j\langle 1|e^{iM\omega\tau - M\gamma^2\tau^2} + \otimes_{j=1+2kM}^{M+2kM}|1\rangle_j\langle 1|\Big)$$

for $k = 0, 1, \ldots, [(L/M)-1]$, where γ denotes the dephasing rate for a single qubit. If we use the QT many times, we can suppress the low-frequency dephasing by employing the mechanism that we described before. To read out the GHZ states, we measure a projection operator defined by $\hat{\mathcal{P}}_{\pm}^{(k)} = |\psi_{\perp}^{(\pm)}\rangle_k\langle\psi_{\perp}^{(\pm)}|$, where $|\psi_{\perp}^{(\pm)}\rangle_k = (1/\sqrt{2})\otimes_{j=1+2kM}^{M+2kM}|0\rangle_j \pm i(1/\sqrt{2})\otimes_{j=1+2kM}^{M+2kM}|1\rangle_j$. We can then estimate the sensitivity in this situation as

$$\delta\omega_{n,t,M}^{(\mathrm{GHZ})} = \frac{\sqrt{\langle\delta\hat{\mathcal{P}}_{\pm}\delta\hat{\mathcal{P}}_{\pm}\rangle}}{|d\langle\hat{\mathcal{P}}_{\pm}\rangle/d\omega|\sqrt{N}} = \frac{e^{M\gamma^2 t^2/n}}{(1-p)^{M(n-1)}\sqrt{MLTt}}, \qquad (1)$$

where $\delta\hat{\mathcal{P}} = \hat{\mathcal{P}} - \langle\hat{\mathcal{P}}\rangle$ and $N \simeq TL/tM$. From this general formula, we can derive many special cases by substituting parameters, which we will describe below. Also, a summary of these results is shown in Table I. By setting $n = 1$, we can reproduce the results discussed in Refs. [58–61] for an entanglement-based sensor with low-frequency dephasing.

For the perfect QT ($p = 0$), we achieve the Heisenberg limit $\delta\omega_{n,t_{\mathrm{opt}},M}^{(\mathrm{GHZ})} = e^{1/4}/\sqrt{c}LT$ when we set $t = T$, $M = cL$, and $n = 4M\gamma^2T^2$, where c denotes a constant number.

TABLE I. Performance of our teleportation-based scheme with L qubits for a given time T. Except with the general form, we show optimized sensitivity by choosing a suitable interaction time (t) and the QT number (n). The uncertainty of the standard Ramsey scheme is given as $\delta\omega_R = e^{1/4}\sqrt{\gamma}/\sqrt{TL}$. With imperfect quantum teleportation that has an error rate of p, we can achieve a sensitivity scaling as $1/T$ using separable states (entangled states with a size M) for a short time scale of $T \ll 1/\sqrt{p}\gamma$ ($T \ll 1/\sqrt{p}\gamma M$). For a longer time scale, if accurate quantum teleportation is available ($p \ll 1$), the sensitivity of our scheme can be still better than the sensitivity of the standard Ramsey scheme. It is worth mentioning that, for the general form, perfect QT, and short T imperfect QT, the sensitivity of the separable sensor can be simply obtained by setting $M = 1$ in that of the entangled sensor.

	General form of sensitivity	Perfect QT	Short T imperfect QT	Long T imperfect QT
Entangled sensor	$\delta\omega \simeq \{[\mathrm{Exp}(M\gamma^2t^2/n)]/[(1-p)^{M(n-1)}\sqrt{MLTt}]\}$	$\delta\omega \simeq [\mathrm{Exp}(1/4)/T\sqrt{ML}]$	$\delta\omega \simeq [\mathrm{Exp}(1/4)/T\sqrt{ML}]$ for $T \ll (1/\sqrt{p}\gamma M)$	$\delta\omega \simeq 2^{3/4}\sqrt{(e\sqrt{p}\gamma/TL)}$ for $T \gg (1/\sqrt{p}\gamma M)$
Separable sensor	$\delta\omega \simeq \{[\mathrm{Exp}(\gamma^2t^2/n)]/[(1-p)^{(n-1)}\sqrt{LTt}]\}$	$\delta\omega \simeq [\mathrm{Exp}(1/4)/T\sqrt{L}]$	$\delta\omega \simeq [\mathrm{Exp}(1/4)/T\sqrt{L}]$ for $T \ll (1/\sqrt{p}\gamma)$	$\delta\omega \simeq 2\sqrt{(e\sqrt{p}\gamma/TL)}$ for $T \gg (1/\sqrt{p}\gamma)$

