New Gauss-Bonnet Black Holes with Curvature-Induced Scalarization in Extended Scalar-Tensor Theories

Daniela D. Doneva[1,2,*] and Stoytcho S. Yazadjiev[1,3,4,†]
[1]Theoretical Astrophysics, Eberhard Karls University of Tübingen, Tübingen 72076, Germany
[2]INRNE—Bulgarian Academy of Sciences, 1784 Sofia, Bulgaria
[3]Department of Theoretical Physics, Faculty of Physics, Sofia University, Sofia 1164, Bulgaria
[4]Institute of Mathematics and Informatics, Bulgarian Academy of Sciences, Acad. G. Bonchev Street 8, Sofia 1113, Bulgaria

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In the present Letter, we consider a class of extended scalar-tensor-Gauss-Bonnet (ESTGB) theories for which the scalar degree of freedom is excited only in the extreme curvature regime. We show that in the mentioned class of ESTGB theories there exist new black-hole solutions that are formed by spontaneous scalarization of the Schwarzschild black holes in the extreme curvature regime. In this regime, below certain mass, the Schwarzschild solution becomes unstable and a new branch of solutions with a nontrivial scalar field bifurcates from the Schwarzschild one. As a matter of fact, more than one branch with a nontrivial scalar field can bifurcate at different masses, but only the first one is supposed to be stable. This effect is quite similar to the spontaneous scalarization of neutron stars. In contrast to the standard spontaneous scalarization of neutron stars, which is induced by the presence of matter, in our case, the scalarization is induced by the curvature of the spacetime.

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Introduction.—The historic direct detection of gravitational waves has opened a new era in physics, giving a powerful tool for exploring the strong-gravity regime, where spacetime curvature is extreme [1]. General relativity (GR) is well tested in the weak-field regime, whereas the strong-field regime still remains essentially unexplored and unconstrained. There are both phenomenological and theoretical reasons for the modification of the original Einstein equations. The attempts to construct a unified theory of all the interactions naturally lead to scalar-tensor-type generalizations of general relativity with an additional dynamical scalar field and with Lagrangians containing various kinds of curvature corrections to the usual Einstein-Hilbert Lagrangian coupled to the scalar field [2–5]. The most natural modifications of this class are the extended scalar-tensor theories, where the usual Einstein-Hilbert action is supplemented with all possible algebraic curvature invariants of second order, with a dynamical scalar field nonminimally coupled to these invariants. We shall focus on the extended scalar-tensor-Gauss-Bonnet (ESTGB) gravity as a natural modification of general relativity and a natural extension of the standard scalar-tensor theories. A very important property of ESTGB gravity is that the field equations are of second order as in general relativity and the theory is free from ghosts. A particular model of ESTGB gravity, the so-called Einstein-dilaton-Gauss-Bonnet gravity (with a coupling function $\alpha e^{\gamma\varphi}$ and vanishing potential for the dilaton field) was extensively studied in the literature [6–14].

In the present Letter, we shall consider a class of ESTGB theories with scalar coupling functions, for which the scalar degree of freedom is excited only in the extreme curvature regime. In particular, we shall show that in the mentioned class of ESTGB theories there exist new black-hole solutions that are formed by spontaneous scalarization of the Schwarzschild black holes in the extreme curvature regime. In contrast to the standard spontaneous scalarization [15–17], which is induced by the presence of matter, in our case, the scalarization is induced by the curvature of the spacetime.

