Dynamical Quantum Phase Transitions in Spin Chains with Long-Range Interactions: Merging Different Concepts of Nonequilibrium Criticality

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We theoretically study the dynamics of a transverse-field Ising chain with power-law decaying interactions characterized by an exponent α, which can be experimentally realized in ion traps. We focus on two classes of emergent dynamical critical phenomena following a quantum quench from a ferromagnetic initial state: The first one manifests in the time-averaged order parameter, which vanishes at a critical transverse field. We argue that such a transition occurs only for long-range interactions $\alpha \leq 2$. The second class corresponds to the emergence of time-periodic singularities in the return probability to the ground-state manifold which is obtained for all values of α and agrees with the order parameter transition for $\alpha \leq 2$. We characterize how the two classes of nonequilibrium criticality correspond to each other and give a physical interpretation based on the symmetry of the time-evolved quantum states.

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Recent experiments with cold atoms [1–9] and trapped ions [10–13] have realized nonequilibrium quantum states with exotic properties that cannot be captured by a thermodynamic equilibrium description. This includes the observation of prethermalization [1–3,11] and many-body localization [5–10]. Despite these remarkable discoveries, it is still a major challenge to reveal universal properties of nonequilibrium quantum states. One possible approach for developing a general understanding of far-from-equilibrium dynamics is to explore concepts of nonequilibrium critical phenomena. However, due to the lack of clear generic principles, different concepts of dynamical criticality have been introduced [14–22].

In this work, we show that two seemingly unrelated nonequilibrium critical phenomena are actually intimately connected. In particular, the first class of nonequilibrium criticality describes dynamical quantum phase transitions (DQPTs) in the asymptotic late-time steady state of an order parameter (DQPT-OP) that is finite in one dynamical phase but vanishes in the other [14,19]. The second class is DQPTs associated with singular behavior in the transient real-time evolution of Loschmidt echoes (DQPT-LO) [22,23]. By studying the quantum dynamics of an initially fully polarized state in a transverse-field Ising chain with power-law decaying interactions, we show that these two types of DQPTs are related in several ways: First, they predict consistent values for the dynamical critical point; see Fig. 1. Second, the singularities in the Loschmidt echo are related to 0's in the time evolution of the order parameter. Third, we argue that the dynamics restores the symmetry breaking imprinted by the initial polarized state only when crossing the DQPT-LO but ceases to do so for quenches within the same dynamical phase.

Model and protocol.—Long-range systems exhibit many interesting properties that have been extensively studied in

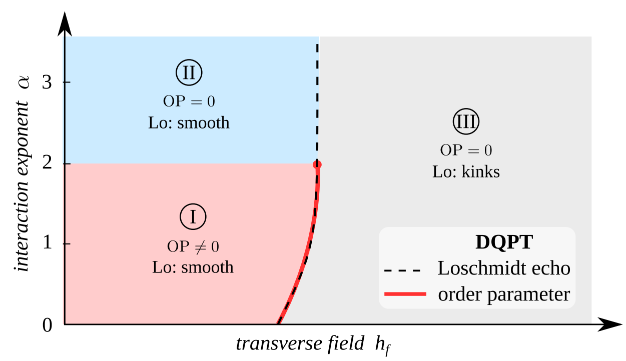

FIG. 1. wDynamical phase diagram. We study the quantum dynamics of an Ising chain with power-law decaying interactions by preparing the system in a fully polarized state and abruptly switching on a finite transverse field h_f. We identify the DQPT through two mechanisms: One introduces an order parameter (DQPT-OP) that is finite only in the dynamical ferromagnetic phase, whereas the other is based on nonanalytic kinks in the Loschmidt rate function (DQPT-LO) that only arise for quenches across the transition. When the interaction exponent $\alpha < 2$, the DQPT occurs simultaneously for both cases along the red line, separating the dynamical ferromagnetic (I) and the dynamical paramagnetic (III) phase. We argue that for $\alpha > 2$ the system does not establish a finite order parameter and thus the DQPT-OP ends at $\alpha = 2$. Yet, the DQPT-LO persists for arbitrarily large α (dashed line) separating two dynamical phases characterized by a monotonic decay (II) and an oscillating decay (III) of the magnetization.

