Analytical Computation of Energy-Energy Correlation at Next-to-Leading Order in QCD

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The energy-energy correlation (EEC) between two detectors in e^+e^- annihilation was computed analytically at leading order in QCD almost 40 years ago, and numerically at next-to-leading order (NLO) starting in the 1980s. We present the first analytical result for the EEC at NLO, which is remarkably simple, and facilitates analytical study of the perturbative structure of the EEC. We provide the expansion of the EEC in the collinear and back-to-back regions through next-to-leading power, information which should aid resummation in these regions.

Introduction.—The energy-energy correlation (EEC) [1] measures particles detected by two detectors at a fixed angular separation χ, weighted by the product of the particles' energies. The EEC is an infrared-safe characterization of hadronic energy flow in e^+e^- annihilation. It has been used for precision tests of quantum chromodynamics (QCD) and measurement of the strong coupling constant α_s [2,3]. In perturbative QCD, the EEC is defined by

$$\frac{d\Sigma}{d\cos\chi} = \sum_{i,j} \int \frac{E_i E_j}{Q^2} \delta(\vec{n}_i \cdot \vec{n}_j - \cos\chi) d\sigma, \qquad (1)$$

where i and j run over all the final-state massless partons, which have four-momenta p_i^μ and p_j^μ (including the case $i = j$ at $\chi = 0$); Q^μ is the total four-momentum of the e^+e^- collision and $d\sigma$ is the differential cross section. The three-vectors $\vec{n}_{i,j}$ point along the spatial components of $p_{i,j}$. The definition Eq. (1) implies the sum rule

$$\frac{1}{\sigma} \int_{-1}^{1} d\cos\chi \frac{d\Sigma}{d\cos\chi} = 1, \qquad (2)$$

where σ is the total cross section for e^+e^- annihilation to hadrons.

The leading order (LO) QCD prediction for the EEC has been available since the 1970s [1]:

$$\frac{1}{\sigma_0} \frac{d\Sigma}{d\cos\chi} = \frac{\alpha_s(\mu)}{2\pi} C_F \frac{3-2z}{4(1-z)z^5}$$
$$\times [3z(2-3z) + 2(2z^2 - 6z + 3)\log(1-z)]$$
$$+ \mathcal{O}(\alpha_s^2), \qquad (3)$$

where σ_0 is the Born cross section for $e^+e^- \to q\bar{q}$, C_F is the quadratic Casimir eigenvalue in the fundamental representation, and we have introduced $z = (1 - \cos\chi)/2$. The cross section is strongly peaked at $\chi = 0$ ($z = 0$) and $\chi = \pi$ ($z = 1$), regions that require resummation of logarithms due to emission of soft and collinear partons. At intermediate angles, higher-order corrections tend to flatten the distribution.

The EEC was first computed numerically at next-to-leading order (NLO) in QCD by several groups in the 1980s and 1990s, originally leading to conflicting results. Different methods were used to handle soft and collinear singularities from real radiation: phase-space slicing [4–7], subtraction methods [6,8–14], or hybrid schemes [6,7,15]. Accurate numerical NLO results are available from the program EVENT2, based on dipole subtraction [13,14]. Quite recently, the EEC has been computed at NNLO in QCD using the CoLoRFulNNLO local subtraction method [16,17].

In perturbation theory, the EEC is singular in both the collinear ($z \to 0$) and back-to-back regions ($z \to 1$), as can be seen explicitly from Eq. (3). The leading-logarithmic collinear behavior can be obtained from the "jet calculus" approach [18,19], in terms of the anomalous dimension matrix of twist-two, spin-three operators [11,19]. Resummation of the EEC in the back-to-back (Sudakov) region has been performed at next-to-leading-logarithmic (NLL) and NNLL accuracy [20–22]. Quite recently, a factorization formula for the EEC has been derived which permits its resummation to N³LL [23]. Possible nonperturbative corrections to the EEC have also been investigated [24].

In $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory, the EEC has been computed analytically at NLO in terms of classical polylogarithms [25], using an approach that bypasses the need for infrared cancellations in intermediate steps [26,27]. In the strong-coupling limit and at large N_c, the EEC in $\mathcal{N} = 4$ SYM theory can be calculated using AdS/CFT duality [28].

