Between Symmetry and Duality in Supersymmetric Quantum Field Theories

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We study two cases of interrelations between the enhancement of symmetries in the infrared (IR) and duality properties of supersymmetric quantum field theories in four dimensions. First, we discuss an $SU(2)$ $\mathcal{N}=1$ model with four flavors, singlet fields, and a superpotential. We show that this model flows to a conformal field theory with $E_6\times U(1)$ global symmetry. The enhancement of the flavor symmetry follows from Seiberg duality. The second example is concerned with an $SU(4)$ gauge theory with matter in the fundamental and antisymmetric representations. We argue that this model has enhanced $SO(12)$ symmetry in the IR, and, guided by this enhancement, we deduce a new IR duality.

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Introduction.—Models with different ultraviolet (UV) properties can flow to the same IR conformal fixed point. In supersymmetric setups there are many examples of such universality properties, with the UV models being gauge theories having different gauge groups and gauge singlets, or having the same gauge groups with different gauge singlet fields and different superpotentials. In the latter case the phenomenon is usually referred to as self-duality. The global symmetry of the two dual models will usually act differently on the gauge nonsinglet fields.

Another interesting phenomenon is that of the global symmetry in the IR being larger than the symmetry in the UV. This often happens when some of the degrees of freedom become free as one approaches the fixed point. However, there are also cases in which the symmetry enhancement happens as the quantum numbers of the states at the IR fixed point align to form representations of a bigger symmetry. The bigger symmetry usually will have the same rank as the symmetry in the UV but with a larger dimension. Enhancements of the rank are also possible, though they will not be discussed here.

We will discuss in this short Letter two cases where self-duality of a certain model can be related to enhancement of symmetry in a similar model. The basic observation is that, in the case of self-duality, one often can add additional gauge singlet operators on the two sides of the duality, without spoiling the IR equivalence, such that the two dual models will have exactly the same field content. The duality will still identify the symmetries of the two models in a nonobvious way, leading to symmetry enhancement. It will also be observed in one of the examples that, in order to obtain a model with enhanced symmetry, the additional gauge singlet fields will break some of the original symmetry.

We will emphasize three important guiding principles. First, breaking global symmetries with interactions might lead to an enhanced symmetry in the IR which is not a subgroup of—and in fact might not contain—the symmetry of the original model. Second, self-dualities of field theories can be utilized to find theories with enhanced flavor symmetry by constructing models which are structurally invariant under dualities, with the effect of the latter being a nontrivial action on the matter. Third, enhancement of symmetry can, in some cases, be a sign of new self-dualities.

E_6 symmetry from duality.—The E_6 model: Let us consider $SU(2)_g$ gauge theory with eight fundamental chiral fields. We split the eight chiral fields into six (Q_A) and two (Q_B). We also introduce gauge invariant operators M_A and M_B coupling as

$$M_AQ_AQ_A+M_BQ_BQ_B. \tag{1}$$

The quiver theory is depicted in Fig. 1, and charges of fields can be obtained in Table I. The choice of gauge singlet fields breaks the symmetry of the model from $SU(8)$ down to $SU(6)_A\times SU(2)_B\times U(1)_h$. We will soon show that this is

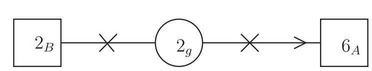

FIG. 1. Model with $E_6\times U(1)$ global symmetry. The cross on the edges denotes the fields M_A and M_B. These are flipping the baryonic operators constructed from the fields associated with the corresponding edge. Flipping chiral operator O means introducing chiral field ϕ_O and coupling it as $\phi_O O$.

TABLE I. Matter and gauge content of the E_6 model.

Field	$SU(2)_g$	$SU(2)_B$	$SU(6)_A$	$U(1)_h$	$U(1)_{\hat{r}}$
Q_A	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{6}$	$\frac{1}{2}$	$\frac{5}{9}$
Q_B	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$-\frac{3}{2}$	$\frac{1}{3}$
M_B	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	3	$\frac{4}{3}$
M_A	$\mathbf{1}$	$\mathbf{1}$	$\overline{\mathbf{15}}$	-1	$\frac{8}{9}$

enhanced to $E_6 \times U(1)_h$. We also note that $SU(8)$ is not subgroup of E_6 and that, for the enhancement to be possible, it is crucial to break the symmetry.

