Gravitational-Wave Luminosity of Binary Neutron Stars Mergers

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We study the gravitational-wave peak luminosity and radiated energy of quasicircular neutron star mergers using a large sample of numerical relativity simulations with different binary parameters and input physics. The peak luminosity for all the binaries can be described in terms of the mass ratio and of the leading-order post-Newtonian tidal parameter solely. The mergers resulting in a prompt collapse to black hole have the largest peak luminosities. However, the largest amount of energy per unit mass is radiated by mergers that produce a hypermassive neutron star or a massive neutron star remnant. We quantify the gravitational-wave luminosity of binary neutron star merger events, and set upper limits on the radiated energy and the remnant angular momentum from these events. We find that there is an empirical universal relation connecting the total gravitational radiation and the angular momentum of the remnant. Our results constrain the final spin of the remnant black hole and also indicate that stable neutron star remnant forms with super-Keplerian angular momentum.

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Gravitational waves (GWs) consistent with the inspiral of a binary neutron star system (BNS) have been observed for the first time on August 17th 2017 during the second observing run of Advanced LIGO and Virgo [1]. The observation sets a lower bound to the total radiated energy, $E_{\rm GW} > 0.025~M_{\odot}c^2$, by considering only a portion of the GW signal corresponding to the inspiral dynamics. The largest GW energy, however, is expected to be radiated during the merger and the subsequent postmerger phases [2,3]; those phases occur at higher GW frequencies, at which the instruments are less sensitive. The only way to theoretically quantify the emitted GW energy is to perform numerical relativity (NR) simulations. NR-based models can be then evaluated on the intrinsic parameters of the binary esimated from the observations to obtain the emitted energy. In this work we study the GW peak luminosity and GW energy emitted by quasicircular BNS mergers using one of the largest sets of NR simulations currently available [2–13].

Compact binary mergers are the most powerful events in the Universe in terms of GW energy. The binary black hole (BBH) mergers observed so far emitted about 1–3 $M_{\odot}c^2$ with peak luminosities reaching 200 $M_{\odot}c^2$ s⁻¹ (about ~3–4 × 10⁵⁶ erg s⁻¹) [14–16]. The largest luminosity is reached for an equal mass and aligned spin configuration, with both holes spinning at maximum rate. Physically, spin-orbit interactions during the dynamics enhance the emission for aligned spin configurations. Fits to the BBH

luminosity and radiated energy as function of mass ratio and spins have been developed in a number of NR-based works, e.g., Refs. [17–20]. By contrast, the total radiated GW energy of BNS systems has been quantified only for particular cases, e.g., Refs. [3,9,10,21], and quantitative models for predicting the properties of the merger remnant are missing.

We consider 100 different BNS simulations that include variation of the gravitational binary mass $M = M_A +$ $M_B \in [2.4, 3.4] M_{\odot}$, the mass ratio $q = M_A/M_B \in$ [1, 2.06], and a sample of 8 equations of state (EOSs) comprising 4 finite-temperature microphysical EOS models. Spin interactions in about 30 BNS are simulated consistently in general relativity following Ref. [22]. Spins are either aligned or antialigned to the orbital angular momentum; the magnitude of each star's spin varies up to $|\mathbf{S}_A|/M^2 \approx 0.15$. A microphysical treatment of neutrino cooling is included in 37 simulations, following the method presented in Ref. [23]. Four simulations also included an effective treatment of turbulent angular momentum transport that may arise from small scale magnetohydrodynamical instabilities in the merger remnant [11]. Most of the BNSs are simulated at multiple grid resolutions for a total of more than 200 data sets that guarantee control on numerical artifacts. Simulations are performed with the BAM [24] and THC codes [25]. Full details on the data are given elsewhere [26]. The GW energy $E_{\rm GW}$ and the binary's angular momentum J are calculated from our simulations

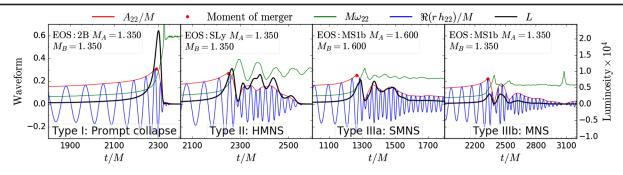


FIG. 1. Merger waveforms and GW luminosity for the three types of mergers. From left to right the GW correspond to a merger ending in a prompt collapse to black hole (type I), a hypermassive neutron star (type II), a supramassive neutron star (type IIIa), and a massive stable neutron star (type IIIb). Note the double y axis.