Basic equations and setting the problem.—The general action of ESTGB theories in vacuum is given by

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}[R - 2\nabla_\mu\varphi\nabla^\mu\varphi + \lambda^2 f(\varphi)\mathcal{R}^2_{\rm GB}], \qquad (1)$$

where φ is the scalar field with a coupling function $f(\varphi)$ depending only on φ, λ is the Gauss-Bonnet coupling constant having dimension of *length*, and $\mathcal{R}^2_{\rm GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta}$ is the Gauss-Bonnet invariant. The action yields the following field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Gamma_{\mu\nu} = 2\nabla_\mu\varphi\nabla_\nu\varphi - g_{\mu\nu}\nabla_\alpha\varphi\nabla^\alpha\varphi, \quad (2)$$

$$\nabla_\alpha\nabla^\alpha\varphi = -\frac{\lambda^2}{4}\frac{df(\varphi)}{d\varphi}\mathcal{R}^2_{\rm GB}, \qquad (3)$$

where ∇_μ is the covariant derivative with respect to the spacetime metric $g_{\mu\nu}$ and $\Gamma_{\mu\nu}$ is defined by

$$\Gamma_{\mu\nu} = -R(\nabla_\mu\Psi_\nu + \nabla_\nu\Psi_\mu) - 4\nabla^\alpha\Psi_\alpha\left(R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}\right) + 4R_{\mu\alpha}\nabla^\alpha\Psi_\nu + 4R_{\nu\alpha}\nabla^\alpha\Psi_\mu - 4g_{\mu\nu}R^{\alpha\beta}\nabla_\alpha\Psi_\beta + 4R^\beta_{\mu\alpha\nu}\nabla^\alpha\Psi_\beta, \quad (4)$$

with $\Psi_\mu = \lambda^2 \frac{df(\varphi)}{d\varphi}\nabla_\mu\varphi$.

We consider further static and spherically symmetric spacetimes as well as static and spherically symmetric scalar field configurations. The spacetime metric then can be written in the standard form

$$ds^2 = -e^{2\Phi(r)}dt^2 + e^{2\Lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (5)$$

The dimensionally reduced field equations are as follows:

$$\frac{2}{r}\left[1 + \frac{2}{r}(1-3e^{-2\Lambda})\Psi_r\right]\frac{d\Lambda}{dr} + \frac{(e^{2\Lambda}-1)}{r^2} - \frac{4}{r^2}(1-e^{-2\Lambda})\frac{d\Psi_r}{dr} - \left(\frac{d\varphi}{dr}\right)^2 = 0, \quad (6)$$

$$\frac{2}{r}\left[1 + \frac{2}{r}(1-3e^{-2\Lambda})\Psi_r\right]\frac{d\Phi}{dr} - \frac{(e^{2\Lambda}-1)}{r^2} - \left(\frac{d\varphi}{dr}\right)^2 = 0, \quad (7)$$

$$\frac{d^2\Phi}{dr^2} + \left(\frac{d\Phi}{dr} + \frac{1}{r}\right)\left(\frac{d\Phi}{dr} - \frac{d\Lambda}{dr}\right) + \frac{4e^{-2\Lambda}}{r}\left[3\frac{d\Phi}{dr}\frac{d\Lambda}{dr} - \frac{d^2\Phi}{dr^2} - \left(\frac{d\Phi}{dr}\right)^2\right]\Psi_r - \frac{4e^{-2\Lambda}}{r}\frac{d\Phi}{dr}\frac{d\Psi_r}{dr} + \left(\frac{d\varphi}{dr}\right)^2 = 0, \quad (8)$$

$$\frac{d^2\varphi}{dr^2} + \left(\frac{d\Phi}{dr} - \frac{d\Lambda}{dr} + \frac{2}{r}\right)\frac{d\varphi}{dr} - \frac{2\lambda^2}{r^2}\frac{df(\varphi)}{d\phi}\left\{(1-e^{-2\Lambda})\left[\frac{d^2\Phi}{dr^2} + \frac{d\Phi}{dr}\left(\frac{d\Phi}{dr} - \frac{d\Lambda}{dr}\right)\right] + 2e^{-2\Lambda}\frac{d\Phi}{dr}\frac{d\Lambda}{dr}\right\} = 0, \quad (9)$$

with $\Psi_r = \lambda^2 \frac{df(\varphi)}{d\varphi}\frac{d\varphi}{dr}$.

