

Chaotic Dynamical Ferromagnetic Phase Induced by Nonequilibrium Quantum Fluctuations

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We investigate the robustness of a dynamical phase transition against quantum fluctuations by studying the impact of a ferromagnetic nearest-neighbor spin interaction in one spatial dimension on the nonequilibrium dynamical phase diagram of the fully connected quantum Ising model. In particular, we focus on the transient dynamics after a quantum quench and study the prethermal state via a combination of analytic time-dependent spin wave theory and numerical methods based on matrix product states. We find that, upon increasing the strength of the quantum fluctuations, the dynamical critical point fans out into a chaotic dynamical phase within which the asymptotic ordering is characterized by strong sensitivity to the parameters and initial conditions. We argue that such a phenomenon is general, as it arises from the impact of quantum fluctuations on the mean-field out of equilibrium dynamics of any system which exhibits a broken discrete symmetry.

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Introduction.—Throwing dice on the table floor is a prototypical random (or pseudorandom) process [1]. Its aleatory nature is a consequence of a few ingredients: the die, the state of being initially out of equilibrium, it dissipates its energy rolling on the table, and hence, it relaxes onto one of few possible equilibrium configurations. In this Letter, we show that the same ingredients play an important role in the physics of quantum many-body systems undergoing a dynamical quantum phase transition (DQPT), leading to the emergence of an intriguing chaotic dynamical phase once the collective dynamics of these systems gets damped by quantum fluctuations.

DQPTs are among the most interesting phenomena occurring in quantum many-body systems after a sudden change of the system parameters (quantum quench) [2], a type of process which can be realized both with ultracold gases [3] and trapped ions [4]. Such DQPTs [5–8] are characterized by the vanishing of a nonequilibrium order parameter (accompanied by critical scaling behavior [9]) and being distinguished from those signaled by nonanalyticities in the temporal evolution of the Loschmidt echo [10] (see Ref. [11] for connections between the two notions). They not only provide a genuine instance of classical and quantum criticality out of equilibrium [9,12,13], but they also demonstrate the emergence of intermediate stages of relaxation with nontrivial time-dependent fluctuations and dynamics [6]. A DQPT separates “phases” characterized by qualitatively different quasistationary states [5,8], anomalous coarsening [14], aging [13–15], as well as by a nontrivial dynamical

evolution of observables and their fluctuations [6–8,11,12]. Because of the lack of spatial and temporal collective scales upon approaching a DQPT, they display features reminiscent of equilibrium critical points.

DQPTs are expected to be strongly affected by quantum fluctuations, and recent investigations beyond mean-field approximations [9,16–18] showed that these fluctuations influence, e.g., the early stages of the evolution [13,14]. In this Letter, we demonstrate a more dramatic effect of fluctuations on the dynamics of the order parameter, which induces a qualitative modification of the dynamical phase diagram that is, in particular, close to the dynamical critical point. We study the nonequilibrium dynamics of an infinite-range (mean-field) ferromagnetic system perturbed by additional short-range interaction terms, which rule the strength of quantum fluctuations. We show that the dynamical phases are robust, whereas the impact of nonequilibrium quantum fluctuations make the dynamical critical point open up in a novel *chaotic dynamical phase* where the dynamics are reminiscent of the that of a coin toss: The asymptotic stationary state displays a finite magnetization whose positive or negative sign is highly sensitive to initial conditions and system parameters, as we show in Fig. 1.

The model.—In this Letter, for the sake of definiteness, we focus on a fully connected quantum Ising ferromagnet in a transverse magnetic field g , in the presence of additional nearest-neighbor couplings in one spatial dimension, governed by the Hamiltonian

$$H = -\frac{\lambda}{N} \sum_{i,j=1}^N \sigma_i^x \sigma_j^x - g \sum_{i=1}^N \sigma_i^z - J \sum_{i=1}^N \sigma_i^x \sigma_{i+1}^x, \quad (1)$$

