Unconventional Universality Class of One-Dimensional Isolated Coarsening Dynamics in a Spinor Bose Gas

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(Received 18 July 2017; published 13 February 2018)

By studying the coarsening dynamics of a one-dimensional spin-1 Bose-Hubbard model in a superfluid regime, we analytically find an unconventional universal dynamical scaling for the growth of the spin correlation length, which is characterized by the exponential integral unlike the conventional power law or simple logarithmic behavior, and numerically confirmed with the truncated Wigner approximation.

DOI: 10.1103/PhysRevLett.120.073002

Introduction.—Coarsening is relaxation dynamics following a sudden change in a system's parameter across a phase transition point. It has been studied in diverse classical systems of immense practical and fundamental importance, such as magnetization processes, metal alloying, mixing of binary liquids, and nucleation in the gas-liquid transition [1,2]. The notable feature of coarsening is the dynamical scaling $C(r,t) = f(r/L_c(t))$, which means that the correlation function $C(r,t)$ is characterized by a single length scale, namely the correlation length $L_c(t)$ with a universal function $f(x)$. The time dependence of $L_c(t)$ classifies coarsening dynamics in various open dissipative systems described by, e.g., Ginzburg-Landau or Cahn-Hilliard equations into several universality classes that depend on fundamental properties of systems such as spatial dimensions and symmetries.

Recently, the relaxation dynamics including coarsening has attracted considerable attention in ultracold atomic gases which emerge as an ideal platform for studying nonequilibrium statistical mechanics in isolated quantum systems [3–5]. Indeed, over the last decade, many theoretical and experimental studies have uncovered a rich variety of relaxation phenomena in isolated quantum systems such as prethermalization [6,7], many-body localization [5,8–10], transport dynamics [11–14], and the Kibble-Zurek mechanism (KZM) [15–18].

Then the following question naturally arises: Are there any unconventional universality classes unique to isolated coarsening dynamics? Recently, coarsening dynamics in two-dimensional (2D) and three-dimensional (3D) multicomponent Bose-Einstein condensates (BECs) have been investigated [19–27], which turn out to belong to the same conventional classes as in open dissipative systems such as the classical binary liquid and the planar spin model [20,21,23,24,27]. As for the 2D coarsening dynamics with domains, this is due to the fact that the curvature and the inertia are the main driving forces promoting the coarsening both for 2D BECs and 2D classical binary liquids. In binary liquids, these forces overcome the effect of the dissipation in an inertial hydrodynamic regime, and the system effectively behaves as an isolated system, showing the characteristic power law $L_c(t) \propto t^{2/3}$. The previous works for the 2D BECs [20,21,23,24,27] confirmed this conventional universality class. Thus, it is still open whether the universality unique to isolated systems exists.

In this Letter, we theoretically investigate a one-dimensional (1D) spin-1 Bose-Hubbard (BH) model to demonstrate that the 1D isolated quantum system exhibits coarsening dynamics that belongs to an unconventional universality class. Unlike 2D and 3D systems, the curvature and the torsion of domain walls are absent in 1D systems, so that a 1D domain-wall interaction is generally weak. In open dissipative systems, such a genuine interaction between 1D topological objects is masked by the effect of dissipation [28]; however, in 1D isolated systems it should become significant. More specifically, while a single 1D domain-wall pair is known to contract by itself in open dissipative systems [29,30], we find that in an isolated system, such a pair undergoes a linear uniform motion without self-contraction. Based on this physical intuition, we obtain an analytical expression of $L_c(t)$ characterized by an exponential integral, and numerically confirm it on the basis of the truncated Wigner approximation (TWA). This behavior is distinct from any power law or simple logarithmic behavior found in open dissipative 1D systems [31–37], and attributed to the genuine interaction between the topological objects under energy conservation and to the absence of the curvature and torsion of a domain wall. Thus, the universality class found here is unique to 1D isolated systems.

Some comments on previous related studies are in order here. The 1D domain-wall dynamics have been investigated numerically and experimentally in multicomponent BECs, where short-time domain dynamics and the KZM have been discussed [38–44]. However, universal coarsening behaviors such as a dynamical scaling have not been

addressed. In contrast to the long-time coarsening dynamics, Nicklas *et al.* have focused on the short-time dynamics after the quench, and they have experimentally investigated the universal dynamical scaling related to critical phenomena [45]. Recently, Maraga *et al.* have studied coarsening in the $O(N)$ model and reported the breakdown of usual dynamical scaling [46]; however, this result is not well understood from the perspective of universality classes.