However, since the entangled state can be teleported to the original site where the entangled state previously interacted with the environment, a correlated error may be induced due to the environmental memory effect. This could happen for $n \geq \tilde{n}_{\rm en}$, where $\tilde{n}_{\rm en}$ denotes the maximum teleportation number of the teleportation without the entangled state being teleported back to the original site. In this case, we have $\tilde{n}_{\rm en} = (2L/M) - 1$. The typical environment has a finite correlation time τ_c. Unless the condition $\tilde{n}_{\rm en}\tau \gg \tau_c \Leftrightarrow c^2L\gamma^2T\tau_c \ll 1$ is satisfied, the error could be correlated [41]. Also, to observe the quadratic decay, we need a condition of $\tau_c \gg t/n$. This means that the correlation time should satisfy these two conflicting conditions. So, although we observe the Heisenberg limit scaling for a small L, the correlated error would begin to hinder the Heisenberg limit as we increase the size of the entangled state.

A natural question is what happens if our QT is imperfect, and so we consider that here. For short times $T \ll 1/\sqrt{p}\gamma M$, the error due to the QT is negligible, and so we obtain the same results as in the perfect QT case by setting $t = T$ and $n = 4M\gamma^2T^2$. In quantum metrology, another interesting regime that is quite often considered is the scaling law in the limit of long T (much greater than the coherence time of the system). We consider this here. We can minimize the uncertainty with $t_{\rm opt}^{(\rm en)} = (\sqrt{n/M}/2\gamma)$ to obtain $\delta\omega_{n,t_{\rm opt}}^{(\rm GHZ)} = (\sqrt{2}e^{1/4}\sqrt{\gamma/\sqrt{Mn}}/(1-p)^{M(n-1)}\sqrt{LT})$ for $p > 0$ and $n > 1$. Furthermore, with $M_{\rm opt} = -1/4\log(1-p) \simeq 1/4p$ and $n_{\rm opt}^{(\rm en)} = 2$, the uncertainty can be minimized as $\delta\omega_{\rm opt}^{(\rm GHZ)} = 2^{3/4}\sqrt{(e\sqrt{p}\gamma/LT)}$. In this case, the condition for the independent error ($\tilde{n}_{\rm en}\tau \geq \tau_c$) is written as $Lp^{3/2}/\gamma \gg \tau_c$ and is satisfied for a large L.

Scheme with separable states.—Now we explore a possibly more practical scheme with separable states, as shown in Fig. 1. We begin by preparing a probe state of $\otimes_{j=1}^{L} |+\rangle_{2j-1}$ located at the site $2j-1$ $(j = 1, 2, \ldots, L)$. Then we let the state evolve for a time $\tau = t/n$ and teleport the state of the probe qubit to the next site using the ancillary qubit. We repeat this step $(n-1)$ times before we finish by allowing our state to evolve for time $\tau = (t/n)$ and reading out the state by measuring $\hat{M}_y = \sum_{j=1}^{L} \hat{\sigma}_y^{(j)}$. We repeat these steps N times during the measurement time T, where $N \simeq T/t$ is the repetition number. We can calculate the sensitivity for this scheme by substituting $M = 1$ in Eq. (1). For an ideal QT, by setting $t = T$ and $p = 0$, we obtain $\delta\omega_{n,T} \simeq e^{1/4}/T\sqrt{L}$ for $n = 4\gamma^2T^2$, and so we can achieve $1/T$ scaling. For $n \geq \tilde{n}$, a correlated error may be induced due to the memory effect where $\tilde{n} = 2L - 1$ denotes the maximum number of teleportations without the qubit state being teleported back to the original site. Fortunately, since the typical environment has a finite correlation time τ_c, such a correlation effect becomes negligible for a large number of qubits to satisfy $\tilde{n}\tau \gg \tau_c \Leftrightarrow L \gg \gamma^2T\tau_c$ [41].