In the present Letter, we are interested in ESTGB theories (ESTGBTs) with coupling function $f(\varphi)$ satisfying the conditions $\frac{df}{d\varphi}(0) = 0$ and $b^2 = \frac{d^2f}{d\varphi^2}(0) > 0$. (We consider here the case when the cosmological value of the scalar field is zero, $\varphi_\infty = 0$.) Without loss of generality, we can put $b = 1$ and this can be achieved by rescaling the coupling parameter $\lambda \to b\lambda$ and by redefining the coupling function $f \to b^{-2}f$. In addition, since the theory depends only on $\frac{df(\varphi)}{d\varphi}$, we can also impose $f(0) = 0$.

The natural and important question is whether the class of ESTGBTs defined above admits (static and spherically symmetric) black-hole solutions. From the dimensionally reduced field equations (6)–(9), it is clear that the usual Schwarzschild black-hole solution is also a black-hole solution to the ESTGBT under consideration with a trivial scalar field $\varphi = 0$. We shall, however, show that the Schwarzschild solution within a certain range of the mass is unstable in the framework of the ESTGBT under consideration. For this purpose, we consider the perturbations of the Schwarzschild solution with mass M within the framework of the described class of ESTGBT. It is not difficult to see that in the considered class of ESTGBTs the equations governing the perturbations of the metric $\delta g_{\mu\nu}$ are decoupled from the equation governing the perturbation $\delta\varphi$ of the scalar field. The equations for the metric perturbations are, in fact, the same as those in the pure Einstein gravity, and therefore, we shall focus only on the scalar field perturbations. The equation governing the scalar perturbations is

$$\Box_{(0)}\delta\varphi + \frac{1}{4}\lambda^2\mathcal{R}^2_{\mathrm{GB}(0)}\delta\varphi = 0, \quad (10)$$

where $\Box_{(0)}$ and $\mathcal{R}^2_{\mathrm{GB}(0)}$ are the D'Alambert operator and the Gauss-Bonnet invariant for the Schwarzschild geometry. Taking into account that the background geometry is static and spherically symmetric, the variables can be separated in the following way: $\delta\varphi = [u(r)/r]e^{-i\omega t}Y_{lm}(\theta,\phi)$, with $Y_{lm}(\theta,\phi)$ being the spherical harmonics. After substituting in (10) and introducing the tortoise coordinate $dr_* = (1-\frac{2M}{r})^{-1}dr$, we obtain the following Schrödinger-like equation

$$\frac{d^2u}{dr_*^2} + [\omega^2 - U(r)]u = 0, \quad (11)$$

with a potential

$$U(r) = \left(1 - \frac{2M}{r}\right)\left(\frac{2M}{r^3} + \frac{l(l+1)}{r^2} - \lambda^2\frac{12M^2}{r^6}\right). \quad (12)$$

A sufficient condition for the existence of an unstable mode is [18]

$$\int_{-\infty}^{+\infty} U(r_*)dr_* = \int_{2M}^{\infty}\frac{U(r)}{1-\frac{2M}{r}}dr < 0. \quad (13)$$

For the spherically symmetric perturbations, the above condition gives $M^2 < \frac{3}{10}\lambda^2$. Therefore, we can conclude that the Schwarzschild black holes with mass satisfying $M^2 < \frac{3}{10}\lambda^2$ are unstable within the framework of the ESTGBT under consideration. Stated differently, the Schwarzschild black holes become unstable when the curvature of the horizon exceeds a certain critical value—in terms of the Kretschmann scalar of the horizon $\mathcal{K}_H$, the instability occurs when $\mathcal{K}_H > \frac{25}{3\lambda^4}$. As a matter of fact, this is only a sufficient condition for instability and the true point of the first bifurcations is actually at a little bit larger masses.

