Nonlinear Decay and Plasma Heating by a Toroidal Alfvén Eigenmode

Z. Qiu,[1,*] L. Chen,[1,2] F. Zonca,[3,1] and W. Chen[4]

[1] *Institute for Fusion Theory and Simulation and Department of Physics, Zhejiang University, Hangzhou, People's Republic of China*

[2] *Department of Physics and Astronomy, University of California, Irvine California 92697-4575, USA*

[3] *ENEA, Fusion and Nuclear Safety Department, C.R. Frascati, Via E. Fermi 45, 00044 Frascati (Roma), Italy*

[4] *Southwestern Institute of Physics, P.O. Box 432 Chengdu 610041, People's Republic of China*

(Received 23 November 2017; published 27 March 2018)

We demonstrate theoretically that a toroidal Alfvén eigenmode (TAE) can parametrically decay into a geodesic acoustic mode and kinetic TAE in a toroidal plasma. The corresponding threshold condition for the TAE amplitude is estimated to be $|\delta B_\perp/B_0| \sim O(10^{-4})$. Here, $\delta B_\perp$ and B_0 are, respectively, the perturbed magnetic field of the pump TAE and the equilibrium magnetic field. This novel decay process, in addition to contributing to the nonlinear saturation of energetic-particle or α-particle driven TAE instability, could also contribute to the heating as well as regulating the transports of thermal plasmas.

DOI: 10.1103/PhysRevLett.120.135001

Introduction.—This work is a theoretical analysis of a novel nonlinear process by which power is transferred nonlocally in velocity space from energetic particles (EPs) to thermal plasma components. This mechanism is mediated by an efficient parametric decay of toroidal Alfvén eigenmodes (TAEs) [1] into geodesic acoustic modes (GAMs) [2] and kinetic TAE (KTAE) [3,4], which control power transfer to thermal ions (α channeling) and thermal electrons (anomalous α-particle slowing-down), respectively. The first part of this Letter presents the nonlinear decay instability and gives an estimate of the threshold condition on the TAE intensity for this novel process to occur. It also computes the resultant anomalous thermal ion and electron heating rates. As the critical TAE intensity threshold is comparable with that of other important nonlinear processes in burning fusion plasmas, we conclude that the new mechanism analyzed here should play an important role in realistic conditions. It could also regulate thermal plasma fluctuation and, thus, transport via the nonlinear generation of GAMs [5]. In the second part of this Letter, we discuss how this novel nonlinear process can be studied by nonlinear gyrokinetic simulations as well as how it can be observed and characterized experimentally.

Energetic particles and their related physics are crucial for burning plasmas in magnetically confined fusion devices as EPs contribute significantly to the total power density. In particular, two important aspects are heating of thermal plasmas and excitation of symmetry breaking collective modes. Plasma heating, especially of bulk ions, is crucial for fusion reactivity. Coulomb collisions preferentially transfer EP energy to electrons at high speed and, thus, means for effectively transferring energy from fusion α's to bulk ions, known as α channeling, have been proposed and explored [6]. Symmetry breaking collective modes excited by EPs, on the other hand, could have deleterious effects on EPs and thermal plasma confinement. Among these, noteworthy are shear Alfvén wave (SAW) instabilities with group velocity nearly aligned with the equilibrium magnetic field and wave-particle resonance condition with 3.5 MeV fusion α's easily satisfied in burning plasmas such as ITER [7]. EP and Alfvén wave physics in fusion plasmas are reviewed in Ref. [8].

Because of equilibrium geometry and/or plasma nonuniformities, SAW instabilities manifest themselves as various Alfvén eigenmodes in magnetically confined plasmas, e.g., TAE [1]. TAE can be driven unstable by EPs at a relatively low threshold [9,10] and lead to EP transport and degrade overall plasma confinement. The transport rate is related to TAE amplitude and spectrum, and, thus, understanding the nonlinear dynamics of TAE is important for assessing the properties of burning plasmas in future reactors.