Despite all of this progress, the analytic computation of the EEC at NLO in QCD has remained an open problem, whose solution is desirable for several reasons. First, the analytical results can settle any remaining discrepancies between different numerical methods, and provide a benchmark for future numerical evaluations. Second, the analytical results allow extraction of the $\mathcal{O}(\alpha_s^2)$ asymptotic behavior in the collinear and back-to-back regions, not just at leading power, but any desired power. Knowledge of the subleading power corrections can be very helpful for improving the understanding of resummation at subleading power [29–38]. Third, no other event-shape variable has been computed analytically at NLO. Calculationally, the EEC appears to be the simplest such observable. Knowing it analytically at NLO marks an important step in the perturbative understanding of event-shape observables, and may pave the way for an analytic computation at NNLO. Recently, progress has been made toward computing the EEC at NLO by linearizing the measurement function [39]. In this Letter, we present the first fully analytic result for the EEC in QCD at NLO.

The calculation.—At LO, calculation of the EEC is straightforward, because only finite phase-space integrals need to be evaluated. At NLO, the renormalized virtual corrections contain explicit infrared (IR) poles, but no singularities from the boundary of phase space. We use the analytical one-loop amplitudes [40,41], and perform the phase-space integration directly. The real radiative corrections represent the most complicated part of this calculation, because the phase-space integrals contain unresolved soft and collinear IR divergences. We apply reverse unitarity [42,43] to write on-shell delta functions as differences of Feynman propagators with opposite signs for $i\varepsilon$, which allows the use of integration-by-parts (IBP) equations [44,45] for multiloop integrals. The EEC measurement function can be written in the same way,

$$\delta(\mathcal{M}_{ij}(\chi)) = \frac{1}{2\pi i}\left(\frac{1}{\mathcal{M}_{ij}(\chi) - i\varepsilon} - \frac{1}{\mathcal{M}_{ij}(\chi) + i\varepsilon}\right), \quad (4)$$

where $\mathcal{M}_{ij}(\chi) = (p_i \cdot Q p_j \cdot Q)(\vec{n}_i \cdot \vec{n}_j - \cos\chi) = (p_i \cdot Q p_j \cdot Q) \times (1 - \cos\chi) - p_i \cdot p_j$. While the application of reverse unitarity to phase-space integrals is now quite standard, Eq. (4) is special in the sense that $\mathcal{M}_{ij}(\chi)$ is a nonlinear function of Lorentz dot products. In addition to the usual IBP equations, an extra equation,

$$[(1 - \cos\chi)(p_i \cdot Q p_j \cdot Q) - p_i \cdot p_j][\delta(\mathcal{M}_{ij}(\chi))]^k$$
$$= [\delta(\mathcal{M}_{ij}(\chi))]^{k-1}, \quad (5)$$

for $k = 1, 2, \ldots$, with $[\delta(\mathcal{M}_{ij}(\chi))]^0 \equiv 0$, has to be added in order to fully reduce the phase-space integrals to master integrals (MIs).

In our calculation, we use QGRAF [46] to generate the squared amplitudes for the LO and real NLO terms. We set all quark masses to zero, and ignore contributions from the top quark, as well as the (tiny) purely axial-vector contributions in the case of e^+e^- annihilation via the Z boson. The color and Dirac algebra is evaluated using FORM [47]. The resulting tree-level matrix elements agree fully with Ref. [40]. The squared matrix elements for the NLO real corrections, ignoring the EEC measurement function, can be divided into three integrand topologies, each consisting of nine Feynman propagators (one in the numerator). Since there are four partons in the real NLO final state, there are $\binom{4}{2} = 6$ different measured pairs to sum over for the EEC. Multiplying the 3 inclusive integrand topologies by the 6 pairs of measurement delta functions gives rise to 18 separate integral topologies. We use LITERED [48,49] to generate the standard IBP equations for these integral families, and then add the additional integral relation from Eq. (5) manually. We then export the resulting IBP relations to FIRE [50,51] to perform the integral reduction, which leads to a total of 40 independent MIs.