Model.—We consider a system of spin-1 bosons in a 1D optical lattice with a lattice constant a. Under the tight-binding approximation, this system is well described by the 1D spin-1 BH model [47]. Representing annihilation and creation operators of bosons with magnetic quantum number m at the jth site as $b_{m,j}$ and $b^{\dagger}_{m,j}$ $(m = 1, 0, -1)$, the Hamiltonian is given by

$$\hat{H} = -J\sum_{m,j}\left(\hat{b}^{\dagger}_{m,j+1}\hat{b}_{m,j} + \text{H.c.}\right) + q\sum_{m,j} m^2 \hat{b}^{\dagger}_{m,j}\hat{b}_{m,j} + \frac{U_0}{2}\sum_j \hat{\rho}_j(\hat{\rho}_j - 1) + \frac{U_2}{2}\sum_j\left(\hat{\mathbf{S}}_j^2 - 2\hat{\rho}_j\right), \quad (1)$$

where J, q, U_0, and U_2 characterize the hopping amplitude, the quadratic Zeeman term, the density-dependent interaction, and the spin-dependent interaction, respectively. The operators for the total particle number and the spin vector at the jth site are given by $\hat{\rho}_j = \sum_m \hat{b}^{\dagger}_{m,j}\hat{b}_{m,j}$ and $\hat{S}_{\alpha,j} = \sum_{m,n}\hat{b}^{\dagger}_{m,j}(S_\alpha)_{mn}\hat{b}_{n,j} (\alpha = x, y, z)$ with the spin-1 spin matrices $(S_\alpha)_{mn}$.

The ground state of this model is either a Mott-insulator phase or a superfluid phase, depending on the parameters [47]. In this work, we focus on a deep superfluid regime, where a dimensionless parameter $\kappa = \rho_f J/U_0$ is much larger than unity. Here, $\rho_f \equiv N/3M$ is the filling factor with the total particle number N and the number of lattice points M.

Numerical result.—We apply the TWA method [48,49] to study the relaxation dynamics dominated by many spin domains. This method can incorporate effects of quantum fluctuations through sampling of initial states. The system is assumed to have a ferromagnetic interaction ($U_2 < 0$), and the parameters in Eq. (1) are set to be $U_0/J = 1/40$, $U_2/U_0 = -1/10$, $N = 40\,000$, and $M = 1024$. Then, κ is about 520 and the system is in a deep superfluid regime. The detailed numerical implementation is described in [50], where we demonstrate that, in a deep superfluid regime, TWA results find good agreement with results obtained by directly solving the Schrödinger equation with the Crank-Nicolson method [56].

To excite many spin domains, we quench the coefficient $q(t)$ for the Zeeman term as

$$q(t) = \begin{cases} -2.4nU_2[1 - (1 + \frac{1}{2.4})\frac{t}{\tau_q}] & (t < \tau_q); \\ nU_2 & (\tau_q \le t), \end{cases} \quad (2)$$

where τ_q is the quench time and $n = N/M$. We choose $q(0)$ such that the initial state is a polar phase. This quench protocol is to cross two phase-transition points from the polar phase to the broken-axisymmetry phase and then to the ferromagnetic phase [47].

Figure 1(a) shows the time evolution of the spin amplitude defined by $S^2_{\text{av},\alpha}(t) = \langle\sum_j \hat{S}^2_{\alpha,j}/n^2M\rangle(t) (\alpha = x, y, z)$ and $S^2_{\text{av,tot}}(t) = \sum_\alpha S^2_{\text{av},\alpha}(t)$, where the bracket means a quantum average $\langle\cdots\rangle(t) = \langle\psi(t)|\cdots|\psi(t)\rangle$ with the state vector $|\psi(t)\rangle$ at time t. In this result, the quench time is set to be $\tau_q = 800\tau$ with $\tau = 4\hbar/J$. At an early stage of the quench protocol, the x and y components of the spin vector rapidly grow because the system is brought to the broken-axisymmetry phase where the dynamical instabilities of the $m = \pm 1$ components lead to an increase in the particle numbers of those components. At a later stage, the instability of the $m = 0$ component becomes strong as the system enters the ferromagnetic phase. Then, the particle number of the $m = 0$ component rapidly decreases and eventually the z component dominates the other components.