the past [24–31]. Experimentally, the real-time dynamics of long-range interacting spin chains in a transverse field can be explored with trapped ions [10,12,32–35] where power-law decaying interactions between the effective spins with exponents $0 \leq \alpha \leq 3$ are mediated by collective vibrations of the underlying ionic crystal [36]. The corresponding Hamiltonian is

$$\hat{H}(h) = -\sum_{i \neq j=1}^{N} V(i-j)\sigma_i^x \sigma_j^x - h\sum_{j=1}^{N} \sigma_j^z, \tag{1}$$

with the transverse field h and the interaction potential $V(x) = Jv(x)/N(\alpha)$. Here, $v(x) = |x|^{-\alpha}$ describes the power-law decaying interactions and J sets the interaction strength. We added a normalization constant $N(\alpha) = [1/(N-1)]\sum_{i\neq j=1}^{N} v(i-j)$ that ensures the intensive scaling of the energy density for any α. For all values of α, this model is known to display an equilibrium quantum phase transition from a ferromagnet to a paramagnet. At finite temperatures, the equilibrium ferromagnetic phase is in one dimension only stable for $\alpha \leq 2$ [37,38].

We are studying the quantum dynamics following a global quantum quench in the transverse field h. To this end, we initially prepare the system in the fully polarized state $|+\rangle = |\rightarrow \ldots \rightarrow\rangle$ and then monitor the ensuing real-time dynamics governed by the Hamiltonian $\hat{H}(h_f)$.

Time evolution of the order parameter.—The first class of dynamical criticality, DQPT-OP, occurs in the long-time asymptotics of a dynamical order parameter, which is finite for quenches within the ordered phase $h_f < h_c$ and 0 for quenches across the dynamical transition $h_f > h_c$. For our model the order parameter is the time-averaged longitudinal magnetization

$$\overline{\sigma^x} = \lim_{T\to+\infty} \frac{1}{T}\int_0^T dt\sigma^x(t), \tag{2}$$

with $\sigma^\beta(t) = \langle S^\beta(t)\rangle$ $(\beta = x,\ y,\ z)$ and $S^\beta = 1/N\sum_i \sigma_i^\beta$ denoting the collective spin operators.

DQPT-OPs have been studied extensively in various integrable quantum many-body systems displaying nonthermal long-time dynamics, such as BCS models [14], models with infinite-range interactions [19,39–44], and field theories in high dimensions [44,45]. An analytically tractable regime of our model is the infinite-range limit, $\alpha = 0$. There, the dynamics of the order parameter corresponds to the precession of a single collective spin S^β, implying that $\sigma^x(t)$ oscillates persistently in time with a single frequency around a mean value set by $\overline{\sigma^x}$. Initializing the system in the ferromagnetic ground state, $h_i < h_c$, a DQPT-OP can occur, characterized by an order parameter $\overline{\sigma^x}$ that remains finite for $h_f < h_c$ but is 0 for $h_f > h_c$. For $\alpha = 0$, the critical value of the field h can be computed analytically, $h_c = J + h_i/2$ [43,46].

For $\alpha > 0$ the dynamics is not exactly solvable. Therefore, we compute the time evolution numerically using a recently developed algorithm based on a time-dependent variational principle applied to matrix product states [47,48]. All presented data are evaluated for bond dimension 100 and time step $0.02/J$. We demonstrate the convergence of our data with the bond dimension in Supplemental Material [49].

For sufficiently small exponent α, we find that the order parameter $\sigma^x(t)$ remains finite within the dynamical ferromagnetic phase $h_f < h_c$, whereas it approaches 0 with damped oscillations within the dynamical paramagnetic phase $h_f > h_c$; see Fig. 2(a) for $\alpha = 1.5$. In this regime, we find that the order parameter increases with system size for $h_f < h_c$ supporting its robustness in the thermodynamic limit. When increasing α further $\sigma^x(t)$ relaxes to 0 regardless of the final transverse field h_f; see Fig. 2(b) for $\alpha = 3$. In that regime the order parameter decays with increasing system size, indicating its approach to 0 in the thermodynamic limit.