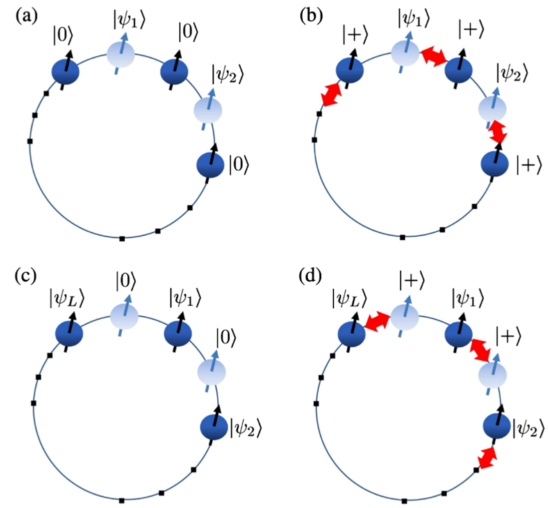

FIG. 1. Schematic illustration of $2L$ qubits in a ring structure to measure globally applied fields with an uncertainty scaling as $1/T$ when we use separable states. (a) Half of the qubits contain information about the target fields as a probe, while the remaining half are used as ancillary qubits for the qubit teleportation. (b) With a controlled phase gate and measurement feedforward operations, we can teleport a quantum state from the original site to the right neighboring site [30,57].(c) After the teleportation, the measured qubit becomes the new ancilla which we initialize into $|0\rangle$. (d) We repeat these steps described in (b) and (c).

We now analyze how imperfect QT affects the performance of our sensing scheme. We can calculate the sensitivity by substituting $M = 1$ and $p > 0$ with Eq. (1). For a short time such as $T \ll 1/\sqrt{p}\gamma$, the error due to QT is negligible, and so we obtain the same results as with perfect QT by setting $t = T$ and $n = 4\gamma^2T^2$, which allows us to achieve uncertainty scaling as $1/T$. We can minimize the uncertainty by setting $t_{\rm opt} = \sqrt{n}/2\gamma$ as long as $T \gg t_{\rm opt}$ is satisfied. In such a case, $\delta\omega_{n,t_{\rm opt}} = [\sqrt{2}e^{1/4}/(1-p)^{n-1}]\sqrt{(\gamma/\sqrt{n}TL)}$, which for $n = 1$ gives the standard Ramsey uncertainty $\delta\omega_R = (e^{1/4}\sqrt{\gamma}/\sqrt{TL})$ [8], where we replace L with $2L$ (because the standard Ramsey scheme can utilize every qubit to probe the target fields without ancillary qubits). For $n \gg 1$. we can treat n as a continuous variable, and we can analytically minimize the uncertainty as $\delta\omega_{\rm opt} \simeq 2\sqrt{(e\sqrt{p}\gamma/LT)}$ for $1/16\gamma^2T^2 \ll p \ll 1$, where we choose $n_{\rm opt} = -1/4\log(1-p) \simeq 1/4p$. The condition required for the error to be independent ($\tilde{n}\tau \geq \tau_c$) is written as $L\sqrt{p}/\gamma \gg \tau_c$ and is satisfied for a large L. In this case, we have a constant factor improvement over the standard Ramsey scheme for a longer T. In fact, as long as $p < 0.0251$, our scheme is better than that standard Ramsey scheme ($\delta\omega_R/\delta\omega_{\rm opt} > 1$). For $p = 10^{-4}$, we obtain

$\delta\omega_R/\delta\omega_{\rm opt} \simeq 3.89$. So our sensor has an advantage with finite errors caused by the imperfect QT.

In conclusion, we have proposed a scheme designed to achieve sensitivity beyond the classical limit and to measure the amplitudes of globally applied fields. We have found that frequent implementations of quantum teleportation provide a suitable circumstance for sensing where the dephasing is suppressed while the information from the target fields is continuously accumulated. If perfect quantum teleportation is available, the uncertainty scales as $1/T$ with our scheme, while any classical sensor shows the uncertainty scaled as $1/\sqrt{T}$. Moreover, even when quantum teleportation is moderately noisy, our protocol still realizes superior quantum enhancement to the standard Ramsey scheme.

This work was supported by JSPS KAKENHI Grant No. 15K17732 and partly supported by MEXT KAKENHI Grant No. 15H05870. S. B. acknowledges support from the EPSRC NQIT Hub, Project No. EP/M013243/1.