Nonlinear mode-mode coupling, as one of the two routes for TAE nonlinear dynamics [11], is relatively less investigated [12–15] than nonlinear wave-EP interactions [16,17]. TAE enhanced coupling to SAW continuum due to downward spectrum cascading via ion induced scattering in the low-β ($\beta \ll \epsilon^2$) and the long wavelength ($k_\perp^2\rho_i^2 < \omega/\Omega_{ci}$) limit is analyzed in Ref. [12]. Here, $\beta \equiv 8\pi n_0 T/B_0^2$ is the ratio of plasma pressure to equilibrium magnetic field pressure with n_0 and T being the equilibrium plasma density and temperature, respectively. B_0 is the equilibrium magnetic field, and $\epsilon \equiv r/R_0$ is the inverse aspect ratio with r and R_0 being the tokamak minor and major radii. $k_\theta \equiv m/r$ is the poloidal wave number with m being the poloidal mode number, and $\rho_i = v_i/\Omega_{ci}$ is the ion gyroradius with v_i being the ion thermal velocity and Ω_{ci} the ion cyclotron frequency. Nonlinear modification of the TAE gap structure by nonlinear distortions of equilibrium magnetic field or density is discussed in Refs. [13,14], respectively. In the former one, in particular, the emphasis is on the compressibility of the $m = 1$ quasimode instead of its heavy ion Landau damping as discussed in Ref. [12]. The nonlinear generation of axisymmetric zero frequency zonal structures (ZFZS) via modulational instability, including both zonal flow and zonal current, is investigated in Refs. [15,18]. Parametric decay of TAE into GAM [2], i.e., the fast varying zonal flow in the acoustic frequency range, and a lower frequency TAE, is investigated in Ref. [19], and it is found that TAE spontaneous decay occurs only when the pump TAE is localized in the upper half of the SAW continuum gap, which generally is not the case for typical tokamak parameters.

In this work, a new novel mechanism of TAE decaying into GAM and small scale lower kinetic TAE (LKTAE) [3,4,20] is proposed. Besides the apparent consequence on TAE nonlinear saturation, the nonlinear process proposed here also has important implications on both thermal plasma heating and confinement. The nonlinearly generated GAM and LKTAE are damped via ion and electron Landau damping, respectively, leading to ion as well as electron heating. On the other hand, GAM as finite frequency zonal flow may interact with other types of turbulence, e.g., drift waves (DWs), leading to cross-scale couplings and potentially improved confinement [5]. Noting that GAM corresponds to finite frequency convective cells in toroidal geometry, while kinetic TAEs can be viewed as standing wave generated due to the coupling of two counterpropagating kinetic Alfvén waves (KAWs) via toroidicity [3], the current work can, thus, be understood qualitatively, as generalization to toroidal geometries of Ref. [21], where finite frequency convective cell generation by KAWs in uniform plasma is discussed.

Parametric decay of TAE.—To investigate the nonlinear interactions among the pump TAE (ω_0, $\mathbf{k}_0$), GAM (ω_G, $\mathbf{k}_G$), and LKTAE (ω_L, $\mathbf{k}_L$), the scalar potential $\delta\phi$ and parallel vector potential $\delta A_\parallel$ are adopted as the field variables. One then has $\delta\phi = \delta\phi_0 + \delta\phi_G + \delta\phi_L$, with the subscripts 0, G, and L denoting pump TAE, GAM, and LKTAE, respectively. The parametric decay of TAE to GAM and LKTAE is then studied within the framework of nonlinear gyrokinetic theory. For TAE and LKTAE with high toroidal mode numbers in magnetized plasmas, the well-known ballooning-mode decomposition [22] in the (r, θ, ϕ) field-aligned flux coordinates is assumed:

$$\delta\phi_0 = A_0 e^{i(n\phi - m_0\theta - \omega_0 t)} \sum_j e^{-ij\theta}\Phi_0(x-j) + \text{c.c.},$$

$$\delta\phi_L = A_L e^{i(n\phi - m_0\theta - \omega_0 t)} e^{-i(\int \hat{k}_G dr - \omega_G t)} \times \sum_j e^{-ij\theta}\Phi_L(x-j) + \text{c.c.}$$

Here, $(m = m_0 + j, n)$ are the poloidal and toroidal mode numbers, m_0 is the reference value of m, $nq(r_0) = m_0$, $q(r)$ is the safety factor, $x = nq - m_0 = nq'(r - r_0)$, $\hat{k}_G$ is the radial envelope wave number due to GAM modulation and $\hat{k}_G \equiv nq'\theta_k$ in the ballooning representation, Φ is the fine radial structure associated with the parallel wave number $k_\parallel$ and magnetic shear, and A is the envelope amplitude. The other notations are standard.