This result naturally leads us to the conjecture that, in our class of ESTGBT and in the interval where the Schwarzschild solution is unstable, there exist black-hole solutions with nontrivial scalar field. In the next sections, we numerically prove that such black-hole solutions really exist and present some of their basic properties.

Numerical setup.—In order to obtain the black-hole solutions with a nontrivial scalar field, we solve numerically via a shooting method the system of reduced field equations (6)–(9). The boundary and the regularity conditions come from the requirements for asymptotic flatness at infinity and regularity at the black-hole horizon $r = r_H$. As usual, the asymptotic flatness imposes the following asymptotic conditions: $\Phi|_{r\to\infty} \to 0, \Lambda|_{r\to\infty} \to 0$, and $\varphi|_{r\to\infty} \to 0$. The very existence of the black-hole horizon requires $e^{2\Phi}|_{r\to r_H} \to 0$ and $e^{-2\Lambda}|_{r\to r_H} \to 0$. The regularity of the scalar field and its first and second derivatives on the black-hole horizon leads to the following condition

$$\left(\frac{d\varphi}{dr}\right)_H = \frac{r_H}{4\lambda^2 \frac{df}{d\varphi}(\varphi_H)}\left[-1 \pm \sqrt{1 - \frac{24\lambda^4}{r_H^4}\left(\frac{df}{d\varphi}(\varphi_H)\right)^2}\right]. \tag{14}$$

In this expression, we have to choose the plus sign, since only in this case can we recover the Schwarzschild solution in the limit $\varphi_H \to 0$. Thus, black holes with a nontrivial scalar field can exist only when

$$r_H^4 > 24\lambda^4 \left(\frac{df}{d\varphi}(\varphi_H)\right)^2. \tag{15}$$

The mass of the black hole M and the dilaton charge D are obtained through the asymptotics of the functions Λ, Φ, and φ: $\Lambda \approx \frac{M}{r} + O(1/r^2)$, $\Phi \approx -\frac{M}{r} + O(1/r^2)$, and $\varphi \approx \frac{D}{r} + O(1/r^2)$.

Results.—In the present Letter, we consider the following coupling function $f(\varphi) = (1/12)[1 - \exp(-6\varphi^2)]$, which is chosen in such a way that we have both non-negligible deviations from the Schwarzschild solution and condition (15) is fulfilled for a large enough range of parameters. This particular choice is quite similar to the coupling function considered in the case of spontaneous scalarization of neutron stars [15]. We have explicitly checked that other choices of $f(\varphi)$ that lead to similar results are of course possible, but exploring a large variety of $f(\varphi)$ functions is out of the scope of the present Letter.

The Schwarzschild solution with a zero scalar field is always a solution of the field equations, but in a certain region of the parameter space, it becomes unstable in the framework of the ESTGBT under consideration and new solutions with a nontrivial scalar field appear. Moreover, there can be regions where more than one solution with a nontrivial scalar field exist and this corresponds, roughly speaking, to the appearance of more than one bound state of the potential in the perturbation equation (11). In the present Letter, we consider only spherically symmetric solutions, and therefore, $l = 0$ in Eq. (12). The different branches of solutions will have a scalar field with a different number of zeros, similar to the eigenfunctions of the perturbation equation (11).

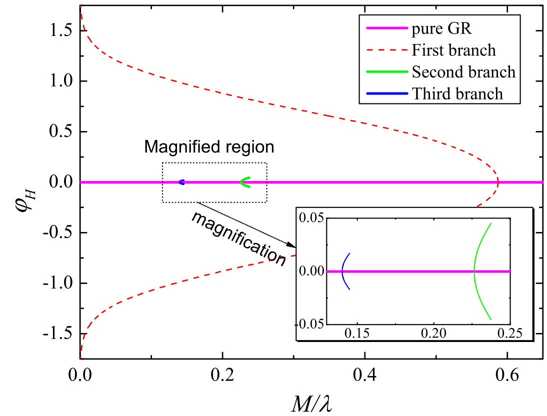

FIG. 1. The scalar field at the horizon as a function of the black-hole mass.