from the GW multipolar waveform, as described in Refs. [2,27]. We work with mass and symmetric mass ratio, $\nu = M_A M_B/M^2$, rescaled quantities, $e_{\rm GW} = E_{\rm GW}/(M\nu)$ and $j = J/(M^2\nu)$. The luminosity peak is computed as $L_{\rm peak} = \max_t \{dE_{\rm GW}(t)/dt\}$. Note that, differently from BBH, the BNS luminosity does depend on the binary mass due to tidal interactions during the dynamics (see below). The conversion factor from geometric units $G = c = M_{\odot} = 1$ used here to CGS units is the Planck luminosity

$$L_P = \frac{c^5}{G} \approx 3.63 \times 10^{59} \text{ erg s}^{-1};$$
 (1)

the typical order of magnitude of $L_{\rm peak}$ for compact binary mergers is $10^{-3}-10^{-4}L_P$.

The BNS merger dynamics is crucially determined by tidal interactions [2,3]. Reference [2] has shown that $e_{\rm GW}$, j, and many other key quantities at the moment of merger (The moment of merger is formally defined as the time of the waveform amplitude's peak, which corresponds to the end of the chirp signal.) can be fully characterized by the sum

$$\kappa_2^T = \kappa_2^A + \kappa_2^B, \tag{2}$$

of the gravitoelectric quadrupolar tidal polarizability coefficients [28]

$$\kappa_2^A = 2\frac{X_B}{X_A} \left(\frac{X_A}{C_A}\right)^5 k_2^A. \tag{3}$$

Above, k_2^A is the quadrupolar Love number describing the static quadrupolar deformation of body A in the gravito-electric field of the companion, C_A is the compactness, and $X_A = M_A/M$. The coefficient κ_2^T parametrizes at leading order the tidal interactions in the general-relativistic two-body Hamiltonian, waveform phase, and amplitude [29]. Larger energy emissions correspond to smaller values of κ_2^T , which, in turn, are favored by larger mass values, more compact NSs, and softer EOS. In what follows we show that a similar characterization holds also for the peak luminosity.

The possible outcomes of a BNS merger are a prompt collapse to black hole (type I), a hypermassive NS (HMNS, type II), a supramassive NS (SMNS, type IIIa), or a stable NS (MNS, type IIIb) [30–32]. We find that the GW peak luminosity is reached during merger and the subsequent dynamical phase and it strongly depends on the merger type. For type I mergers the luminosity peak just follows the moment of merger, similarly to the BBH case. Type II mergers have multiple peaks of comparable luminosity on a time scale of $\mathcal{O}(100M)$ (few ms). The peaks following the moment of merger are related to the HMNS emission and can be of comparable or stronger magnitude. Type III mergers are qualitatively similar to type II, but the peak luminosities are lower. Four representative simulations are presented in Fig. 1.

The BNS peak luminosity can be characterized by a simple function of the tidal polarizability coefficients, Eq. (3). In the post-Newtonian (PN) description of the inspiral dynamics, tidal effects contribute to the luminosity with a leading order 5PN term $\delta L_{\rm Tidal} = \frac{32}{5} \nu^2 x^{10} \kappa_2^L$ [33], where $x = (\pi M f_{\rm GW})^{2/3}$ is the PN expansion parameter, $f_{\rm GW}$ is the GW frequency and

$$\kappa_2^L = 2\left(\frac{3 - 2X_A}{X_B}\kappa_2^A + (A \leftrightarrow B)\right). \tag{4}$$

The perturbative parameter κ_2^L captures the strong-field dynamics behavior for $L_{\rm peak}$ as shown in Fig. 2. Our irrotational BNS sample can be fit by (Similar results are obtained also using κ_2^T since $X_A \sim X_B \sim 1/2$.)

$$L_{\rm peak}(\nu, \kappa_2^L) \approx L_0 \frac{\nu^2}{q^2(\nu)} \frac{[1 + n_1 \kappa_2^L + n_2 (\kappa_2^L)^2]}{(1 + d_1 \kappa_2^L)}, \qquad (5)$$

with $L_0 = 2.178 \times 10^{-2}$, $n_1 = 5.2(4) \times 10^{-4}$, $n_2 = -9.3(6) \times 10^{-8}$, $d_1 = 2.7(7) \times 10^{-2}$, and a coefficient of determination $R^2 = 0.944$. The maximal residuals are of the order of 30% (with one outlier at ~39%). Note that the prediction using BBH fits would overestimate L_{peak} of, at least, a factor 4. Our fit does not make use of spinning BNS data, but it captures them. For the spin magnitudes