For the (secondary) generated GAM we assume it is predominantly electrostatic, with both the usual mesoscale structure and an additional fine-scale radial structure [18] due to the radially localized structure of the pump TAE, thus,

$$\delta\phi_G = A_G e^{i(\int \hat{k}_G dr - \omega_G t)} \sum_j \Phi_G(x-j) + \text{c.c.}$$

Here, Φ_G is the fine-scale structure of GAM [18], and the summation over j is the summation over the radial positions where the pump TAE poloidal harmonics are localized. As a result, $\mathbf{k}_G = \hat{\mathbf{k}}_G - i\partial_r \ln \Phi_G \hat{\mathbf{e}}_r$, and one typically has $|\partial_r \ln \Phi_G| \gg |\hat{k}_G|$.

The nonlinear GAM equation can be determined from the nonlinear gyrokinetic vorticity equation, and one obtains

$$\mathcal{E}_{G^*}\delta\phi_{G^*} = i(c/B_0\omega_G)k_G k_{\theta,0} \times [\Gamma_0 - \Gamma_L - (\hat{b}_L - \hat{b}_0)k_{\parallel,0}^2 V_A^2 \sigma_{0^*}\sigma_L/(\omega_0\omega_L)]\delta\phi_{0^*}\delta\phi_L. \quad (1)$$

The two terms on the right-hand side of Eq. (1) are, respectively, the generalized Reynolds and Maxwell stresses, valid for arbitrary $k_\perp\rho_i$. Here, $\Gamma_k \equiv \langle J_k^2 F_0/n_0\rangle$ with $\langle\cdots\rangle \equiv \int(\cdots)d^3\mathbf{v}$ denoting velocity space integration, $J_k \equiv J_0(k_\perp\rho)$ with J_0 being the Bessel function of zero index, $\rho = v_\perp/\Omega$, F_0 is the equilibrium particle distribution function, $k_\parallel \equiv (nq - m)/(qR_0)$ is the parallel wave number, $\hat{b} = k_\perp^2\rho_i^2/2$, $\sigma_k \equiv 1 + \tau - \tau\Gamma_k$, $\tau \equiv T_e/T_i$, and $\sigma_k \neq 1$ denotes finite parallel electric field $\delta E_\parallel$ and, thus, deviation from the ideal magnetohydrodynamic (MHD) condition $\delta E_\parallel = 0$ due to kinetic effects. Furthermore, $\mathcal{E}_G$ is the linear GAM dispersion function, defined as [23]

$$\mathcal{E}_G \equiv \langle(1 - J_G^2)F_0/n_0\rangle - T_i \sum_s \overline{\langle q_s J_G \omega_d \delta H_G^L\rangle}/(n_0 e^2 \omega \overline{\delta\phi}_G),$$

with δH_k being the nonadiabatic component of the guiding center distribution function [24] and $\omega_d = (v_\perp^2 + 2v_\parallel^2)/(2\Omega R_0)(k_r \sin\theta + k_\theta \cos\theta)$ being the magnetic drift frequency.

Equation (1) has two radial scales due to the weak ballooning nature of TAE [18]. Assuming $\Phi_{G^*} \equiv \Phi_{0^*}\Phi_L$ as the fast radial varying component [18] of GAM, one then derives the envelope equation of GAM:

$$\mathcal{E}_{G^*}A_{G^*} = i(c/B_0\omega_G)k_{\theta,0}\hat{\alpha}_G A_{0^*}A_L. \quad (2)$$

Here, introducing radial integration as averaging over length scales intermediate between the fine radial scale and envelope mesoscale discussed above, we have $\hat{\alpha}_G \equiv (\int \Phi_{0^*}\Phi_L dr)^{-1} \int \Phi_{0^*}\Phi_L k_G[\Gamma_0 - \Gamma_L - (\hat{b}_L - \hat{b}_0)k_\parallel^2 V_A^2 \sigma_{0^*}\sigma_L/(\omega_0\omega_L)]dr$.