where σ_i^α are the standard Pauli matrices at lattice site i . In the limit $J \rightarrow 0$, H maps to the exactly solvable Lipkin-Meshkov-Glick (LMG) model [19] and displays both a quantum critical point in equilibrium [20] at $g = 2\lambda$ and a DQPT after a quench [5,21], with the longitudinal global magnetization $S_x(t)$ —with $S_\alpha \equiv \langle \sum_i \sigma_i^\alpha \rangle / N$ —being the dynamical order parameter of the DPT. For example, for quenches starting from the ferromagnetic ground state at $g_0 = 0$, $S_x(t)$ evolves periodically with a period set by the postquench values g and λ of the couplings. In particular, the time average $\bar{S}_x = \lim_{T \rightarrow \infty} \int_0^T dt S_x(t) / T$ vanishes for $g > \lambda$ (because the oscillations of $S_x(t)$ are symmetric around zero) while it does not for $g < \lambda$ (because the oscillations do not change the sign of S_x), corresponding to the dynamically paramagnetic and ferromagnetic “phases,” respectively. At the dynamical critical point, $g = \lambda$, the order parameter decays exponentially to zero with $S_x(t) \sim e^{-gt}$ for $t \gg g^{-1}$. Within mean-field theory, which is an exact treatment of the LMG model in the thermodynamic limit [21], the DQPT can be rationalized [8,9,21] in terms of the motion of a classical particle with position $S_x(t)$ in an effective, double-well even potential $\mathcal{U}(S_x)$. If $S_x(t=0)$ is such that $\mathcal{U}(S_x(0)) > \mathcal{U}(0)$, then $S_x(t)$ explores both wells and $\bar{S}_x = 0$; otherwise the motion is localized within one well and $\bar{S}_x \neq 0$.

In this Letter, we study how this mean-field nonequilibrium phase diagram is affected by quantum fluctuations. While for $J = 0$ all spins perform a coherent collective motion, turning on a short-range perturbation is expected to affect the persistent classical oscillations of $S_x(t)$, altering the features of the mean-field evolution and inducing relaxation towards a stationary and eventual thermal state.

In order to address these questions we develop a spin wave theory in the reference frame aligned with the instantaneous average total spin, with the spin-coherent state in this direction representing the instantaneous spin wave vacuum. While for $J = 0$ the length of the total spin is constantly maximal, i.e., $|S(t)| \equiv 1$, in the presence of a small short-range perturbation $J \neq 0$ a finite density $\epsilon(t)$ of spin wave excitations is generated by the precessing collective spin, yielding $|S(t)| = 1 - \epsilon(t)$. As long as $\epsilon(t) \ll 1$, the nonlinear, inelastic scattering among spin waves is negligible and thermalization is expected to occur at longer times. Accordingly, the temporal regime with $\epsilon(t) \ll 1$, within which the mean-field motion receives correction from having $J \neq 0$ while keeping its nonequilibrium features (e.g., the DQPT), can be qualified as being prethermal, in analogy with similar cases [22].

Outline of results.—In the presence of quantum fluctuations, one would expect the collective motion of $S_x(t)$ to be damped by the generation of spin wave excitations with a

finite rate, leading to the breakdown of the approximation $\epsilon(t) \ll 1$ (throughout the paper we fix energy units such that $\bar{\lambda} \equiv \lambda + J = 1$). We find, instead, that for small $J \lesssim 0.25$, $\epsilon(t)$ always saturates, implying that the dynamical paramagnetic and ferromagnetic phases indicated by A and B, respectively, in Fig. 1 are stable. In particular, $S_x(t)$ approximately oscillates with a period which is perturbatively close to the mean-field one. A numerical analysis based on the time-dependent variational principle (MPS-TDVP) [23,24] indicates that this stability extends to larger values of J where the density of spin waves is no longer small. This implies that H inherits the dynamical phase diagram of the classical case with $J = 0$. However, the presence of spin wave excitations makes the dynamical critical point at $\lambda = g$ for $J = 0$ fan out in a chaotic dynamical ferromagnetic phase, denoted by C in Fig. 1. Within C, the nonequilibrium quantum fluctuations in the form of spin waves act effectively as a self-generated bath responsible for the localization of the system, initially with