After the quench, many domain walls are formed as shown in Fig. 1(b), which is a spatiotemporal distribution of $S_{z,j}$ obtained by a single classical trajectory of the TWA calculation. The encircled regions show where spin

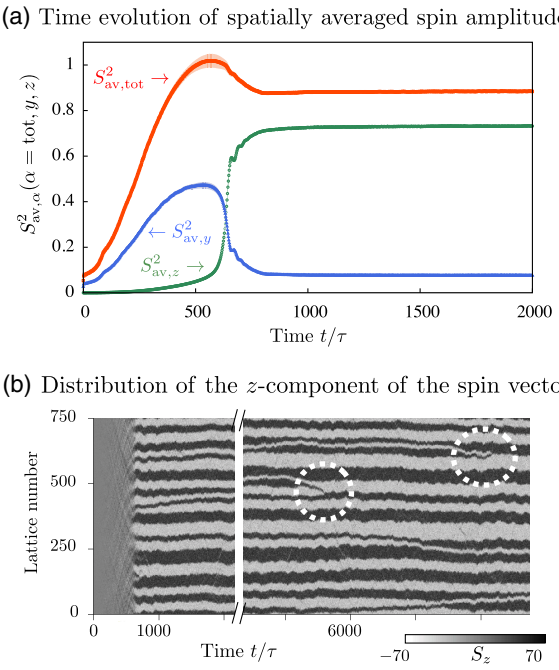

FIG. 1. (a) Time evolution of $S^2_{\text{av},\alpha}(\alpha = \text{tot}, y, z)$ for quench time $\tau_q = 800\tau$. Color bands show $3\sigma/\sqrt{N_{\text{sam}}}$ error bars in the TWA calculation, where σ is the standard deviation and N_{sam} is the number of samples. The behavior of the x component (not shown) is almost the same as that of the y component. (b) Spatiotemporal distribution of $S_{z,j}$ corresponding to a single classical trajectory in the TWA calculation. Two dashed-white circles indicate where domains merge.

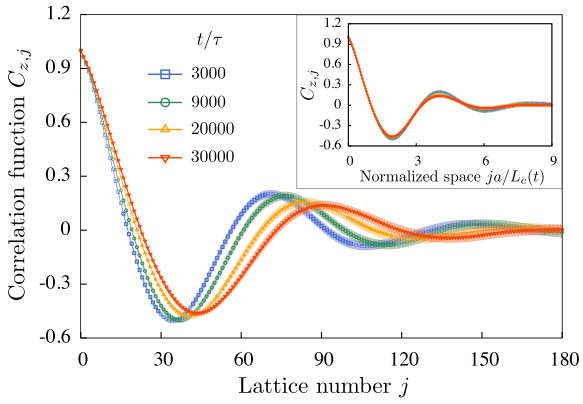

FIG. 2. Time evolution of the correlation function $C_{z,j}$ with $t/\tau = 3000$, 9000, 20 000, and 30 000. Color bands show $3\sigma/\sqrt{N_{\rm sam}}$ error bars. The first zero-crossing point of each curve defines the correlation length $L_c(t)$. The inset shows $C_{z,j}$ with the x axis normalized by $L_c(t)$. All curves converge to a single one.

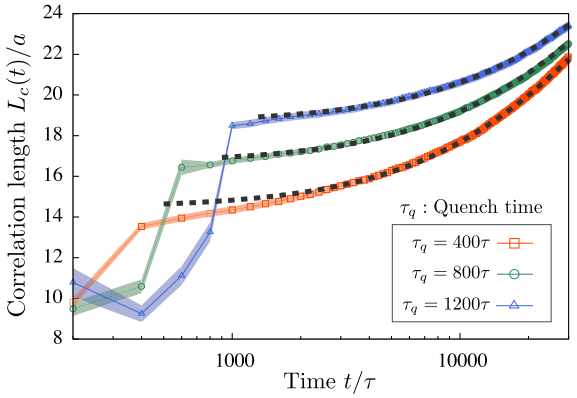

FIG. 3. Time evolution of the correlation length $L_c(t)$ for quench times $\tau_q/\tau = 400$, 800, and 1200. Color bands show error bars. Black-dashed curves show analytic results of Eq. (8).

domains merge. This merging process enlarges domain structures.