The nonlinear LKTAE generation due to the coupling between the pump TAE and GAM is described by

$$\mathcal{E}_L\delta\phi_L = i\frac{c}{B}k_G k_{\theta,0}\left(\frac{\Gamma_0 - \Gamma_G}{\omega_L} + \frac{1-\Gamma_L}{\sigma_L\omega_0}\sigma_0\right)\delta\phi_{G^*}\delta\phi_0. \quad (3)$$

Here, $\mathcal{E}_L \equiv (1-\Gamma_L) - k_\parallel^2 V_A^2 \sigma_L \hat{b}_L/\omega_L^2$ is the WKB dispersion function of LKTAE, and the radial eigenmode dispersion relation of LKTAE can be derived noting that $k_\parallel^2 V_A^2 \propto (1 - \epsilon_0 \cos\theta)$ with $\epsilon_0 \equiv 2(r/R_0 + \Delta')$, Δ' representing finite Shafranov shift and $\sigma_L = 1 + \tau - \tau\Gamma_L \neq 1$ due to ion finite Larmor radii effects. Noting that $\omega_L = \omega_0 - \omega_G$, the nonlinear coupling coefficient of Eq. (3) recovers that of Eq. (10) of Ref. [21] for KAW lower sideband generation by pump KAW beating with finite frequency convective cell, when only the electrostatic convective cell generation is considered.

Noting that $\Phi_G = \Phi_0\Phi_{L^*}$, and proceeding as for Eq. (2) to remove fine-scale fast radial variations, the eigenmode equation of LKTAE can be derived as

$$\hat{\mathcal{E}}_L A_L = i(c/B_0)k_{\theta,0}\hat{\alpha}_L A_{G^*}A_0, \quad (4)$$

with $\hat{\mathcal{E}}_L \equiv \int dr|\Phi_L|^2\mathcal{E}_L$ and $\hat{\alpha}_L \equiv \int dr|\Phi_0|^2|\Phi_L|^2 k_G [(\Gamma_0 - \Gamma_G)/\omega_L + (1-\Gamma_L)\sigma_0/(\sigma_L\omega_0)]$. For LKTAE with even mode structure, the eigenmode dispersion relation can be written as [4,20] $\hat{\mathcal{E}}_L \equiv (\pi k_\theta^2\rho_i^2\omega_A^2\hat{D}_L)/[2^{2\hat{\xi}+1}\Gamma^2(\hat{\xi}+1/2)\omega_L^2]$, with $\hat{D}_L = -2\sqrt{2}\Gamma(\hat{\xi}+1/2)/[\hat{\alpha}\Gamma(\hat{\xi})] - \delta W_f$, δW_f being the normalized potential energy due to thermal plasma contribution, $\Gamma(\hat{\xi})$ and $\Gamma(\hat{\xi}+1/2)$ being Euler gamma functions, $\hat{\xi} \equiv 1/4 - \Gamma_+\Gamma_-/(4\sqrt{\Gamma_-\hat{s}^2\hat{\rho}_K^2})$, $\Gamma_\pm \equiv \omega_L^2/\omega_A^2 \pm \epsilon_0\omega_L^2/\omega_A^2 - 1/4$, $\omega_A^2 \equiv V_A^2/(q^2R_0^2)$, $\hat{\alpha}^2 = 1/(2\sqrt{\Gamma_-\hat{s}^2\hat{\rho}_K^2})$, $\hat{s} \equiv r\partial_r q/q$ being the magnetic shear, and $\hat{\rho}_K^2 \equiv (k_\theta^2\rho_i^2/2)[3/4 + (T_e/T_i)(1 - i\delta_e)]$ denoting the kinetic effects associated with finite ion Larmor radii and electron parallel dynamics. In particular, δ_e describes dissipative effects associated with electrons, e.g., Landau damping.

The nonlinear dispersion relation can then be derived from Eqs. (2) and (4):

$$\hat{\mathcal{E}}_L\mathcal{E}_{G^*} = -(ck_{\theta,0}/B_0)^2(\hat{\alpha}_G\hat{\alpha}_L/\omega_G)|A_0|^2. \quad (5)$$

In the long wavelength ($k_\perp^2\rho_i^2 \lesssim 1$) limit, Eq. (5) recovers Eq. (17) of Ref. [19], where a pump TAE decaying into a GAM and a TAE lower sideband (still a gap mode and not a damped eigenmode of the discretized continuous spectrum as in the present case) is discussed.