We solve for the MIs by the method of differential equations (DEs) [52,53], and convert the DE systems into a canonical form [54]. Some of the DE systems can be converted to canonical form using the original variable z; for others, an algebraic change of variable to $x = \sqrt{z}$ or $y = i\sqrt{z}/\sqrt{1-z}$ is required. After identifying the appropriate variable for each integral family, the conversion to a canonical basis can be automated by the *Mathematica* package FUCHSIA [55]. The resulting symbol alphabet, characterizing the arguments of the polylogarithms, is $\{1 - x, y, 1 - y, 1 + y\}$. Note that z, $1 - z$, x and $1 + x$ also appear, but are not multiplicatively independent, since $1/(1 - y^2) = 1 - x^2 = 1 - z$, etc., so we do not count them as separate symbol letters. This alphabet implies that the solution to the DEs can be written fully in terms of harmonic polylogarithms (HPLs) [56], which can be manipulated conveniently using the *Mathematica* package HPL [57]. Our final NLO result contains at most weight 3 HPLs, which can all be reduced to classical polylogarithms.

The most intricate part of the calculation is the determination of the constants of integration for the DEs, which requires combining several different constraints. First, we require that the leading power expansion z^α of each MI in the collinear limit $z \to 0$ has the correct power α, which can be predicted by simple power counting. We find that all the MIs in our problem have at most a z^{-1} pole. (Some have z^0 or z^1 as their leading behavior.) Requiring the absence of

z^{-2} or worse poles strongly constrains the boundary constants. The second constraint is the $z \to -\infty$ limit: Before converting to the canonical basis, MIs that are pure functions of uniform transcendental weight should vanish in this limit. The third constraint comes from performing a weighted integration over z, which allows the removal of the measurement constraint, according to the integral relation

$$\int dPS^{(4)} \hat{z}_{ij}^n (1 - \hat{z}_{ij})^m \mathcal{I}(\{p\})$$
$$= \int_0^1 dz \, z^n (1 - z)^m \int dPS^{(4)} \mathcal{I}(\{p\})$$
$$\times 2p_i \cdot Q \, p_j \cdot Q \, \delta(\mathcal{M}_{ij}(\chi)). \quad (6)$$

Here $dPS^{(4)}$ is the four-particle Lorentz-invariant phase-space measure in D dimensions, $\hat{z}_{ij} = Q^2 p_i \cdot p_j (2p_i \cdot Q p_j \cdot Q)^{-1}$, and $\mathcal{I}(\{p\})$ denotes a MI integrand. We choose the integers n and m to be sufficiently positive that the particular integration over z converges, and $n \leq 1$, $m \leq 1$ to keep the IBP reduction tractable. The integral on the left-hand side can be reduced to known inclusive four-particle phase-space integrals [58], if we multiply the integrand on both sides by $(p_i \cdot Q p_j \cdot Q)^{n+m}$. The last constraint we apply is to demand that the full NLO real corrections, after substituting in the results for the MIs, have at most a z^{-1}

pole. This gives extra constraints, beyond the constraints applied to the individual MIs. A similar method has been applied to fix constants of integration for DEs for auxiliary EEC MIs [39].

The result.—After combining the real and virtual corrections, and adding the counterterm to renormalize α_s, we obtain our final result for the EEC at NLO. We write the differential distribution as

$$\frac{1}{\sigma_0} \frac{d\Sigma}{d\cos\chi} = \frac{\alpha_s(\mu)}{2\pi} A(z) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2$$
$$\times \left(\beta_0 \log \frac{\mu}{Q} A(z) + B(z)\right) + \mathcal{O}(\alpha_s^3), \quad (7)$$

where the LO coefficient $A(z)$ has already been given in Eq. (3), and $\beta_0 = 11 C_A/3 - 4 N_f T_f/3$. For QCD with N_f flavors of quarks, $C_A = N_c = 3$, $C_F = (N_c^2 - 1)/(2N_c) = 4/3$, and $T_f = 1/2$. The NLO coefficient $B(z)$ can be further decomposed into different color structures,

$$B = C_F^2 B_{\rm lc} + C_F(C_A - 2C_F)B_{\rm nlc} + C_F N_f T_f B_{N_f}. \quad (8)$$