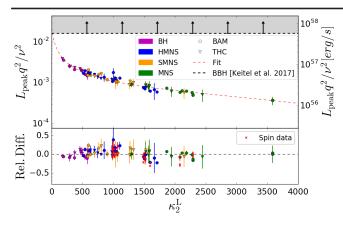


FIG. 2. GW luminosity peak as a function of the tidal parameter κ_2^L . The error bars are calculated from simulations performed at different resolutions. Second panel: fit's residuals with errors. Red crosses correspond to the residuals of the spinning data when compared to the fit.

considered here, the spin-orbit contribution to $L_{\rm peak}$ is within the fit and numerical uncertainties. As an example of application, a fiducial equal-mass BNS with $M=2.8~M_{\odot}$ and $\kappa_2^A=\kappa_2^B\sim 92~(\kappa_2^L\sim 1472)$ has $L_{\rm peak}\sim 8.168\times 10^{-4}$ ($\sim 1.852\times 10^{55}~{\rm erg~s^{-1}}$). The application of Eq. (5) to GW170817 is also straightforward and just requires evaluating the posteriors for the likely distribution of the mass ratio and the tidal parameters.

The L_{peak} analysis also highlights that the threshold between type I and type II mergers is approximately controlled by the value of κ_2^L (or κ_2^T). Prompt collapse happens above a mass threshold $M > M_{pc} = c_{pc} M_{max}^{TOV}$, where $M_{\text{max}}^{\text{TOV}}$ is the maximum gravitational mass of a nonrotating NS and $1.3 \lesssim c_{\rm pc} \lesssim 1.6$ is a constant that depends only weakly on the binary's mass ratio. Both $c_{\rm pc}$ and $M_{\rm max}^{\rm TOV}$ depend on the EOS [32,34,35]. For a given EOS, the prompt collapse threshold translates into limiting values of $\kappa_{2 \text{ pc}}^T$ (or $\kappa_{2 \text{ pc}}^L$), that can be computed by considering all the possible pairs of NS such that $M_A + M_B =$ $M_{\rm pc}$ (with 1.1 $M_{\odot} < M_{\rm A} < M_{\rm max}^{\rm TOV}$). The residual dependence on q is approximately quadratic and introduces a ≤35% increase when extremely unequal mass binaries are considered. For our set of 8 EOS we find that type I mergers are characterized by $\kappa_{2~pc}^{T} \sim 80~(\kappa_{2~pc}^{L} \sim 600)$, where the value can vary by about $\delta\kappa_{2~pc}^{T} \lesssim 40~(\delta\kappa_{2~pc}^{L} \lesssim 200)$. Such predictions are verified by our NR sample, although no common threshold can be found for all the considered EOS. Conversely, narrow, nonoverlapping bands of values of $\kappa_{2 \text{ pc}}^{T}$ and $\kappa_{2 \text{ pc}}^{L}$ are observed for most EOSs, as long as $q \lesssim 1.3$.

The most luminous BNSs do not correspond, in general, to the BNS that radiate the largest amount of energy. That is yet another difference with respect to BBH. The largest GW energies per unit mass are radiated by type II mergers over typical timescales of few tens of milliseconds after the moment of merger [36]. The remnant HMNS undergoing

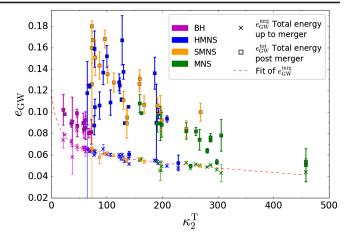


FIG. 3. Reduced GW energy at merger (times) and total (square) as a function of the tidal parameter κ_2^T .

gravitational collapse is a very efficient emitter of GWs; about twice the energy emitted during the inspiral and merger can be emitted during the postmerger phase. Figure 3 shows the total energy, $e_{\rm GW}^{\rm tot}$, and the energy radiated up to the moment of merger, $e_{\rm GW}^{\rm mig}$, as a function of κ_2^T for our irrotational BNS sample. While $e_{\rm GW}^{\rm mrg}$ tightly correlates with κ_2^T , the total energy has a more complex behavior. Our results set an upper bound of $e_{\rm GW}^{\rm tot} \lesssim 0.18$ obtained for $100 \lesssim \kappa_2^T \lesssim 200$, e.g. for the fiducial M = $2.8~M_{\odot}$ BNS discussed above. Hence, if two different BNSs with $M \sim 2.8~M_{\odot}$ and $\nu \sim 1/4$ are type I and type II, respectively (for example, if the NS matter is softer or stiffer in one case), then the former might be more luminous and the latter might emit more energy. Given similar total masses, type II mergers can have energies per unit mass larger than a factor $e_{\rm GW}^{\rm tot}({\rm type\,II})/e_{\rm GW}^{\rm tot}({\rm type\,I})\sim 1.8$ with respect to type I, and than a factor ~3 with respect to type III. However, not all type II are more energetic than type I. Sufficiently large individual NS masses in type I mergers can rescale $e_{\rm GW}^{\rm tot}$ to larger absolute energies than those of type II. Combined measurements of the luminosity and energy radiated at merger around the threshold values for prompt collapse, $\kappa_{2 pc}^T$ (or $\kappa_{2 pc}^L$), could, in principle, distinguish between different EOSs, provided that $q \lesssim 1.3$.