In Table I, $U(1)_{\hat{r}}$ is the superconformal R symmetry obtained by a maximization [1], and the conformal anomalies are $c = \frac{29}{24}$ and $a = \frac{13}{16}$. To study the protected spectrum of the theory, it is very useful to compute the supersymmetric index [2]. Using the standard definitions, this is given as an expansion in the superconformal fugacities q and p [3],

$$1 + \overline{\mathbf{27}} h^{-1} (qp)^{4/9} + h^3 (qp)^{2/3} + \cdots + (-\mathbf{78} - 1) qp + \cdots . \tag{2}$$

The boldface numbers are representations of E_6, as we will elaborate on momentarily, and h is the fugacity for $U(1)_h$. We remind the reader that the power of qp is half of the R charge for scalar operators and we observe that all of the operators are above the unitarity bound. Let us count some of the operators contributing to the index. The relevant operators of the model are $Q_B Q_A$ and M_A, which comprise the $(\mathbf{2}, \mathbf{6})$ and $(\mathbf{1}, \overline{\mathbf{15}})$ representations of $(SU(2)_B, SU(6)_A)$, which gives $\overline{\mathbf{27}}$ of E_6. We also have M_B, a singlet of non-Abelian symmetries. At order qp, assuming that the theory flows to an interacting conformal fixed point, the index gets contributions only from marginal operators minus conserved currents for global symmetries [4]. The operators contributing at order qp are gaugino bilinear $\lambda\lambda$ $[(\mathbf{1}, \mathbf{1})]$, $Q_A \bar{\psi}_{Q_A}$ $[(\mathbf{1}, \mathbf{35} + \mathbf{1})]$, and $Q_B \bar{\psi}_{Q_B}$ $[(\mathbf{3} + \mathbf{1}, \mathbf{1})]$. These operators give the contribution

$$1 - (\mathbf{1}, \mathbf{35}) - 1 - (\mathbf{3}, \mathbf{1}) - 1,$$

which gives the conserved currents for the symmetry we see in the Lagrangian. Here, $\bar{\psi}_F$ is the complex conjugate Weyl fermion in the chiral multiplet of the scalar F. We also have the operators $\bar{\psi}_{M_A} M_A$, $\bar{\psi}_{M_B} M_B$, $M_B Q_B Q_B$, and $Q_A Q_A M_A$, which cancel out in the computation since the first two are fermionic while the second two are bosonic, but they both have the same representations of flavor symmetry pairwise. Finally, we have $Q_A^3 Q_B$ $[(\mathbf{2}, \mathbf{70})]$ and $Q_A \bar{\psi}_{M_A} Q_B$ $[(\mathbf{2}, \mathbf{20} + \mathbf{70})]$. These two contribute

$$-(\mathbf{2}, \mathbf{20})$$

to the index, which, combined with the above, forms the character of the adjoint representation of the $E_6 \times U(1)_h$ symmetry. We emphasize that the fact that we see $-\mathbf{78}$ in order qp of the index is a proof following from the representation theory of the superconformal algebra that the symmetry of the theory is enhanced to E_6, where the only assumption, in addition to the absence of accidental $U(1)$ values, is that the theory flows to an interacting fixed point. We also observe that the conformal manifold here is a point.

Symmetry and duality: The enhancement of symmetry to E_6 follows from a well-known IR duality [5]. Note that we can reorganize the gauge charged matter into two groups of four chiral fields. We take four out of the six Q_A values and call them $\tilde{Q}$, then combine the other two with Q_B and call them Q. This also decomposes the symmetry $SU(6)_A$ to $SU(4)_A \times SU(2)_A \times U(1)_{h'}$, with a combination of $U(1)_{h'}$ and $U(1)_h$ being the baryonic symmetry (see Fig. 2). IR duality [5] without the gauge singlet fields will then map the baryonic symmetry to itself while conjugating the two $SU(4)$ symmetries and adding 16 gauge singlet mesonic operators. With our choice of gauge singlet fields, the flipper fields of Fig. 2 are flipping eight of the baryons, and the bifundamental gauge invariant operators form half of the mesons. Thus, the duality removes the half of the mesons which connect $SU(2)_B$ with $SU(4)_A$ and attaches the other half between the $SU(4)_A$ and $SU(2)_A$ quiver flavor nodes. This transformation acts only on the symmetry, leaving the quiver structurally unchanged. The action on the symmetry is as the Weyl transformation, which enhances the $SU(6)_A \times SU(2)_B$ symmetry to E_6.