We have calculated each coefficient in the color decomposition analytically. The leading-color correction $B_{\rm lc}$ reads,

$$B_{\rm lc} = + \frac{122400 z^7 - 244800 z^6 + 157060 z^5 - 31000 z^4 + 2064 z^3 + 72305 z^2 - 143577 z + 63298}{1440(1-z)z^4}$$
$$- \frac{-244800 z^9 + 673200 z^8 - 667280 z^7 + 283140 z^6 - 48122 z^5 + 2716 z^4 - 6201 z^3 + 11309 z^2 - 9329 z + 3007}{720(1-z)z^5} g_1^{(1)}$$
$$- \frac{244800 z^8 - 550800 z^7 + 422480 z^6 - 126900 z^5 + 13052 z^4 - 336 z^3 + 17261 z^2 - 38295 z + 19938}{720(1-z)z^4} g_2^{(1)}$$
$$+ \frac{4 z^7 + 10 z^6 - 17 z^5 + 25 z^4 - 96 z^3 + 296 z^2 - 211 z + 87}{24(1-z)z^5} g_1^{(2)}$$
$$+ \frac{-40800 z^8 + 61200 z^7 - 28480 z^6 + 4040 z^5 - 320 z^4 - 160 z^3 + 1126 z^2 - 4726 z + 3323}{120 z^5} g_2^{(2)}$$
$$- \frac{1 - 11z}{48 z^{7/2}} g_3^{(2)} - \frac{120 z^6 + 60 z^5 + 160 z^4 - 2246 z^3 + 8812 z^2 - 10159 z + 4193}{120(1-z)z^5} g_4^{(2)}$$
$$- 2(85 z^4 - 170 z^3 + 116 z^2 - 31 z + 3) g_1^{(3)} + \frac{-4 z^3 + 18 z^2 - 21 z + 5}{6(1-z)z^5} g_2^{(3)} + \frac{z^2 + 1}{12(1-z)} g_3^{(3)}, \quad (9)$$

where the $g_m^{(n)}$ are pure functions of uniform transcendental weight n. Their explicit definitions are

$$g_1^{(1)} = \log(1-z), \qquad g_2^{(1)} = \log(z),$$

$$g_1^{(2)} = 2[\mathrm{Li}_2(z) + \zeta_2] + \log^2(1-z),$$

$$g_2^{(2)} = \mathrm{Li}_2(1-z) - \mathrm{Li}_2(z),$$

$$g_3^{(2)} = -2\mathrm{Li}_2(-\sqrt{z}) + 2\mathrm{Li}_2(\sqrt{z}) + \log\left(\frac{1-\sqrt{z}}{1+\sqrt{z}}\right)\log(z),$$

$$g_4^{(2)} = \zeta_2,$$

$$g_1^{(3)} = -6\left[\mathrm{Li}_3\left(-\frac{z}{1-z}\right) - \zeta_3\right]$$
$$- \log\left(\frac{z}{1-z}\right)[2(\mathrm{Li}_2(z)+\zeta_2)+\log^2(1-z)],$$

$$g_2^{(3)} = -12\left[\mathrm{Li}_3(z) + \mathrm{Li}_3\left(-\frac{z}{1-z}\right)\right]$$
$$+ 6\mathrm{Li}_2(z)\log(1-z) + \log^3(1-z),$$

$$g_3^{(3)} = 6\log(1-z)[\mathrm{Li}_2(z)-\zeta_2] - 12\mathrm{Li}_3(z) + \log^3(1-z).$$

$$\tag{10}$$

Note that B_{lc} contains explicit dependence on $\sqrt{z}$ through the function $g_3^{(2)}$ and its coefficient, whose product is even under $\sqrt{z} \to -\sqrt{z}$. This property also holds in $\mathcal{N}=4$ SYM theory [25]. To describe B_{lc}, we need just two weight 1, four weight 2, and three weight 3 transcendental functions. To express B_{nlc} and B_{N_f} requires two more weight 3 transcendental functions. The NLO EEC in $\mathcal{N}=4$ SYM theory [25], after some rearrangement, can be expressed in terms of a subset of the transcendental functions needed for QCD. Individual virtual and real contributions contain HPLs with argument $y = i\sqrt{z}/\sqrt{1-z}$. However, they cancel out in the final physical result. The explicit expressions for B_{nlc} and B_{N_f} can be found in the Supplemental Material [59] for this Letter. In an ancillary file, we provide computer-readable expressions for all these functions, as well as their behavior in various limits.