Noting that the frequency difference between neighboring LKTAEs is rather small [4], and that GAM frequency depends on k_G and thus $k_{r,L}$ due to finite Larmor-radius and drift orbit-width effects [23], the impact of frequency mismatch on the parametric decay process is, in general, negligible. This, of course, further requires that local GAM continuum frequency be smaller than $\omega_0 - \omega_L$. Taking $\mathcal{E}_{G^*} = -2i\hat{b}_G(\gamma+\gamma_G)/\omega_G$, with γ_G being the collisionless damping rate of GAM [23], while $\hat{\mathcal{E}}_L \simeq i\partial_{\omega_L}\hat{\mathcal{E}}_{Lr}(\gamma+\gamma_L)$, with γ_L being the radiative damping rate of LKTAE [4,20], $\hat{\mathcal{E}}_{Lr}$ being the real part of $\hat{\mathcal{E}}_L$ and $\partial_{\omega_L}\hat{\mathcal{E}}_{Lr} \equiv \partial\hat{\mathcal{E}}_{Lr}/(\partial\omega_L)$, we then obtain the desired dispersion relation of the parametric decay process:

$$(\gamma+\gamma_G)(\gamma+\gamma_L) = -\left(\frac{c}{B_0}k_{\theta,0}\right)^2\frac{\hat{\alpha}_G\hat{\alpha}_L|A_0|^2}{2\hat{b}_G\partial_{\omega_L}\hat{\mathcal{E}}_{Lr}}. \quad (6)$$

The condition for the spontaneous excitation of the parametric instability is then

$$-\left(\frac{c}{B_0}k_{\theta,0}\right)^2\frac{\hat{\alpha}_G\hat{\alpha}_L|A_0|^2}{2\hat{b}_G\partial_{\omega_L}\hat{\mathcal{E}}_{Lr}} > \gamma_L\gamma_G, \quad (7)$$

i.e., the nonlinear drive by the pump TAE overcomes the threshold condition due to GAM and LKTAE damping.

Equation (7), generally, requires a numerical solution due to its complex dependence on the mode structures and, thus, equilibrium geometry. Analytical estimations can be made in the simplified limits, e.g., for $\hat{b}_L \ll 1$. Noting that for $|\hat{b}_k| \ll 1$, $\Gamma_k(\hat{b}_k) \simeq 1 - \hat{b}_k - 3\hat{b}_k^2/4$ and $\sigma_k \simeq 1 + \tau(\hat{b}_k + 3\hat{b}_k^2/4)$, one then has $\hat{\alpha}_G \simeq k_G(\hat{b}_L - \hat{b}_0)[1 - \omega_A^2/(4\omega_0\omega_L)] < 0$ and $\hat{\alpha}_L \simeq (k_G/\omega_L)\{\hat{b}_G - \hat{b}_0 + \hat{b}_L[(1+\tau\hat{b}_0)/(1+\tau\hat{b}_L)][(\omega_0 - \omega_G)/\omega_0]\} > 0$. So the right-hand side of Eq. (6) has a positive sign. Noting that $|\delta B_{r,0}| \sim |ck_\theta k_{\parallel,0}A_0/\omega_0|$, the threshold condition on pump TAE amplitude can be estimated as

$$\left(\frac{\delta B_r}{B_0}\right)^2 \sim \frac{\gamma_L\gamma_G}{\omega_0^2}\frac{k_{\parallel,0}^2}{k_L^2}\frac{4}{\epsilon_0} \sim O(10^{-9}), \quad (8)$$

with typical parameters such as $|\gamma_L/\omega_0| \sim |\gamma_G/\omega_0| \sim 10^{-2}$ [4,23], $k_L\rho_i \lesssim 1$, and $k_\parallel\rho_i \sim 10^{-3}$. The nonlinear cross section of the analyzed nonlinear decay instability is comparable to other channels for TAE nonlinear saturation via wave-wave coupling investigated in the short wavelength ($k_\perp^2\rho_i^2 > \omega/\Omega_{ci}$) limit [25], e.g., ZFZS generation [15,19].