The largest GW energy that a BNS can emit can be inferred from our data set, we find

$$E_{\rm GW}^{\rm tot} \lesssim 0.126 \frac{M}{2.8} \ M_{\odot} c^2.$$
 (6)

Our results imply that current LIGO-Virgo GW searches at kHz frequencies are insensitive to the postmerger signal (cf. Fig. 1 of Ref. [37]).

Finally, we show that the total radiated energy uniquely determines the angular momentum of the merger remnant, cf. Fig. 4. All the BNS remnants are characterized by values that lay on a given $e_{\rm GW}^{\rm tot}(j_{\rm rem})$ curve. That happens rather independently from the binary's intrisinc parameters but

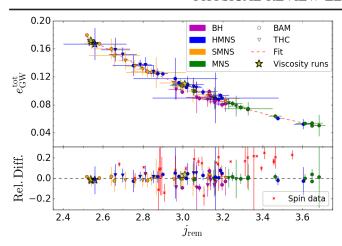


FIG. 4. Reduced total GW energy vs final angular momentum of the remnant. Simulations including magnetically driven viscosity effects [11] are marked with stars, the empty star refers to a control run with zero viscosity. Lower panel: residuals of the fit and residuals of spin data, with relative errors.

also from the particular physics simulated in the postmerger. Notably, the simulations employing viscosity and neutrino cooling (marked with stars in the plot) lay on the same curve of simulations employing a purely hydrodynamical prescription for the matter [11]. This fact suggests that the emission of gravitational radiation is the dominant mechanism determining the dynamics on the dynamical time scales after merger, $T_{\rm dyn} \sim 20$ ms. The irrotational NR data are well described by the relation

$$e_{\rm GW}^{\rm tot} \approx c_2 j_{\rm rem}^2 + c_1 j_{\rm rem} + c_0, \tag{7}$$

where $c_0 = 0.9(4)$, $c_1 = -0.4(3)$, $c_2 = 0.05(3)$, with fit residuals below 20%. Spinning data increase fit residuals to 30% with a small but systematic drift for large and aligned spin configurations.

For type I and II mergers, the final spin of the remnant black hole can be estimated from the angular momentum of the remnant system (BH or HMNS + disk) at the end of the initial, GW dominated phase. Thus, Eq. (7) could be used to estimate the final BH spin from the measurement of the energy radiated by the binary in GWs, which might be possible with third-generation GW observatories. The value of $J_{\rm rem}/M^2$ provides an upper limit for the remnant BH dimensionless spin, we predict $0.6 \lesssim J_{\rm rem}/M^2 \lesssim 0.9$ for moderately spinning BNS. Type I mergers produce the smallest disks ($\sim 10^{-3} M_{\odot}$), carrying a negligible amount of angular momentum [12,38-40]. Thus, the remnant and final BH angular momenta coincide and 0.75≲ $(J/M^2)_{\rm BH,type\,I} \lesssim 0.8$, where the fastest spinning black holes are associated with larger values of κ_2^L . For type II mergers, we estimate that a disk of baryon mass $M_{b,{
m disk}} \sim$ $0.1~M_{\odot}$ contains 10%-15% of $J_{\rm rem}$. Viscosity-driven disk ejecta can carry away a large fraction of this momentum over the disk lifetime while we evaluate

 $\Delta(J/M^2)_{
m BH,type\,II}\lesssim 0.03$ by accretion. For the final BH dimensionless spin we predict $0.6\lesssim (J/M^2)_{
m BH,type\,II}\lesssim 0.85$, where the slowest spinning BHs are produced by light, symmetric BNSs.

The dimensionless angular momentum at the end of the initial, GW dominated, phase of the postmerger evolution for type III binaries is in the range $0.62 \lesssim J_{\rm rem}/M^2 \lesssim 0.82$. We compare $J_{\rm rem}$ for each type III binary to that of sequences of uniformly rotating NSs having the same rest mass. We find that $J_{\rm rem}$ exceeds, in most cases significantly, the Keplerian limit. Type III remnants are thus super-Keplerian. This suggests that the subsequent viscous evolution is likely to be accompanied by massive outflows [13,41].

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