Finding the solutions with nontrivial scalar field might sometimes be numerically difficult, and it is of great help to know the exact points of bifurcation. That is why after employing the methodology developed in [17] and finding numerically the eigenvalues of the perturbation equation (11), we determined the regions of the parameter space where the Schwarzschild solution is stable and where one or more unstable modes are present. This means that we have determined the points of bifurcation of the Schwarzschild solution.

The obtained black-hole solutions are plotted in Fig. 1, where only the first three bifurcations of the Schwarzschild solution are shown. We will call the Schwarzschild solution the trivial branch of solutions, while the rest of the branches of black holes with a nontrivial scalar field will be called nontrivial branches. As one can see, all the nontrivial branches start from a bifurcation point at the trivial branch and they span either to $M = 0$ (the first nontrivial branch) or they are terminated at some nonzero M (all other nontrivial branches), because beyond this mass the condition (15) is violated. One can also notice that Fig. 1 is symmetric with respect to the x axis. This is because the theory is invariant under the symmetry $\varphi \to -\varphi$. Thus, for a fixed M, the solutions with positive and negative values of φ_H would naturally have opposite signs of the dilaton charge, but they have the same metric functions and thus mass.

For the first branch, there are no zeros of φ, the next one has one zero, and the third one has two zeros, as one can see in Fig. 2. For smaller values of M, there are more bifurcation points, but our investigations show the corresponding nontrivial branches would be even shorter and

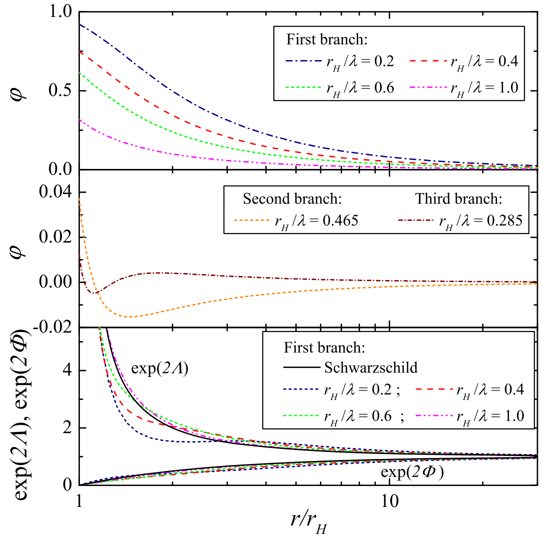

FIG. 2. (Top) The scalar field as a function of the normalized radial coordinate r/r_H for several black-hole solutions belonging to the first nontrivial branch. (Middle) The scalar field as a function of the normalized radial coordinate r/r_H for black-hole solutions belonging to the second and third nontrivial branches. (Bottom) The g_{tt} and g_{rr} components of the metric as functions of the normalized radial coordinate r/r_H for several black-hole solutions belonging to the first nontrivial branch.

that is why we have not plotted them. Moreover, it is expected that only the first nontrivial branch characterized by a scalar field without zeros will be stable, while the rest of the branches correspond to unstable solutions. The components of the metric g_{tt} and g_{rr} for some representative solutions with different r_H/λ are also plotted in the bottom panel of Fig. 2 for the first nontrivial branch. As one can see, they can deviate significantly from the Schwarzschild one. We have not plotted g_{tt} and g_{rr} for the other nontrivial branches, since they are practically indistinguishable from the pure general relativistic case.

The dilaton charge as a function of the mass is shown in Fig. 3. While the dependence $\varphi_H(M/\lambda)$ is monotonic for the first nontrivial branch and φ_H increases significantly for small masses, $D(M/\lambda)$ has an extremum (either minimum or maximum depending on the sign of φ_H) and tends to zero for small masses.