Deformation to Wess-Zumino (WZ) model: Let us deform our model by adding M_B to the superpotential, which entails a vacuum expectation value for $Q_B Q_B$. This will result in the Higgs mechanism in the gauge group, and we will be left with a WZ model of 27 chiral fields connected through a superpotential (see Fig. 3). In fact, this model is the same as the one discussed in Ref. [6] (see Ref. [7] for related observations). The superpotential can be thought of as a determinant of a 3×3 Hermitian octonionic matrix, exactly the form that the E_6 symmetry preserves. Although this is not too interesting of a model in four dimensions, all of our arguments admit a generalization by reducing a circle to three dimensions, where this WZ model flows to an interacting fixed point.

Relation to E string compactifications: The theory we have discussed can be related to compactifications of the E

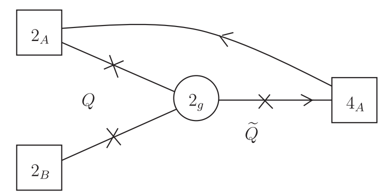

FIG. 2. An equivalent way to represent the theory of Fig. 1.

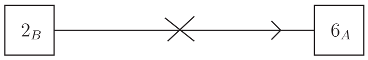

FIG. 3. Model obtained upon deforming the quiver by admixing M_B to the superpotential.

string, a superconformal theory in six dimensions with E_8 flavor symmetry. In Fig. 4 we depict the field theory one obtains by compactifying the rank 1 E string on a torus with a half unit of flux breaking E_8 to $E_6 \times U(1)$ [8]. Giving a vacuum expectation value to the flip fields of this model gives us the exact model of Fig. 1, up to the singlet field M_B, which does not effect the enhancement of symmetry. This relation provides a geometric explanation for the enhancement of symmetry.

We thus summarize that starting from $SU(2)_g$ gauge theory with four flavors and adding gauge singlet fields breaking the $SU(8)$ symmetry to $U(1)_h \times SU(6)_A \times SU(2)_B$, the renormalization group flow leads to a model with $E_6 \times U(1)_h$ symmetry. This statement assumes only that the flow leads to interacting fixed points, and we see no evidence to the contrary. The enhancement of symmetry discussed here is very closely related to the enhancement of symmetry of the sequence of $USp(4n)_g$ gauge theories discussed in Ref. [9], which follows Ref. [10]. See also Ref. [11] for related examples in a different context.

Duality from $SO(12)$ symmetry.—The second case we study is $SU(4)_g$ gauge theory with four flavors, Q and $\tilde{Q}$, and two chiral fields, X, in the antisymmetric representation. The fields and the charges are in the upper half of Table II.

This model and the one discussed in the previous section are two first entries, $N = 2$, 1, in a sequence of $SU(2N)_g$ gauge theories with four flavors in fundamental representation and a field in antisymmetric representation. All models in this sequence are known to have self-dual descriptions [12,13]. We will not detail the dualities here and will simply mention that the two $SU(4)$ symmetry groups are manifest in all dual frames. We can construct a model with $SU(4)_g$ gauge symmetry which is dual to itself by turning on the very particular collection of gauge singlet fields listed in the bottom half of Table II. We couple the gauge singlet fields to the following superpotential:

$$\tilde{Q}QX^2M + QQXB + \phi X^2. \qquad (3)$$

As all of the dualities preserve the global symmetry and the superpotential also does not break it, we naively do not expect any non-Abelian enhancement of symmetry following from these dualities. However, computing the index, we find that all the states fall into representations of $SO(12) \times SU(2) \times U(1)^2$,

$$1 + (\mathbf{1}, \mathbf{32}')a^{-2}(qp)^{1/3} + (\mathbf{2}, \mathbf{12})b^{-2}(qp)^{1/2} + \cdots + (-(\mathbf{1}, \mathbf{66}) - (\mathbf{3}, \mathbf{1}) - 1 - 1)qp + \cdots. \qquad (4)$$