Impact on plasma heating.—The process discussed here, besides its apparent impact on TAE saturation, also has an effect on plasma heating, since the generated GAM and LKTAE would be dissipated through ion and electron Landau damping, respectively. Thus, the GAM ion Landau damping provides an additional channeling of fusion-α

power density to bulk ion heating [6,26], whereas LKTAE electron Landau damping contributes to anomalous α-particle slowing-down. The heating rate can be estimated by Eqs. (2) and (4) with the help of an additional equation describing the feedback of the two sidebands to the pump TAE, which can be obtained closely following the derivation of Eq. (3):

$$\hat{\mathcal{E}}_0 A_0 = -i(c/B_0)k_{\theta,0}\hat{\alpha}_0 A_G A_L, \tag{9}$$

with $\hat{\mathcal{E}}_0 \equiv \int dr|\Phi_0|^2[(1-\Gamma_0) - k_\parallel^2 V_A^2 \sigma_0 \hat{b}_0/\omega_0^2]$ being the eigenmode dispersion function of pump TAE, and $\hat{\alpha}_0 \equiv \int dr|\Phi_0|^2|\Phi_L|^2 k_G[(\Gamma_L - \Gamma_G)/\omega_0 + (1-\Gamma_0)\sigma_L/(\sigma_0\omega_0)]$. The three-wave nonlinear dynamic equations can then be cast as

$$(\partial_t - \gamma_0)A_0 = -\frac{c}{B_0 \partial_{\omega_0}\hat{\mathcal{E}}_{0r}} k_{\theta,0}\hat{\alpha}_0 A_G A_L, \tag{10}$$

$$(\partial_t + \gamma_{G^*})A_G = -\frac{c}{2B_0 \hat{b}_G} k_{\theta,0}\hat{\alpha}_G A_{0^*} A_L, \tag{11}$$

$$(\partial_t + \gamma_L)A_L = \frac{c}{B_0 \partial_{\omega_L}\hat{\mathcal{E}}_{Lr}} k_{\theta,0}\hat{\alpha}_L A_{G^*} A_0. \tag{12}$$

Here, γ_0 is the linear growth rate of pump TAE due to resonant EP drive. The LKTAE and GAM amplitudes can be estimated from the fixed point solution of the above equations, and one obtains $|A_L|^2 = -2\gamma_0\gamma_G \hat{b}_G \partial_{\omega_0}\hat{\mathcal{E}}_{0r}/[(c/B_0)^2 k_{\theta,0}^2 \hat{\alpha}_0\hat{\alpha}_G]$ and $|A_G|^2 = \gamma_0\gamma_L \partial_{\omega_L}\hat{\mathcal{E}}_{Lr}\partial_{\omega_0}\hat{\mathcal{E}}_{0r}/[(c/B_0)^2 k_{\theta,0}^2 \hat{\alpha}_0\hat{\alpha}_L]$, respectively. Thus, the power of ions heating by GAM Landau damping is then $P_i = 2\gamma_G\omega_G\partial_{\omega_G}\hat{\mathcal{E}}_{Gr}|A_G|^2$ and the electron heating power by LKTAE is $P_e = 2\gamma_L\omega_L\partial_{\omega_L}\hat{\mathcal{E}}_{Lr}|A_L|^2$. Note that, "GAM channeling" was proposed in Ref. [26], where the ion Landau damping of the GAM resonantly excited by EPs was investigated. However, due to the low GAM frequency compared to the high characteristic frequencies of fusion α's, this process is, in general, inefficient in burning plasmas.

Numerical or experimental verification.—To verify the nonlinear process proposed and analyzed here, one can resort to either numerical simulations or experiments. In order to properly account for all the relevant physics including nonlinear coupling in the short wavelength limit and dissipation due to electron dynamics such as collisionless Laudau damping, numerical simulation codes with nonlinear gyrokinetic treatment of ions and drift kinetic treatment of electrons are required.