*matsuzaki.yuichiro@lab.ntt.co.jp

[1] D. Budker and M. Romalis, Nat. Phys. **3**, 227 (2007).
[2] G. Balasubramanian *et al.*, Nature (London) **455**, 648 (2008).
[3] J. Maze *et al.*, Nature (London) **455**, 644 (2008).
[4] C. L. Degen, F. Reinhard, and P. Cappellaro, Rev. Mod. Phys. **89**, 035002 (2017).
[5] J. Simon, Adv. Phys. **48**, 449 (1999).
[6] A. Chang, H. Hallen, L. Harriott, H. Hess, H. Kao, J. Kwo, R. Miller, R. Wolfe, J. Van der Ziel, and T. Chang, Appl. Phys. Lett. **61**, 1974 (1992).
[7] M. Poggio and C. Degen, Nanotechnology **21**, 342001 (2010).
[8] S. F. Huelga, C. Macchiavello, T. Pellizzari, A. K. Ekert, M. B. Plenio, and J. I. Cirac, Phys. Rev. Lett. **79**, 3865 (1997).
[9] R. S. Said, D. W. Berry, and J. Twamley, Phys. Rev. B **83**, 125410 (2011).
[10] B. L. Higgins, D. W. Berry, S. D. Bartlett, H. M. Wiseman, and G. J. Pryde, Nature (London) **450**, 393 (2007).
[11] G. Waldherr, J. Beck, P. Neumann, R. Said, M. Nitsche, M. Markham, D. Twitchen, J. Twamley, F. Jelezko, and J. Wrachtrup, Nat. Nanotechnol. **7**, 105 (2012).
[12] R. Demkowicz-Dobrzanski, J. Kolodynski, and M. Guta, Nat. Commun. **3**, 1063 (2012).
[13] D. Gottesman, Quantum information science and its contributions to mathematics, in *Proceedings of Symposia in Applied Mathematics* (American Mathematical Society, Washington, DC, 2009).
[14] L. Viola, E. Knill, and S. Lloyd, Phys. Rev. Lett. **82**, 2417 (1999).
[15] J. Taylor, P. Cappellaro, L. Childress, L. Jiang, D. Budker, P. Hemmer, A. Yacoby, R. Walsworth, and M. Lukin, Nat. Phys. **4**, 810 (2008).
[16] G. de Lange, D. Ristè, V. V. Dobrovitski, and R. Hanson, Phys. Rev. Lett. **106**, 080802 (2011).
[17] W. Dür, M. Skotiniotis, F. Frowis, and B. Kraus, Phys. Rev. Lett. **112**, 080801 (2014).
[18] D. A. Herrera-Martí, T. Gefen, D. Aharonov, N. Katz, and A. Retzker, Phys. Rev. Lett. **115**, 200501 (2015).
[19] G. Arrad, Y. Vinkler, D. Aharonov, and A. Retzker, Phys. Rev. Lett. **112**, 150801 (2014).
[20] E. M. Kessler, I. Lovchinsky, A. O. Sushkov, and M. D. Lukin, Phys. Rev. Lett. **112**, 150802 (2014).
[21] L. Cohen, Y. Pilnyak, D. Istrati, A. Retzker, and H. S. Eisenberg, Phys. Rev. A **94**, 012324 (2016).
[22] T. Unden *et al.*, Phys. Rev. Lett. **116**, 230502 (2016).
[23] S. Zhou, M. Zhang, J. Preskill, and L. Jiang, Nat. Commun. **9**, 78 (2018).
[24] S. Schmitt *et al.*, Science **356**, 832 (2017).
[25] J. Boss, K. Cujia, J. Zopes, and C. Degen, Science **356**, 837 (2017).
[26] B. Misra and E. G. Sudarshan, J. Math. Phys. (N.Y.) **18**, 756 (1977).
[27] W. M. Itano, D. J. Heinzen, J. J. Bollinger, and D. J. Wineland, Phys. Rev. A **41**, 2295 (1990).
[28] P. Facchi, H. Nakazato, and S. Pascazio, Phys. Rev. Lett. **86**, 2699 (2001).
[29] H. Nakazato, M. Namiki, and S. Pascazio, Int. J. Mod. Phys. B **10**, 247 (1996).
[30] R. Raussendorf and H. J. Briegel, Phys. Rev. Lett. **86**, 5188 (2001).
[31] J. P. Barjaktarevic, R. H. McKenzie, J. Links, and G. J. Milburn, Phys. Rev. Lett. **95**, 230501 (2005).
[32] M. Silva, V. Danos, E. Kashefi, and H. Ollivier, New J. Phys. **9**, 192 (2007).
[33] S. Olmschenk, D. Matsukevich, P. Maunz, D. Hayes, L.-M. Duan, and C. Monroe, Science **323**, 486 (2009).
[34] M. Baur, A. Fedorov, L. Steffen, S. Filipp, M. P. da Silva, and A. Wallraff, Phys. Rev. Lett. **108**, 040502 (2012).
[35] D. V. Averin, K. Xu, Y. P. Zhong, C. Song, H. Wang, and S. Han, Phys. Rev. Lett. **116**, 010501 (2016).
[36] K. Hornberger, in *Entanglement and Decoherence* (Springer, New York, 2009), pp. 221–276.
[37] G. De Lange, Z. Wang, D. Riste, V. Dobrovitski, and R. Hanson, Science **330**, 60 (2010).
[38] F. Yoshihara, K. Harrabi, A. O. Niskanen, Y. Nakamura, and J. S. Tsai, Phys. Rev. Lett. **97**, 167001 (2006).
[39] K. Kakuyanagi, T. Meno, S. Saito, H. Nakano, K. Semba, H. Takayanagi, F. Deppe, and A. Shnirman, Phys. Rev. Lett. **98**, 047004 (2007).
[40] Y. Kondo, Y. Matsuzaki, K. Matsushima, and J. G. Filgueiras, New J. Phys. **18**, 013033 (2016).
[41] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.120.140501 for the details of the calculations, which includes Refs. [36,42–53].
[42] H. Hasegawa, Phys. Rev. E **83**, 021104 (2011).
[43] A. Carcaterra and A. Akay, Phys. Rev. E **84**, 011121 (2011).
[44] Y. Romach *et al.*, Phys. Rev. Lett. **114**, 017601 (2015).
[45] F. Wilhelm, M. Storcz, U. Hartmann, and M. R. Geller, in *Manipulating Quantum Coherence in Solid State Systems* (Springer, New York, 2007), pp. 195–232.
[46] L. T. Hall, J. H. Cole, and L. C. L. Hollenberg, Phys. Rev. B **90**, 075201 (2014).