Dynamical transition in the order parameter.—In the following we focus on the order parameter $\overline{\sigma^x}$ obtained

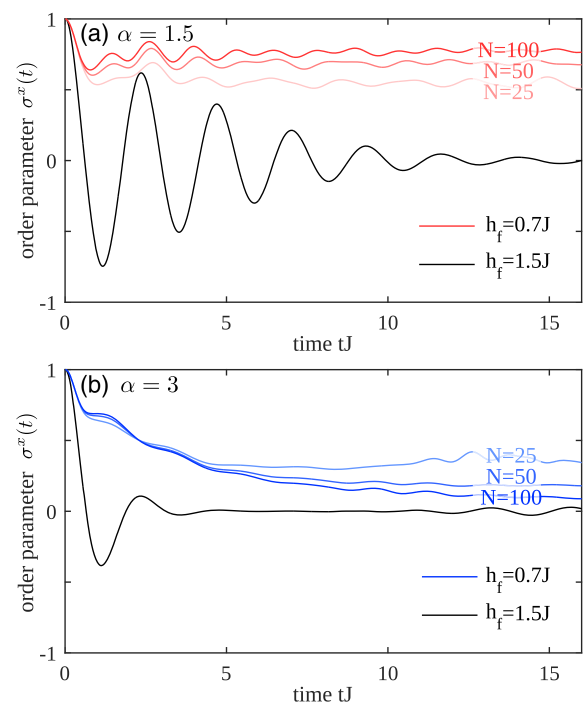

FIG. 2. Time evolving the order parameter. (a) For $\alpha = 1.5$ and $h_f = 0.7J$ the order parameter $\sigma^x(t)$ approaches a finite value, describing a dynamical symmetry-broken state with ferromagnetic order, whereas it decays to 0 with strong oscillations for quenches to $h_f = 1.5J$. (b) Even though for shorter-ranged interactions $\alpha = 3$ the order parameter reaches 0 for all values of h_f, the nature of the decay is very different: For small fields $h_f = 0.7J$ it decays with a timescale much longer than the microscopic scales, whereas for large fields $h_f = 1.5J$ it oscillates around 0 and decays rapidly.

from our numerical data by integrating $\sigma^x(t)$ over a time window of $5/J$ around half of the first finite-size recurrence time, which scales approximately linearly with system size [49]. The order parameter $\overline{\sigma^x}$ displays the same qualitative crossover in finite-size systems: a monotonic decrease of $\overline{\sigma^x}$ from 1 to 0 as the final transverse field h_f is increased; see Fig. 3. Analyzing the finite-size flow we, however, observe a markedly different behavior when tuning the value of α. At small α and moderate fields $h_f \lesssim J$, Figs. 3(a) and 3(b), our numerics indicate that $\overline{\sigma^x}$ increases with system size N and the finite-size flow suggests a critical point close to $h_c \approx J$ in the thermodynamic limit. By contrast, for large α, Fig. 3(c), the order parameter $\overline{\sigma^x}$ rapidly vanishes with increasing system size, suggesting the absence of a transition. A finite-size scaling analysis of the dynamical transition would require a two-parameter scaling, which depends both on the system size and on time. In addition the dynamical critical point is not known for our system. Because of our limited amount of data, such an analysis is therefore not feasible. However, it would be a direction for future research.

One might expect that our model is not integrable and hence thermalizes for any $\alpha \in (0, \infty)$, which would turn the DQPT-OP into a conventional thermal transition. We argue that this is not the case. To this end, we first consider the integrable point $\alpha = 0$. In that limit, we find that the thermal transition occurs at $h_c^{\mathrm{th}} = \sqrt{2}J$, which is significantly larger than the critical field of the dynamical transition $h_c = J$ [49]. For small $\alpha = 0.1$ our numerical data suggest that the DQPT is also located at $h_c \approx J$; see Fig. 3(a). Assuming now that the critical field of the thermal transition changes only perturbatively for weak α, our numerical evidence for $h_c \approx J$ is inconsistent with the transition being thermal also for $\alpha = 0.1$. Determining the value of α at which the system starts to thermalize remains a challenging open question.