For experimental observations, meanwhile, noting that the pump TAE and, thus, the generated GAM and LKTAE are localized in the tokamak center, where the TAE drive (by EPs) is maximum, and that $|k_G| \sim |k_{r,L}| \sim O([\epsilon_0\rho_i^2/(n^2 q'^2)]^{-1/4})$, diagnostics with high resolution in both radial structure and frequency for local (not line averaged) fluctuations are required. Possible diagnostics could be phase contrast imaging [27], beam emission spectroscopy [28], or electron cyclotron emission imaging [29]. Better signal-to-noise ratio is required for clearer demonstration of the nonlinear decay process. One option is to set up the experiments in the condition for minimized threshold conditions, which generally requires $q\sqrt{7/4 + T_e/T_i} > 1$ for weak GAM Landau damping and $3\% \gtrsim \beta_i > [\lambda\epsilon/(2q)]^2$ for weak TAE or LKTAE Landau damping, and $\omega_G > \omega_0 - \omega_\ell$ due to the frequency matching constraint. Here, ω_ℓ is the lower accumulational point frequency of the toroidicity induced SAW continuum gap and λ expresses the fraction of $\omega_0 - \omega_\ell$ in units of the gap width. For the process discussed here to dominate over other processes in long wavelength limits [12–14], one further requires $k_\perp^2\rho_i^2 > \omega_0/\Omega_{ci}$, which corresponds to $(T_i/T_E)^{1/2}/(q\epsilon^{1/2}) > \omega_0/\Omega_{ci}$ for EP driven TAEs. Another possibility is using antenna excitation of TAE [30,31] such that there is no wave-EP interactions, and one can focus on the decay process. The threshold TAE amplitude can then be measured directly by scanning the antenna strength. Further insights and understanding can be achieved by antenna excitation of TAE and LKTAE at the same time to observe the generation of GAM. The excitation of TAE and KTAE at the same time has more applications than purely academic study, since in burning plasmas TAE and KTAE (more likely upper KTAE) can be excited by EPs at the same time, and the generation of GAM can directly influence plasma confinement as well as α channeling, as we have discussed above. The relative intensity ratio of LKTAE to pump TAE can be roughly estimated from the fixed point solutions of Eqs. (10)–(12), and is given by $A_L^2/A_0^2 = (\gamma_0/\gamma_L)(\partial_{\omega_0}\hat{\mathcal{E}}_{0,r}/\partial_{\omega_L}\hat{\mathcal{E}}_{L,r}) \times (\hat{\alpha}_L/\hat{\alpha}_0)$, which, for typical joint European Torus (JET) parameters, can be estimated as $A_L^2/A_0^2 \sim 0.1$.

Conclusion and discussion.—In this Letter, TAE decaying into a GAM and a LKTAE with the same poloidal and toroidal mode numbers of the pump TAE is investigated as a possible channel for TAE nonlinear saturation. This channel is possible when the GAM frequency is larger than the difference between the pump TAE frequency and the lower accumulation point frequency of the toroidicity induced SAW continuum gap; i.e., $\beta q^2 \gg \epsilon^2$. The nonlinear dispersion relation for the decay instability is derived, valid for arbitrary wavelength. The conditions for the decay instability to take place, i.e., the threshold condition for the pump TAE amplitude to overcome GAM and LKTAE damping, is given by Eq. (7). This threshold condition needs, generally, numerical solution with the mode structure and geometry carefully accounted for. However, in the $k_\perp^2\rho_i^2 \lesssim 1$ limit, it can be analytically simplified and estimated to be $|\delta B_r/B_0|^2 \sim 10^{-9}$, comparable to other channels for TAE nonlinear saturation via wave-wave coupling in the short wavelength ($k_\perp^2\rho_i^2 > \omega/\Omega_{ci}$) limit.

Besides the impact on TAE saturation, the decay process discussed in this Letter will also contribute to the channeling of EP or fusion-α power density to bulk thermal plasma heating. The GAM Landau damping will lead to bulk ion heating and, thus, has direct impact on steady-state operation of a fusion reactor, whereas LKTAE is predominantly damped by electron kinetics, and therefore will contribute to the anomalous slowing-down of fusion α's. An estimate of the power to ion and electron heating are derived, respectively.

As a final remark, it is worth noting that the GAM radial structures generated by the novel mechanism proposed in this work are typically comparable with the DW radial correlation length. The nonlinear generated GAM, thus, provide additional benefits of regulating the DW turbulence and, consequently, improved confinement. In this respect, as noted above, EP could be regarded as an effective "nonlinear free-energy source" for the GAM fluctuations. Thus, the present nonlinear mechanism may be considered as an example and evidence of the unique role played by EP as mediators of cross-scale couplings in burning plasmas of fusion interest [32].

This work is supported by U.S. DOE Grant, the National Science Foundation of China under Grants No. 11575157 and No. 11235009, the ITER-CN under Grants No. 2013 GB104004 and No. 2013 GB111004, Fundamental Research Fund for Chinese Central Universities under Grant No. 2017FZA3004, and EUROfusion Consortium under Grant Agreement No. 633053.