We have performed a number of checks on the results. First, the individual virtual and real corrections are IR
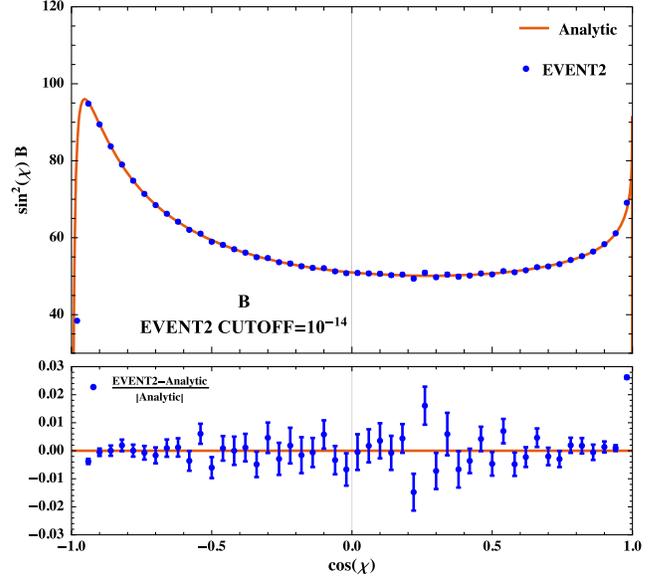
FIG. 1. Analytical results for $\sin^2(\chi)B$ are compared with numerical results from EVENT2 [13,14]. The EVENT2 prediction is obtained after sampling over 10^{10} points, with the internal CUTOFF set to 10^{-14}. Error bars represent EVENT2 statistical uncertainties.

divergent, but the divergent terms cancel after summing virtual and real, as required for any IR-safe observable. Second, in Fig. 1 we compare our analytical results with numerical predictions from EVENT2, which is based on the dipole subtraction method [13,14]. We find excellent agreement with EVENT2 over a large range; the apparent discrepancy in the rightmost bin is mainly due to the finite bin width used in EVENT2. The $z \to 0$ and $z \to 1$ limits of the analytical results are in perfect agreement with those predicted respectively by jet calculus [11,19] and soft-gluon resummation [22,23], as we discuss in the next section.

Discussion.—It is interesting to study the end-point asymptotic limits of the EEC, which provide useful information for resummation and for constructing more accurate parton showers. Expanding our results in the $z \to 0$ limit gives

$$B(z) = C_F\left\{\frac{1}{z}\left[\log(z)\left(-\frac{107C_A}{120} + \frac{25C_F}{32} + \frac{53N_fT_f}{240}\right) + C_A\left(-\frac{25\zeta_2}{12} + \frac{\zeta_3}{2} + \frac{17683}{2700}\right)\right.\right.$$
$$\left. + C_F\left(\frac{43\zeta_2}{12} - \zeta_3 - \frac{8263}{1728}\right) - \frac{4913N_fT_f}{3600}\right] + \log(z)\left[C_A\left(\frac{33\zeta_2}{2} - \frac{703439}{25200}\right)\right.$$
$$\left. + C_F\left(\frac{42109}{1200} - 21\zeta_2\right) + N_fT_f\left(\frac{86501}{12600} - 4\zeta_2\right)\right] + C_A\left(\frac{213\zeta_2}{5} - \frac{101\zeta_3}{2} - \frac{26986007}{5292000}\right)$$
$$\left. + C_F\left(-\frac{1541\zeta_2}{30} + 65\zeta_3 + \frac{18563}{2700}\right) + N_fT_f\left(-\frac{46\zeta_2}{3} + 12\zeta_3 + \frac{2987627}{330750}\right)\right\} + \mathcal{O}(z),$$