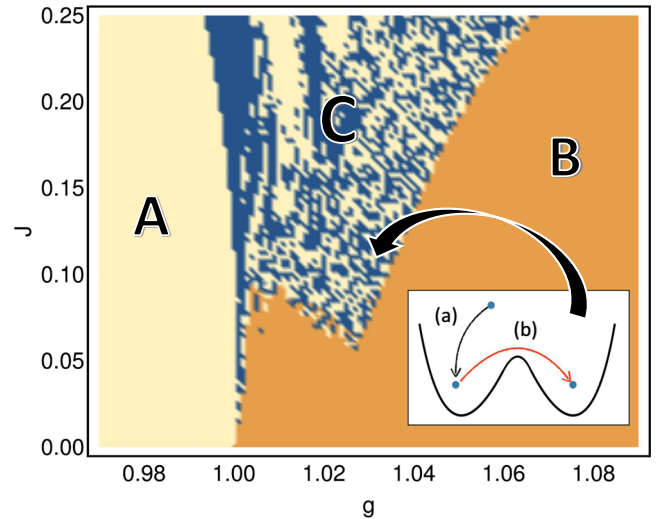


FIG. 1. A dynamical phase diagram of the model in Eq. (1) after a quantum quench starting from the ferromagnetic ground state with $g = 0$ and positive expectation value $S_x > 0$ of the global magnetization, in the plane of the postquench value g of the transverse field and J of the nearest-neighbor coupling (here $N = 100$). We consider here the range of values of g and J within which the low-density spin wave expansion is applicable, and units are chosen such that $\bar{\lambda} \equiv \lambda + J = 1$. The color of each point of the diagram is determined by the value of long-time average \bar{S}_x of S_x : light yellow for $\bar{S}_x > 0$, orange for $\bar{S}_x = 0$, and blue for $\bar{S}_x < 0$. Regions A and B correspond to the dynamic ferromagnetic and paramagnetic phase, respectively, of the mean-field model ($J = 0$). Upon increasing J at a fixed g close to the mean-field critical point, i.e., $g \simeq \bar{\lambda}$, a new chaotic dynamical ferromagnetic phase C arises, exhibiting relaxation from an initial paramagnetic behavior to symmetry-broken sectors [process (a) in the inset] sometimes followed by assisted hopping between the two sectors with opposite signs of \bar{S}_x [process (b) in the inset]. See Fig. 2 for an illustration of the dynamics in region C.

$S_x > 0$, into one of the two minima of \mathcal{U} , with either sign and for the possible hopping of the collective spin S_x between the two of them; these processes are sketched as (a) and (b), respectively, in the inset of Fig. 1. The strong sensitivity of the long-time ferromagnetic ordering to the values of g , λ , and J , and of the initial data, can be regarded as a signature of a collective chaotic behavior. The stability of this picture upon increasing N in both the analytic and the numerical approaches leads us to conclude that such behavior carries over to the thermodynamic limit.

Time-dependent spin wave theory.—We now briefly outline the nonequilibrium spin wave theory at the core of this work [25]. We first introduce a time-dependent reference frame $\mathcal{R} = (\hat{X}, \hat{Y}, \hat{Z})$ in the spin space, with its \hat{Z} axis following the collective motion of $\vec{S}(t)$. The change of frame is implemented by the time-dependent global rotation operator $V(\theta(t), \phi(t)) = \exp(-i\phi(t)\sum_i \sigma_i^z/2) \times \exp(-i\theta(t)\sum_i \sigma_i^y/2)$ parametrized by the angles $\theta(t)$ and $\phi(t)$, which are eventually determined in such a way that $S_x(t) \equiv S_y(t) \equiv 0$. For $J = 0$, when H is a function of the total spin only, this requirement translates into a closed pair of (classical) ordinary differential equations, the solution of which determines the evolution of the order parameter $S_x(t) = \sin\theta(t) \cos\phi(t)$ [21]. For $J \neq 0$, the additional short-range interaction renders H a function of not only the total spin, i.e., the $k = 0$ Fourier mode of the spins, but also of all the k modes of the spins, which now contribute to the dynamics. In order to make the equations of motion tractable and to set up a systematic expansion, we introduce the canonically conjugated spin wave variables q_i and p_i at site i with respect to the instantaneous \hat{Z} axis via the Holstein–Primakoff (HP) transformation

$$\frac{\sigma_i^X}{2} \simeq \sqrt{s} q_i, \quad \frac{\sigma_i^Y}{2} \simeq \sqrt{s} p_i, \quad \frac{\sigma_i^Z}{2} = s - \frac{q_i^2 + p_i^2}{2}. \quad (2)$$