In the limit of the nearest-neighbor Ising model, $\alpha = \infty$, it is well established that $\overline{\sigma^x} = 0$ for all $h_f > 0$ [50]. Constructing perturbatively a generalized Gibbs ensemble around this point [51] suggests the absence of a steady-state transition also in the vicinity of $\alpha = \infty$ [49]. Our numerics provides strong evidence that the dynamical phase transition is absent for all $\alpha > 2$. We estimate $\alpha = 2$ as an upper bound for the DQPT-OP, in analogy to the equilibrium finite-temperature transition that can only occur for $\alpha < 2$. However, we also find a crossover region $2 \leq \alpha \lesssim 2.4$ where the finite-size flow of our simulations is not fully indicative [49].

In a recent work, a different interpretation has been proposed for $\alpha > 2$: Based on extrapolating transient dynamics ($tJ < 10$) of the order parameter to infinite times, a prethermal ordered phase has been conjectured to exist for all values of α [52]. This approach is, however, inconsistent with the arguments for the absence of a DPQT-OP near $\alpha = \infty$ and might be explained by the fact that the error of the extrapolation procedure cannot be estimated in an unbiased way, unless the functional form of the decay (or at least the timescales involved) is known.

Dynamical transition in the Loschmidt echo.—The second class of dynamical transitions we consider is DQPT-LOs, which arise as singularities in Loschmidt amplitudes $\mathcal{G}(t) = \langle\Psi| \exp[-iH(h_f)t]|\Psi\rangle$ as a function of time [22], where $|\Psi\rangle$ denotes the initial state. Formally, Loschmidt amplitudes at imaginary times resemble equilibrium boundary partition functions [22,53,54]. Therefore, it is suitable to introduce a dynamical counterpart of the free energy density, which is the Loschmidt rate function (or large deviation function [54]) $g(t) = -N^{-1} \log[\mathcal{G}(t)]$. Similarly to equilibrium free energies being nonanalytic at conventional phase transitions, the Loschmidt rate function $g(t)$ can display nonanalyticities, which define the DQPT-LO. Such DQPT-LOs have been studied in different models [22,23,55–70] and measured in recent experiments [67,71].

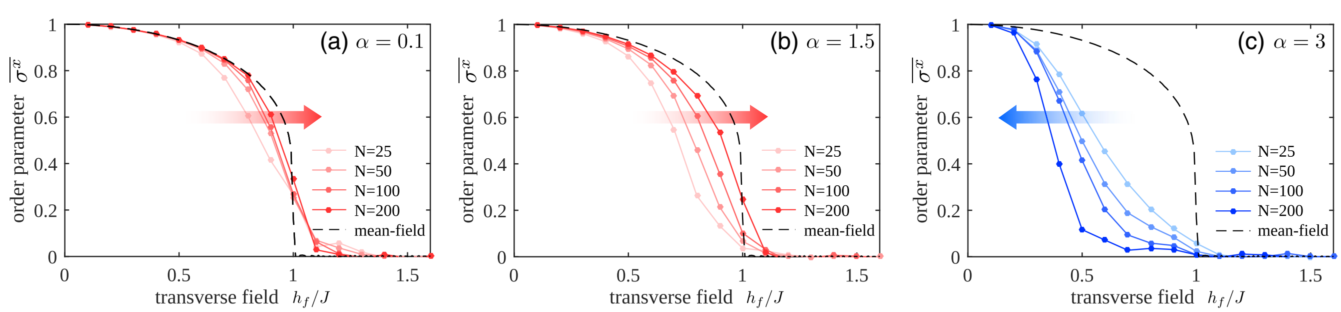

FIG. 3. Dynamical phase diagram of the order parameter. We estimate the asymptotic value of the order parameter $\bar{\sigma}^x$ as a function of the quenched transverse field h_f for different system sizes N and interaction exponents (a) $\alpha = 0.1$, (b) $\alpha = 1.5$, and (c) $\alpha = 3$. For both values of $\alpha < 2$ we find that the finite-size flow of the order parameter indicates a DQPT-OP with the critical point $h_c \sim J$. For very long-ranged interactions, (a), the order parameter $\overline{\sigma^x}$ approaches the mean-field predictions, $\alpha = 0$ (dashed lines), with increasing system size. By contrast, for relatively short-ranged interactions (c) $\alpha = 3$, the finite-size flow suggests that the order parameter $\bar{\sigma}^x$ flows toward 0 in the thermodynamic limit for all values of the transverse field.