$$\tag{11}$$

where we have expanded through $\mathcal{O}(z^0)$. Note that individual terms in Eq. (9) are far more singular as $z \to 0$ than is the total Eq. (11). The EEC in the $z \to 0$ limit is dominated by collinear splitting. The leading-logarithmic term $\log(z)/z$ has been

predicted [11,19] using jet calculus [18,19]. The result is expressed as a product of two 2×2 (quark-gluon) anomalous dimension matrices for twist 2, spin 3 operators, plus a contribution due to the running coupling. It agrees fully with the coefficient of $\log(z)/z$ in Eq. (11).

In the back-to-back limit, $z \to 1$, we find that the expansion of $B(z)$ to next-to-leading power reads

$$
\begin{aligned}
B(z) = C_F \Bigg\{ \frac{1}{1-z} \Bigg[&+ \frac{1}{2} C_F \log^3(1-z) + \log^2(1-z) \left(\frac{11 C_A}{12} + \frac{9 C_F}{4} - \frac{N_f T_f}{3} \right) \\
&+ \log(1-z) \left(C_A \left(\frac{\zeta_2}{2} - \frac{35}{72} \right) + C_F \left(\zeta_2 + \frac{17}{4} \right) + \frac{N_f T_f}{18} \right) + C_A \left(\frac{11 \zeta_2}{4} + \frac{3 \zeta_3}{2} - \frac{35}{16} \right) \\
&+ C_F \left(3 \zeta_2 - \zeta_3 + \frac{45}{16} \right) + N_f T_f \left(\frac{3}{4} - \zeta_2 \right) \Bigg] + \left(\frac{C_A}{2} + C_F \right) \log^3(1-z) \\
&+ \log^2(1-z) \left(\frac{27 C_A}{8} + \frac{13 C_F}{2} - \frac{N_f T_f}{2} \right) + \log(1-z) \Bigg[C_A \left(22 \zeta_2 - \frac{2011}{72} \right) \\
&+ C_F (47 - 19 \zeta_2) + N_f T_f \left(\frac{361}{36} - 4 \zeta_2 \right) \Bigg] + C_A \left(\frac{6347 \zeta_2}{80} - 21 \zeta_2 \log(2) - \frac{137 \zeta_3}{4} - \frac{3305}{72} \right) \\
&+ C_F \left(-\frac{1727 \zeta_2}{20} + 42 \zeta_2 \log(2) + \frac{121 \zeta_3}{2} + \frac{3437}{96} \right) + N_f T_f \left(-\frac{1747 \zeta_2}{120} + 12 \zeta_3 + \frac{2099}{144} \right) \Bigg\} + \mathcal{O}(1-z). \quad (12)
\end{aligned}
$$

All the terms enhanced by $(1-z)^{-1}$ were predicted previously [22], in full agreement with Eq. (12). The next-to-leading power terms are new. They will provide useful information for resumming large Sudakov logarithms beyond leading power [29–38]. We note the appearance of $\zeta_2 \log(2)$ in the constant term at next-to-leading power, which originates solely from B_{nlc}.

Summary.—We have presented the analytical result for the EEC in QCD at NLO. Our calculation was enabled by using the IBP equations in a novel way. The final result turns out to be rather simple; only 11 transcendental functions are required to describe the QCD results, and these functions are no more complicated than the ones in the $\mathcal{N} = 4$ SYM results [25]. In contrast, the polynomial prefactors are of considerably higher degree for QCD. We have checked our results against EVENT2 numerically and found full agreement. We have also expanded the EEC to next-to-leading power in the collinear and back-to-back limits. The simplicity of the full NLO result provides encouragement for trying to compute the EEC at NNLO analytically. It will also be interesting to apply our method to other event-shape variables, such as the C parameter (which does appear to require elliptic functions, even at LO) [40].

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[59] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.120.102001 for detailed analytic formulae for the remaining coefficients in the color decomposition of the EEC at NLO, their expansion in various kinematics limits, and further cross checks against numerical results.