To investigate this coarsening behavior quantitatively, we calculate a spatial correlation function $C_{z,j}(t)$ for $\hat{S}_{z,i}$ defined by

$$C_{z,j}(t) = \frac{\sum_{k=1}^{M} \langle \hat{S}_{z,j+k} \hat{S}_{z,k} \rangle(t)}{\sum_{k=1}^{M} \langle \hat{S}_{z,k} \hat{S}_{z,k} \rangle(t)}. \tag{3}$$

Figure 2 shows the time evolution of $C_{z,j}$, and the inset shows the same curves with the abscissa normalized by the correlation length $L_c(t)$ defined by the first zero crossing point of the correlation function. We find that all curves are rescaled into a single universal curve, showing a dynamical scaling characteristic of coarsening dynamics. A small deviation from the single curve is expected to be caused by density and spin waves excited upon merging of the domains, which cannot dissipate in the isolated system and weaken long-range correlations. Actually, in dissipative 1D systems, clear dynamical scaling without a small deviation has been confirmed [57,58].

To understand the universality class, we examine the time evolution of $L_c(t)$. Figure 3 shows $L_c(t)$, which exhibits behavior quite different from the conventional logarithmic and power laws [31–37].

Analytic result.—We show that the growth law of $L_c(t)$ in Fig. 3 is characterized by the exponential integral. As shown in Fig. 1(b), the size of a domain grows only through merging of two domain-wall pairs. This suggests that a domain-wall pair plays a key role here.

To analyze the domain-wall pair dynamics, we note that this system can be transformed to a continuum model similar to the spinor Gross-Pitaevskii equation because the width of the domain wall $2\lambda = 2a\sqrt{J/|q_F|} = 6.4a$ at $q_F = q(\tau_q)$ is larger than the lattice constant a. Then, we can derive a spin-hydrodynamic equation [59–62], i.e., the Landau-Lifshitz (LL) equation, given by

$$\frac{\partial}{\partial t}\mathbf{S}(x,t) = -\mathbf{S}(x,t) \times \mathbf{B}(x,t), \tag{4}$$

where $\mathbf{B}(x,t) = -J'\nabla^2\mathbf{S}(x,t) - q'S_z\hat{\mathbf{e}}_z$, $J' = Ja^2/\hbar$ and $q' = -q_F/\hbar$. The derivation of Eq. (4) is described in [50].

Applying the singular perturbation method [29,30,63] to Eq. (4), we find that a domain-wall pair undergoes a linear uniform motion with velocity $V(l)$ given by

$$V(l) = 4\sqrt{J'q'}e^{-l/\lambda}\sin(\phi_1 - \phi_2). \tag{5}$$

Here, l is the distance between the two domain walls, and the phase ϕ_j is the azimuthal angle of $\mathbf{S}_j$ at the center of the domain wall labeled by $j = 1$, 2. The derivation is described in [50], where Eq. (5) is compared with numerical results of Eq. (4).

Next, we investigate the correlation length $L_c(t)$ by assuming that many domain-wall pairs randomly move and that the merging of domains occurs through collisions between the domain-wall pairs. Let us examine a situation where there are $N_d(t)$ domain walls, and the average distance between the walls at time t is denoted as $l_d(t)$. First, we note that the average collision time $\tau_c(t)$ is given by $l_d(t)/V_{\rm av}(t)$. Here the average velocity is represented as $V_{\rm av}(t) = V_0\exp(-l_d(t)/\lambda)$ because of Eq. (5). However, we cannot determine the proportionality constant V_0 since the distribution of $\phi_1 - \phi_2$ is complicated. Then, assuming only two properties of a Poisson process [64], we derive the time derivative of $N_d(t)$:

$$\frac{d}{dt}N_d(t) = -\frac{\alpha}{\tau_c(t)} = -\frac{\alpha V_{\rm av}[l_d(t)]}{l_d(t)}, \tag{6}$$

where α is a positive constant.