We then express all the spin operators in H , see Eq. (1), in terms of the spin wave coordinates in Fourier space \tilde{q}_k, \tilde{p}_k , and retain up to quadratic terms in the spatial fluctuations modes $(\tilde{q}_k, \tilde{p}_k)$ with $k \neq 0$ (i.e., we neglect collisions among spin waves). After averaging the Heisenberg equations of motion of the spins over the nonequilibrium state [25], we find that θ and ϕ evolve according to (recall $\bar{\lambda} \equiv \lambda + J$)

$$\begin{aligned} \frac{d\theta}{dt} &= 4[\bar{\lambda}\rho(t) - J\delta^{pp}(t)] \sin\theta \cos\phi \sin\phi \\ &\quad + 4J\delta^{qp}(t) \cos\theta \sin\theta \cos^2\phi, \\ \frac{d\phi}{dt} &= -2g + 4[\bar{\lambda}\rho(t) - J\delta^{qq}(t)] \cos\theta \cos^2\phi \\ &\quad + 4J\delta^{qp}(t) \sin\phi \cos\phi, \end{aligned} \quad (3)$$

where $\delta^{\alpha\beta}(t) \equiv \sum_{k \neq 0} \Delta_k^{\alpha\beta} \cos k/(Ns)$ with $\alpha, \beta \in \{p, q\}$ is the quantum “feedback” given by the correlation functions of the spin waves,

$$\begin{aligned} \Delta_k^{qq}(t) &\equiv \langle \tilde{q}_k(t) \tilde{q}_{-k}(t) \rangle, & \Delta_k^{pp}(t) &\equiv \langle \tilde{p}_k(t) \tilde{p}_{-k}(t) \rangle, \\ \Delta_k^{qp}(t) &\equiv \langle \tilde{q}_k(t) \tilde{p}_{-k}(t) + \tilde{p}_k(t) \tilde{q}_{-k}(t) \rangle / 2. \end{aligned} \quad (4)$$

The relevance of these spin wave excitations is controlled by the quantity

$$\epsilon(t) \equiv \frac{1}{N/2} \sum_{k \neq 0} (\Delta_k^{qq} + \Delta_k^{pp} - 1) / 2, \quad (5)$$

i.e., by the total number of spin waves divided by $N/2$. In Eq. (3), $\rho(t) = 1 - \epsilon(t)$ is the ratio between the expectation value of the modulus of the total spin of the system and its maximal value $N/2$, which is conserved by the dynamics only when $J = 0$ [21]. The evolution of $\Delta_k^{\alpha\beta}$ in Eq. (4) is ruled by a system of linear differential equations involving $\theta(t)$ and $\phi(t)$ [25]. The quadratic approximation is justified as long as the density of excited spin waves is small, i.e., $\epsilon(t) \ll 1$. For a quench starting from the spin coherent state fully polarized in the \hat{x} direction, considered here (i.e., from $g = 0$) the initial data of Eqs. (3) are $\theta(0) = \pi/2$, $\phi(0) = 0$ with $\Delta_k^{qq}(0) = \Delta_k^{pp}(0) = 1/2$, and $\Delta_k^{qp}(0) = 0$ for $k \neq 0$; in particular, $\epsilon(0) = 0$ (note that at time $t = 0$ the mobile \hat{Z} axis is aligned with the fixed \hat{x} direction). Equation (3) includes the feedback terms $\delta^{\alpha\beta}(t)$ from quantum fluctuations, which both “dress” the value of $\bar{\lambda}$ and generate new terms of pure quantum origin, in addition to the classical mean-field dynamics corresponding to $J = 0$ in Eq. (3).