The area of the black-hole horizon, $A_H = 4\pi r_H^2$, is plotted as functions of the mass in the top panel of Fig. 4 for all of the considered branches of solutions. Only the first branch of nontrivial solutions differs significantly from the Schwarzschild case, and the deviations are the largest for intermediate masses. This observation is similar to the behavior of the dilaton change.

In order to have an indicator for the stability of the black-hole branches, one can study the entropy of the black holes. The black-hole entropy in the presence of a Gauss-Bonnet term in the action (1) is not just one fourth of the horizon area and its definition is a little bit more complicated. We

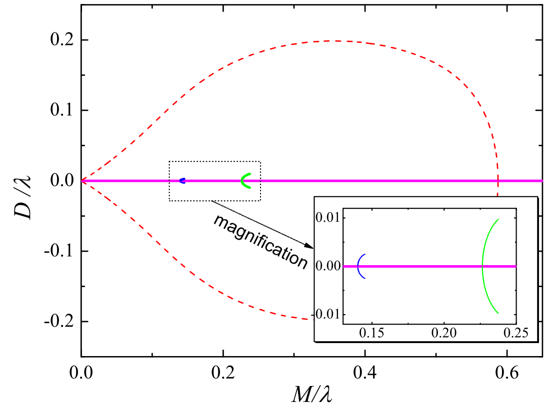

FIG. 3. The dilaton charge of the black hole as a function of its mass. The notations are the same as in Fig. 1.

adopt the entropy formula proposed by Wald and coworker in [19,20], namely,

$$S_H = 2\pi \int_H \frac{\partial \mathcal{L}}{\partial R_{\mu\nu\alpha\beta}} \epsilon_{\mu\nu}\epsilon_{\alpha\beta}, \tag{16}$$

where $\mathcal{L}$ is the Lagrangian density and $\epsilon_{\alpha\beta}$ is the volume form on the two-dimensional cross section H of the horizon. In our case, we find $S_H = \frac{1}{4}A_H + 4\pi\lambda^2 f(\varphi_H)$. The entropy as a function of the black hole's mass is plotted in the bottom panel of Fig. 4. The first nontrivial branch has an entropy larger than the Schwarzschild one and it is therefore thermodynamically more stable. This is an expected result since, for masses smaller than the point of the first bifurcation, the Schwarzschild solution will get unstable and there should be another one. The second and the third nontrivial branches, on the other hand, have lower entropy compared to the pure general relativistic case, which means that they are most probably unstable. The same is expected to apply for the rest of nontrivial branches

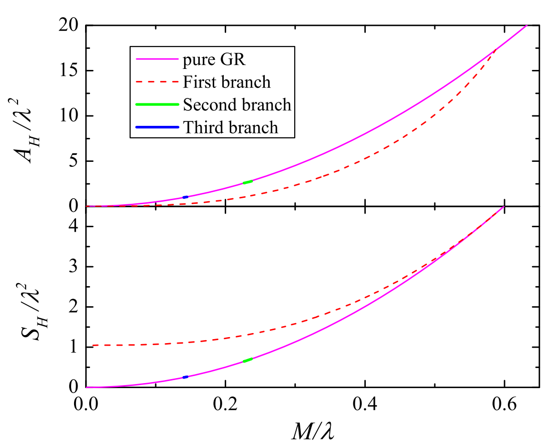

FIG. 4. (Top) The area of the black-hole horizon A_H as a function of the mass. (Bottom) The entropy of the black hole as a function of its mass.

that exist for smaller masses. The dynamical stability of our black-hole solutions will be investigated in a future work.