Secondly, we use the fact that $N_d(t)$ is inversely proportional to $l_d(t)$, which leads to $N_d(t) = \beta/l_d(t)$ with a positive constant β. We substitute it into Eq. (6), obtaining

$$\frac{d}{dt} l_d(t) = \gamma l_d(t) \exp[-l_d(t)/\lambda], \tag{7}$$

where $\gamma = \alpha V_0/\beta$.

The solution to this equation is expressed by the exponential integral $\mathrm{Ei}[x] = \int_{-\infty}^{x} \exp(t)/t\, dt$ [65]. Using this function and the fact that $l_d(t)$ is proportional to the correlation length $L_c(t) = \eta l_d(t)$ with the proportionality constant η, we arrive at

$$L_c(t) = \eta\lambda \mathrm{Ei}^{-1}[\gamma(t - t_0) + \mathrm{Ei}[L_c(t_0)/\eta\lambda]]. \tag{8}$$

Here $\mathrm{Ei}^{-1}[b] = a$ is the inverse function of $\mathrm{Ei}[a] = b$. We note that this equation asymptotically gives a logarithmic law after a sufficiently long time.

In Fig. 3, we plot Eq. (8) as dashed curves, which are in excellent agreement with the numerical results. The deviations of the data ($\tau_q/\tau = 400$) in the early time are due to partial breakdown of Eq. (5) because it becomes a good approximation only when the distance l is large. Note that Eq. (8) has two constants γ and η. In Fig. 3, we use $(\gamma\tau,\eta) = (0.000380, 1.72)$, $(0.000385, 1.70)$, $(0.000380, 1.69)$ for $\tau_q/\tau = 400$, 800, 1200, respectively. Thus, all numerical data can be fitted with almost the same γ and η.

Finally, we comment on the relation between our result of Eq. (8) and the previous works concerning the 1D coarsening [31–37]. In these works, the energy is dissipated, so that two domain walls forming a wall pair contract by themselves. Such a self-contraction was confirmed in Refs. [29,30], and it is different from the coarsening process (merging of two domain-wall pairs) in our study. Thus, these systems do not obey Eq. (8), although both previous studies and ours show the same logarithmic behavior in the long-time limit. As an exception, there is a convective Cahn-Hilliard equation, where a domain-wall pair undergoes a linear uniform motion [36,66]. Thus we expect that this system obeys Eq. (8), though it was not derived in previous literature.

Discussion.—We first discuss why Eq. (8) is universal. As can be seen from the derivation, this law originates from the mechanism where a domain-wall pair moves at an average velocity proportional to $\exp(-l/\lambda)$ without the self-contraction. The exponential dependence of the velocity on l and the absence of self-contraction are due to the interaction between the 1D domain walls and isolation from the environment [67]. Thus, Eq. (8) reflects the nature of a 1D isolated system. A typical example that satisfies these conditions is the 1D LL equation, which is a universal effective equation in 1D spin systems. Thus, we expect that the growth law of Eq. (8) is universal in a 1D isolated spin system if the domain is stable and the domain merging occurs.

Next, we discuss possible experimental situations. A difficulty of observing Eq. (8) is a limited lifetime of trapped atoms. In a 1D system, the interaction between domain walls is weak due to the exponential decay of $\mathbf{S}(x)$ for the domain walls, so that the relaxation time is very long. However, Eq. (8) may be observed if we prepare a 1D system with ^{7}Li ($F = 1$).

We consider quasi-1D systems of ^{7}Li atoms in a 1D optical lattice, where atoms are tightly confined in a radial direction as shown in inset (a) of Fig. 4. The parameters used are $a_{\mathrm{exp}} = 0.387\ \mu$m [68], $M_{\mathrm{exp}} = 128$, $N_{\mathrm{exp}} = 5000$, the radial trapping frequency $\omega_r = 2\pi \times 4500/\mathrm{s}$, and the depth of the lattice $V_d = 5E_r$ with $E_r = \hbar^2/8Ma_{\mathrm{exp}}^2$ being the recoil energy. Then, this system behaves as a quasi-1D system, and a 1D calculation can be justified since the condition $\hbar\omega_{\mathrm{r}} \sim 2.8\mu$ with the chemical potential μ is satisfied and excitations in the radial direction are suppressed.