Nonequilibrium quantum phase diagram.—Via a joint numerical integration of Eq. (3) and of the evolution equations [see Eqs. (26) in Ref. [25]] of $\Delta_k^{\alpha\beta}$ in Eq. (4), for a range of postquench values of g and J , we obtained the dynamical “phase” diagram portrayed in Fig. 1. In particular, for each integration, we compute the direction $(\theta(t), \phi(t))$ of the total spin \vec{S} and the density $\epsilon(t)$ of spin waves, verifying that the latter always settles around a small value at long times within the range of parameters considered here. Then, we compute the long-time average \bar{S}_x of $S_x(t)$ and color the corresponding point in light yellow if $\bar{S}_x > 0$, in orange if $\bar{S}_x = 0$, and in blue if $\bar{S}_x < 0$. The results of this procedure, in Fig. 1, show that the two dynamical ferromagnetic and paramagnetic phases present for $J = 0$ survive at $J > 0$: for $g \ll \bar{\lambda}$ the order parameter has nonzero time average (its value being perturbatively close to the mean-field one), while for $g \gg \bar{\lambda}$, it vanishes for all of the values of J within the range considered here. In particular, the persistent oscillations of S_x , characteristic of the mean-field solution, are not wiped out by the spin wave bath, which does not produce a significant noise during the prethermal stage of dynamics and leaves the overall motion perturbatively close to perfect coherence for all observation times.

Near the dynamical transition point $g \simeq \bar{\lambda}$, the system becomes extremely sensitive to the nonequilibrium quantum fluctuations, realizing a peculiar intermediate phase, the existence of which is intimately related to the conservation of the energy. In a typical point of this region, the dynamics of S_x is characterized by the processes illustrated by the inset of Fig. 1 and by Fig. 2: the decay from a transient paramagnetic behavior to one of the two ferromagnetic sectors, possibly followed by one or more hops between them. Typically, after an initial transient with the energy of the macroscopic total spin slightly above the barrier $\mathcal{U}(0)$ separating the two ferromagnetic wells of the effective potential \mathcal{U} , the production of spin waves causes the dynamics to get trapped within one of the two. The system thus shows ferromagnetic order at long times, though it might occasionally hop to the opposite well, assisted by the absorption of energy from the spin waves. The asymptotic sign of $S_x(t)$, and hence of \bar{S}_x , sensitively depends on the specific values of the parameters in a large part of this novel ferromagnetic region (indicated by C in Fig. 1), implying a collective chaotic character of the dynamics within it, as illustrated in Fig. 2. Unlike the quantum critical cone emanating from equilibrium quantum critical points at finite temperature [27], the boundaries of region C are expected to

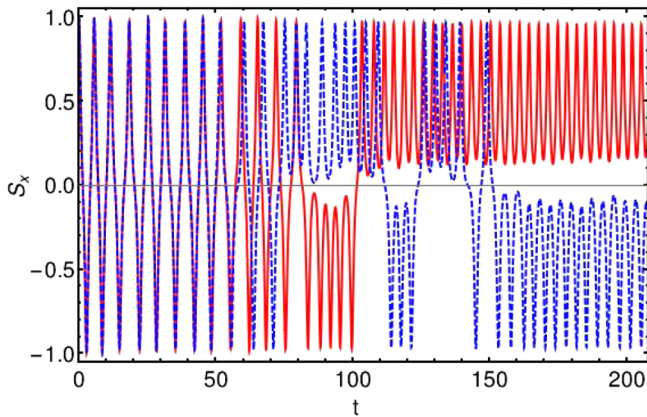


FIG. 2. The evolution of the order parameter $S_x(t)$ in the chaotic dynamical ferromagnetic phase (indicated by C in Fig. 1) for $\bar{\lambda} \equiv \lambda + J = 1$, $g = 1.03$, with $J = 0.1$ (solid red) and $J = 0.1001$ (dashed blue), i.e., two very close points in the non-equilibrium phase diagram, located at the ending point of the black arrow in Fig. 1 (here $N = 200$). The dynamical order parameter $S_x(t)$ initially displays a paramagnetic behavior, with a gradual loss of energy in favor of the creation of spin waves, witnessed by a growth of $\epsilon(t)$. This makes the orbit fall into one of the two ferromagnetic wells, corresponding to process (a) of Fig. 1. However, it might later reabsorb some spin waves and hop to the opposite ferromagnetic sector, corresponding to process (b) of Fig. 1. The two lines are practically on top of each other during the initial paramagnetic transient, but show completely different fates at the onset of the critical process (a) and they eventually end up in distinct wells. [In both cases $\epsilon(t)$ grows from $\epsilon(t=0) = 0$ to values around 0.04 in the final stage.] Such extreme sensitivity illustrates the “mosaic” appearance of the region C in Fig. 1.

be sharp both towards the ferromagnetic and the paramagnetic phases, signaling two transitions expected to be characterized by diverging time scales.