Conclusions.—In the present Letter, we have studied black-hole solutions in a particular class of ESTGB theories described by a coupling function $f(\varphi)$ that satisfies the conditions $\frac{df}{d\varphi}(0) = 0$ and $\frac{d^2f}{d\varphi^2}(0) > 0$. We have shown that for such theories an effect similar to the spontaneous scalarization of neutron stars exist—the Schwarzschild solution becomes unstable below a certain mass and new branches of black-hole solutions with nontrivial scalar field appear that bifurcate from the Schwarzschild one at certain masses. The first branch of nontrivial solutions is characterized by a scalar field that has no zeros, while the scalar field has one zero for the second branch, two zeros for the third branch, and so on. The main difference with the standard spontaneous scalarization of neutron stars is that the scalar field is not sourced by the matter, but instead by the extreme curvature of the spacetime around black holes. This places the considered solutions among the very few examples of scalarized black holes.

We have explicitly constructed such solutions with a nonzero scalar field. The scalarized black-hole solutions tend to the Schwarzschild one for very small masses and for larger masses close to the bifurcation point, and the maximum deviation is observed for intermediate masses. Actually, this is true only for the first nontrivial branch characterized by a scalar field without zeros. The rest of the branches are terminated at some nonzero mass because, beyond that mass, they violate condition (15).

We have studied the behavior of the black-hole entropy and the results show that the first nontrivial branch has higher entropy than the Schwarzschild black holes and it is thermodynamically more stable, while the rest of the branches have lower entropy. Thus, the general expectation is that the first branch of solutions is stable and it is the one that would be realized in practice because of the instability of the Schwarzschild solutions. The other nontrivial branches are supposed to be unstable. In a future publication, we plan to study the linear stability of the solutions with nontrivial scalar field.

The results presented in the current Letter are for a particular coupling function that can produce non-negligible deviations from pure general relativity. We have tested, though, several other functions satisfying the above given conditions for $f(\varphi)$ and the results are qualitatively very similar.

The black holes we have considered posses a nontrivial scalar field, and thus they have scalar "hair." When the branch of the solution is fixed, then this hair is secondary, which means that the dilaton charge is not an independent parameter, but instead it depends on black-hole mass. However, the number of the branches is an independent parameter introducing a new hair of discrete type. One may adopt the view that only the stable branch has to be considered, in this way getting rid of the discrete hair. In our opinion, the classification of black-hole solutions presented in the present work is rather subtle and needs a much deeper analytical investigation.

A similar effect of scalarization is observed also for neutron stars [21,22]. The presence of nontrivial scalar field in this case is strongly constrained by the binary pulsar observations, so a natural question to ask is whether the coupling function considered in the present Letter is in agreement with these observations. Let us consider a more general form of the coupling function $f(\varphi) = (1/2\beta)[1 - \exp(-\beta\varphi^2)]$. Based on our nonperturbative results in [22], we obtain that the spontaneous scalarization can give rise to stable neutron stars with a nontrivial scalar field only for $\lambda/M_{\odot} > 14.5$ and $\beta > 32$. These estimates are made for the MPA1 equation of state [23], but similar bounds should hold for other modern realistic equations of state. Thus, it is evident that the coupling function used in the present Letter is well within the observational bounds imposed by the binary pulsar observations. In other words, for the chosen coupling function $f(\varphi)$ with $\beta = 6$, one can expect large effects for black holes and no effects in binary pulsars.

At the end, let us comment on the importance of our results in view of the recent detection of gravitational wave emission by binary black-hole mergers. One of the most prominent effects would come from the fact that if the black holes are scalarized there will be an additional channel of energy loss during the inspiral phase via the emission of dipole scalar field radiation. As a result, the inspiral will be faster in comparison with the pure general relativistic case, similar to the mergers of scalarized binary neutron stars [24,25]. Of course, a full numerical calculation of the waveform including the nonlinear phase of the merger would give us even a more powerful tool for constraining the ESTGB gravity.

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Note added in proof.—Recently, two papers studying a similar model for BHs appeared in the Refs. [21,26].

*daniela.doneva@uni-tuebingen.de
†yazad@phys.uni-sofia.bg

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