Under the above setup, we have performed a TWA calculation for the 1D system and confirmed Eq. (8) as shown in Fig. 4. The characteristic time τ_{exp} is about 0.156 ms, and the calculation terminates at about 4.7 s, which is accessible in current experiments. To measure the correlation function, we need the spatial resolution of about 1 μm, which is available in an *in situ* imaging method [45]. When experiments continue until 9 s, we can obtain 4 data points for $L_c(t)$ [69]. In this time evolution, $L_c(t)$ is completely different from any power and logarithmic laws. Thus, we can distinguish Eq. (8) from the conventional laws.

Finally, we discuss a finite-size effect and three-body loss. As for the former, we note that the number of domain walls is not large as shown in inset (b) of Fig. 4. Thus, in the long-time dynamics, the coarsening should be suppressed.

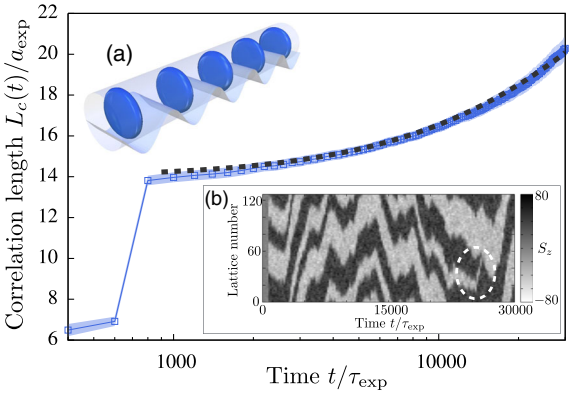

FIG. 4. Time evolution of the correlation length $L_c(t)$ in the experimental setup. A black-dashed curve is the analytic result of Eq. (8) with $(\gamma\tau_{\mathrm{exp}}, \eta) = (0.000280, 1.70)$, where a color band shows an error bar. Inset. (a) Possible experimental setup. (b) Spatiotemporal distribution of $S_{z,j}$ for a single classical trajectory in the TWA calculation.

However, we confirm Eq. (8) until about 4.7 s. After this time, the finite-size effect may be significant. As for the latter, the central density at each site is about 2.23×10^{14} /cm^3. Thus, if a three-body loss rate of ^{7}Li is 6×10^{-31} cm^6/s [70], the particle loss until 9 s is below 19%, which allows experimental test of our predictions.

Conclusion.—The relaxation dynamics, described by the 1D spin-1 BH model, have been analytically and numerically studied. Our numerical calculation based on the TWA method has revealed that the system in a deep superfluid regime exhibits coarsening with the dynamical scaling that belongs to the universality class different from conventional classes. We have analytically obtained the universal domain-growth law of Eq. (8), which is in remarkable agreement with the numerical data.

We are grateful to I. Danshita, S. Furukawa, I. Ichinose, S. Inouye, K. Kasamatsu, Y. Kuno, T. Matsui, and H. Takeuchi for useful discussions and comments on this work. This work was supported by KAKENHI Grant No. JP26287088 from the Japan Society for the Promotion of Science, and a Grant-in-Aid for Scientific Research on Innovative Areas Topological Materials Science (KAKENHI Grant No. JP15H05855), and the Photon Frontier Network Program from MEXT of Japan, ImPACT Program of Council for Science, Technology and Innovation (Cabinet Office, Government of Japan). K. F. was supported by JSPS fellowship (JSPS KAKENHI Grant No. JP16J01683). R. H. was supported by the Japan Society for the Promotion of Science through Program for Leading Graduate Schools (ALPS) and JSPS fellowship (JSPS KAKENHI Grant No. JP17J03189).

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[70] The three-body parameter of ^{7}Li is not reported in literature. For ^{87}Rb and ^{23}Na, the parameters are 4.3×10^{-29} cm^6/s and 6.6×10^{-30} cm^6/s for a noncondensed cloud [71]. In the BEC cases, the values are 6 times lower than the noncondensed cases. Generally, an atom with a small radius has a small three-body parameter, so that we speculate that the parameter of ^{7}Li is smaller than 10^{-30} cm^6/s.

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