The Loschmidt amplitude is not uniquely defined when the ground-state manifold of the initial Hamiltonian is degenerate. In order to maintain the connection of DQPT-LOs to macroscopic observables and therefore potentially to DQPT-OPs, the proper generalization is the probability to stay in the ground-state manifold [23],

$$P(t) = \sum_n |\langle \Psi_n(h_i)| e^{-iH(h_f)t} |\Psi_0(h_i)\rangle|^2, \qquad (3)$$

which reduces to the Loschmidt echo $\mathcal{L}(t) = |\mathcal{G}(t)|^2$ in the limit of a single ground state. Here, $\{|\Psi_n(h_i)\rangle\}$ denotes the degenerate states at h_i and $|\Psi_0(h_i)\rangle$ is the chosen initial condition. In our case we have that $|\Psi_0(h_i)\rangle = |+\rangle$ and $|\Psi_1(h_i)\rangle = |-\rangle = |\leftarrow \ldots \leftarrow\rangle$. Consequently, we obtain $P(t) = P_+(t) + P_-(t)$ with $P_+(t) = |\langle + | + (t)\rangle|^2$ and $P_-(t) = |\langle - | + (t)\rangle|^2$. After our work appeared, the Loschmidt echo $\mathcal{L}(t)$ itself was also computed for the long-range Ising model [72–74].

Merging the different concepts of DQPT.—Let us now establish the connection between the two concepts of dynamical criticality. For this purpose we first consider the limit of $\alpha = 0$ where the dynamics is described by semiclassical Bloch equations for the collective spin $\vec{\sigma}(t) = \{\sigma^x(t), \sigma^y(t), \sigma^z(t)\}$. In that case, the individual probabilities $P_\pm(t) = \exp[-N\lambda_\pm(t)]$ with $\lambda_\pm(t) = -\log[(1 \pm \vec{\sigma}(t)\cdot\vec{\sigma}(0))/2]$ exhibit a particularly illustrative form: for the fully polarized state, $\vec{\sigma}(t)\cdot\vec{\sigma}(0)$ measures the projection of $\vec{\sigma}(t)$ onto the x axis [75].

DQPT-LOs can occur in $P(t)$ because the individual probabilities $P_\pm(t) = \exp[-N\lambda_\pm(t)]$ show an exponential dependence on system size N. Therefore, in the thermodynamic limit only one of the two dominates such that $P(t) = \exp[-N\lambda(t)]$ with $\lambda(t) = \min_{\eta=\pm}\lambda_\eta(t)$ [23]. While at short times $\lambda(t) = \lambda_+(t)$ due to the initial condition, $\lambda_-(t)$ can take over at a critical time, which leads to a kink in $\lambda(t)$. For the concrete case of $\alpha = 0$ this can be traced back to $\vec{\sigma}(t)$ crossing the equator of the Bloch sphere, $\sigma^x = 0$, because then $\lambda_+ = \lambda_-$. As we have seen in Fig. 2, this can happen only when the DQPT-OP is crossed, i.e., for $h_f > h_c$. Therefore, a DQPT-LO occurs only when crossing the DQPT-OP, which manifests itself in a vanishing long-time magnetization $\bar{\sigma^x} = 0$. In this way the $\mathbb{Z}_2$ symmetry, broken explicitly by the initial state, is restored in the long-time limit as well as at the critical times at which the DQPT-LO occur.