*zqiu@zju.edu.cn

[1] C. Cheng, L. Chen, and M. Chance, Ann. Phys. (N.Y.) **161**, 21 (1985).
[2] N. Winsor, J. L. Johnson, and J. M. Dawson, Phys. Fluids **11**, 2448 (1968).
[3] R. R. Mett and S. M. Mahajan, Phys. Fluids B **4**, 2885 (1992).
[4] F. Zonca and L. Chen, Phys. Plasmas **3**, 323 (1996).
[5] T. S. Hahm, M. A. Beer, Z. Lin, G. W. Hammett, W. W. Lee, and W. M. Tang, Phys. Plasmas **6**, 922 (1999).
[6] N. Fisch and M. Herrmann, Nucl. Fusion **34**, 1541 (1994).
[7] K. Tomabechi, J. Gilleland, Y. Sokolov, R. Toschi, and ITER Team, Nucl. Fusion **31**, 1135 (1991).
[8] L. Chen and F. Zonca, Rev. Mod. Phys. **88**, 015008 (2016).
[9] L. Chen, in *Theory of Fusion Plasmas*, edited by J. Vaclavik, F. Troyon, and E. Sindoni (Association EUROATOM, Bologna, 1988), p. 327.
[10] G. Y. Fu and J. W. Van Dam, Phys. Fluids B **1**, 1949 (1989).
[11] L. Chen and F. Zonca, Phys. Plasmas **20**, 055402 (2013).
[12] T. S. Hahm and L. Chen, Phys. Rev. Lett. **74**, 266 (1995).
[13] F. Zonca, F. Romanelli, G. Vlad, and C. Kar, Phys. Rev. Lett. **74**, 698 (1995).
[14] L. Chen, F. Zonca, R. Santoro, and G. Hu, Plasma Phys. Controlled Fusion **40**, 1823 (1998).
[15] L. Chen and F. Zonca, Phys. Rev. Lett. **109**, 145002 (2012).
[16] H. L. Berk and B. N. Breizman, Phys. Fluids B **2**, 2246 (1990).
[17] F. Zonca, L. Chen, S. Briguglio, G. Fogaccia, G. Vlad, and X. Wang, New J. Phys. **17**, 013052 (2015).
[18] Z. Qiu, L. Chen, and F. Zonca, Nucl. Fusion **57**, 056017 (2017).
[19] Z. Qiu, L. Chen, and F. Zonca, Europhys. Lett. **101**, 35001 (2013).
[20] F. Zonca and L. Chen, Phys. Plasmas **21**, 072121 (2014).
[21] F. Zonca, Y. Lin, and L. Chen, Europhys. Lett. **112**, 65001 (2015).
[22] J. W. Connor, R. J. Hastie, and J. B. Taylor, Phys. Rev. Lett. **40**, 396 (1978).
[23] F. Zonca and L. Chen, Europhys. Lett. **83**, 35001 (2008).
[24] E. A. Frieman and L. Chen, Phys. Fluids **25**, 502 (1982).
[25] L. Chen and F. Zonca, Europhys. Lett. **96**, 35001 (2011).
[26] M. Sasaki, K. Itoh, and S.-I. Itoh, Plasma Phys. Controlled Fusion **53**, 085017 (2011).
[27] M. Porkolab, J. C. Rost, N. Basse, J. Dorris, E. Edlund, L. Lin, Y. Lin, and S. Wukitch, IEEE Trans. Plasma Sci. **34**, 229 (2006).
[28] W. Mandl, R. C. Wolf, M. G. von Hellermann, and H. P. Summers, Plasma Phys. Controlled Fusion **35**, 1373 (1993).
[29] B. Tobias, C. Domier, T. Liang, X. Kong, L. Yu, G. Yun, H. Park, I. J. Classen, J. Boom, A. Donné *et al.*, Rev. Sci. Instrum. **81**, 10D928 (2010).
[30] A. Fasoli, D. Borba, G. Bosia, D. Campbell, J. Dobbing, C. Gormezano, J. Jacquinot, P. Lavanchy, J. Lister, P. Marmillod *et al.*, Phys. Rev. Lett. **75**, 645 (1995).
[31] P. Lauber, S. Günter, and S. Pinches, Phys. Plasmas **12**, 122501 (2005).
[32] F. Zonca, L. Chen, S. Briguglio, G. Fogaccia, A. V. Milovanov, Z. Qiu, G. Vlad, and X. Wang, Plasma Phys. Controlled Fusion **57**, 014024 (2015).