The dynamical phases discussed here turn out to be robust against the perturbation of J , even for values $J \simeq 0.67$, at which the low-density spin wave expansion need not be accurate. To show this, we performed numerical simulations by using a time-dependent variational principle on the matrix product state manifold (MPS-TDVP) [23,24], resulting in the evolution reported in Fig. 3; this approach allows us to explore the dynamics of $S_x(t)$ up to times of the order $\sim 60\bar{\lambda}^{-1}$. For $g \lesssim \bar{\lambda}$ we find a ferromagnetic region with $\bar{S}_x \neq 0$ of the same sign as the initial magnetization $S_x(t=0)$. For large values of $g \gtrsim \bar{\lambda}$, instead, we find a paramagnetic phase with $\bar{S}_x = 0$ [25], while for intermediate values, \bar{S}_x does not vanish, but it may have a sign opposite to that of $S_x(0)$; this observation is consistent with what observed at smaller values of J , see Fig. 3. In addition, in this regime, the final value of \bar{S}_x may sensibly depend on the system size N : For $N \approx 100$ (see caption of Fig. 3), we observe $\bar{S}_x \neq 0$ of the same sign as $S_x(t=0)$, while for a slightly larger system $N = 125$, $S_x(t)$ at long times has the opposite sign, which is eventually observed also in a system with $N = 400$, as shown by the inset of Fig. 3. This is consistent with the sensitivity to the parameters predicted by the spin wave approach, see Fig. 1. These numerical simulations of the exact quantum evolution fully confirm—and even extend to a larger region of the phase diagram—the scenario outlined by the time-dependent spin wave theory, i.e., the robustness of the two dynamical phases and the emergence of a chaotic region in between.

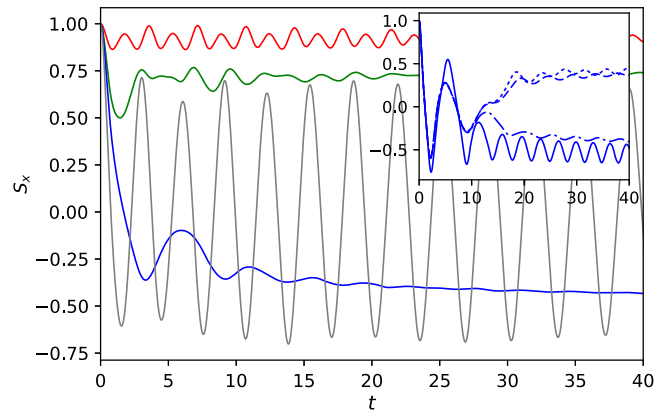


FIG. 3. The evolution of S_x for $J = 0.67$, $g = 0.5, 0.83, 1, 1.33$ (red, green, blue, gray), with $\bar{\lambda} = 1$ and $N = 400$, as obtained from MPS-TDVP simulations. Inset: The sensitivity of $S_x(t)$ to the system size N in the chaotic dynamical ferromagnetic phase, for a system with $J = 0.5$, $\bar{\lambda} = 1$, $g = 1.1$, and a bond dimension $D = 128$. S_x approaches a positive value for small $N = 123, 124$ (dashed, dotted). However, upon adding just one spin ($N = 125$, dash-dotted), S_x reverses its sign, and \bar{S}_x converges to a negative value, which is also observed in larger systems with $N = 400$ (solid line). For further details see [25].

Perspectives.—In summary, the nonequilibrium quantum fluctuations due to spin wave excitations qualitatively modify the mean-field phase diagram, turning the $J = 0$ quantum critical point into a phase with unusual dynamical properties. The nonequilibrium spin wave theory at the core of this work can be straightforwardly extended to a wide variety of spin systems, in higher dimensions, with other types of integrability breaking terms (of short or long-range character) or nonequilibrium protocols: a chaotic dynamical phase always arises whenever a mean-field system undergoing a ferromagnetic transition is subject to the impact of out-of-equilibrium quantum fluctuations [28]. In addition, the phenomena discussed here could be within experimental reach, considering recent progress in realizing spin models [29] as well as in highlighting universal scaling behavior close to dynamical critical points using cold gases [30].

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