Although these considerations address a fine-tuned limit of $\alpha = 0$, we show in Fig. 4 based on our numerical data that the relation between DQPT-LO and DQPT-OP is robust and extends to $\alpha > 0$. DQPT-LOs in the form of kinks occur whenever the system is quenched sufficiently strongly such that $h_f > h_c$ whereas for $h_f < h_c$ the rate function $\lambda(t)$ stays smooth. In addition we compare the period of the kinks (τ_{LE}) with the period of the 0's of $\sigma^x(t)$ (τ_{OP}), which are obtained by the distance between successive kinks in $\lambda(t)$ and successive 0's in $\sigma_x(t)$, respectively. Specifically, we plot in the insets of Fig. 4 the inverse of this timescale and find within the error bars, which denote the standard error of the mean, good agreement over a wide range of h_f, supporting the close connection of $\sigma^x(t)$ and DQPT-LOs for generic values of α. The precise location of the 0's in $\sigma^x(t)$ exhibits a small, essentially constant shift compared to the kinks in $\lambda(t)$ [22,23]. This is illustrated in the Bloch spheres of Fig. 4, where the kinks (black dots) appear slightly later in time than the zero crossings of the order parameter $\sigma^x = 0$. Moreover, we emphasize that the connection between DQPT-LOs and the 0's of $\sigma^x(t)$ is also valid for $\alpha > 2$ where no DQPT-OP occurs. The field h_c marking the appearance of DQPT-LOs for $h_f > h_c$ then separates a regime of monotonic decay of σ^x for $h_f < h_c$ from oscillatory decay for $h_f > h_c$; see Fig. 4(b).

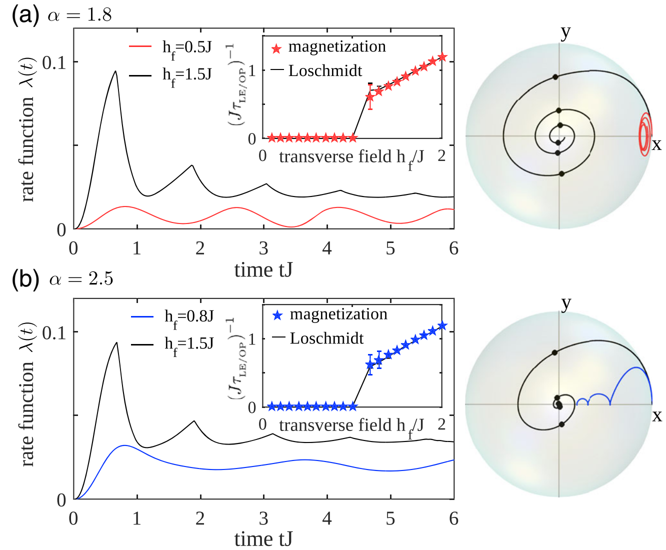

FIG. 4. Dynamical quantum phase transitions in the Loschmidt echo. We compute an extension of the Loschmidt echo, which is the return probability to the degenerate ground-state manifold, Eq. (3), for different values of the transverse field h_f and interaction exponent (a) $\alpha = 1.8$ and (b) $\alpha = 2.5$. We observe nonanalyticities in the associated rate function $\lambda(t)$ for arbitrary values of the interaction exponent α, provided the final transverse field h_f is sufficiently large. The insets compare the typical rate of kinks in $\lambda(t)$, solid line, with the zero crossings of the order parameter $\sigma^x(t)$. The right panels show the evolution of the magnetization $\vec{\sigma}(t)$ projected onto the xy plane of the Bloch sphere. When quenching across the dynamical transition, the magnetization spreads over both hemispheres (black curves) whereas it remains located on one hemisphere for quenches within the same dynamical phase (blue and red curves), indicating a bifurcation of the dynamics.

Conclusions and outlook.—We have studied dynamical quantum phase transition in a transverse-field Ising chain with power-law decaying interactions. We have argued that two seemingly different concepts of nonequilibrium criticality, specifically dynamical transitions in the order parameter and dynamical transitions in the Loschmidt echo, are actually intimately related in the following ways. (i) We find that both of them predict consistent values for the dynamical critical point for interaction exponent $\alpha < 2$. (ii) For generic values of α, the period of kinks in the Loschmidt rate function agrees with the period of 0's in the order parameter. (iii) The order parameter restores symmetry imprinted by the initial polarized state, only for quenches across the dynamical quantum phase transition, but ceases to do so for quenches within the same dynamical phase.

In future studies, it would be interesting to extract the dynamical critical exponents of the order parameter. Furthermore, studying in detail the scaling of order parameter fluctuations with system size could establish for which values of the interaction exponent our system is thermalizing according to the eigenstate thermalization hypothesis.

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Note added.—During the review process of our work, recent experiments observed some of our findings on DQPT in the order parameter [13] and the Loschmidt echo [71].

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