[47] F. H. L. Koppens, D. Klauser, W. A. Coish, K. C. Nowack, L. P. Kouwenhoven, D. Loss, and L. M. K. Vandersypen, Phys. Rev. Lett. **99**, 106803 (2007).
[48] C. W. Gardiner and P. Zoller, *Quantum Noise* (Springer, Berlin, 2004).
[49] R. Barends *et al.*, Nature (London) **508**, 500 (2014).
[50] C. J. Ballance, T. P. Harty, N. M. Linke, M. A. Sepiol, and D. M. Lucas, Phys. Rev. Lett. **117**, 060504 (2016).
[51] R. Blume-Kohout, J. K. Gamble, E. Nielsen, K. Rudinger, J. Mizrahi, K. Fortier, and P. Maunz, Nat. Commun. **8**, 14485 (2017).
[52] C. Neill *et al.*, arXiv:1709.06678.
[53] G. Popkin, Science **354**, 1090 (2016).
[54] G. M. Palma, K. A. Suominen, and A. K. Ekert, Proc. R. Soc. A **452**, 567 (1996).
[55] A. Shaji and C. M. Caves, Phys. Rev. A **76**, 032111 (2007).
[56] M. Bohmann, J. Sperling, and W. Vogel, Phys. Rev. A **91**, 042332 (2015).
[57] D. E. Browne and H. J. Briegel, arXiv:quant-ph/0603226.
[58] M. Schaffry, E. M. Gauger, J. J. L. Morton, J. Fitzsimons, S. C. Benjamin, and B. W. Lovett, Phys. Rev. A **82**, 042114 (2010).
[59] J. A. Jones, S. D. Karlen, J. Fitzsimons, A. Ardavan, S. C. Benjamin, G. A. D. Briggs, and J. J. Morton, Science **324**, 1166 (2009).
[60] Y. Matsuzaki, S. C. Benjamin, and J. Fitzsimons, Phys. Rev. A **84**, 012103 (2011).
[61] A. W. Chin, S. F. Huelga, and M. B. Plenio, Phys. Rev. Lett. **109**, 233601 (2012).