Here $(\mathfrak{R}_1, \mathfrak{R}_2)$ denotes the characters of representation $\mathfrak{R}_1 \times \mathfrak{R}_2$ of $SU(2) \times SO(12)$, and a and b are fugacities for $U(1)_a$ and $U(1)_b$, respectively. The superconformal R symmetry here is given as

$$\hat{r} = r + 0.057\mathfrak{q}_b + 0.142\mathfrak{q}_a,$$

and, to evaluate the index, we used the R charge $r + \frac{1}{6}\mathfrak{q}_a$, which is close to the superconformal one. The two $SU(4)$ symmetry groups are imbedded in $SO(12)$ as $SO(6) \times SO(6)$. Moreover, at the order of the index in which the conserved currents and marginal deformations contribute, we observe a term in adjoint representation of the $SO(12)$, indicating that the symmetry enhances to this in the IR. The decomposition into $SO(6) \times SO(6)$ representations of the adjoint is

$$\mathbf{66} = (\mathbf{15}, \mathbf{1}) + (\mathbf{1}, \mathbf{15}) + (\mathbf{6}, \mathbf{6}). \qquad (5)$$

The first two terms come from $Q\bar{\psi}_Q$ and $\tilde{Q}\bar{\psi}_{\tilde{Q}}$. We have an operator in $(\mathbf{6}, \mathbf{6})$ residing in the bosonic operator $\tilde{Q}^2Q^2X^2$, and two fermionic operators, $\tilde{Q}^2\bar{\psi}_BX$ with $\tilde{Q}Q\bar{\psi}_M$. The combined effect of these is to contribute $(\mathbf{6}, \mathbf{6})$ to the enhancement of the conserved current.

Duality from symmetry: The enhancement to $SO(12)$ does not follow from the dualities of Ref. [12], as these preserve the two $SU(4)$ symmetries. However, the Weyl group for $SO(12)$ permutes the Cartan generators residing

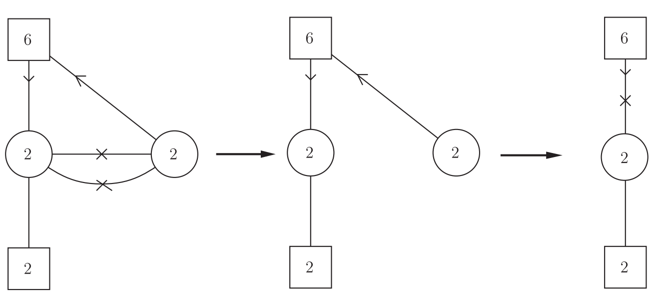

FIG. 4. The model on the left corresponds to compactification of a rank 1 E string on a torus with a particular flux for the global symmetry. Giving vacuum expectation values to the flip fields, one obtains the model in the middle, which is equivalent under Seiberg duality to the third model.

TABLE II. Matter and gauge content of the $SO(12)$ model.

Field	$SU(4)_g$	$SU(4)$	$SU(4)$	$SU(2)$	$U(1)_a$	$U(1)_b$	$U(1)_r$
Q	**4**	**4**	**1**	**1**	−1	1	$\frac{1}{2}$
$\tilde{Q}$	$\bar{\mathbf{4}}$	**1**	**4**	**1**	−1	−1	$\frac{1}{2}$
X	**6**	**1**	**1**	**2**	2	0	0
M	**1**	$\bar{\mathbf{4}}$	$\bar{\mathbf{4}}$	**1**	−2	0	1
B	**1**	**6**	**1**	**2**	0	−2	1
ϕ	**1**	**1**	**1**	**3**	−4	0	2

TABLE III. Decomposition of the $SU(4)$ representations into $SU(2)\times SU(2)\times U(1)$ subgroups.

Field	$SU(2)_L$	$\widetilde{SU}(2)_L$	$U(1)_L$	$SU(2)_R$	$\widetilde{SU}(2)_R$	$U(1)_R$
Q^-	**2**	**1**	−1	**1**	**1**	0
Q^+	**1**	**2**	1	**1**	**1**	0
$\tilde{Q}^-$	**1**	**1**	0	**2**	**1**	−1
$\tilde{Q}^+$	**1**	**1**	0	**1**	2	1
X	**1**	**1**	0	**1**	**1**	0

in the two $SU(4)$ symmetries. Enhancement of symmetry can be taken as an indication that there is a dual description of the original theory in which the two $SU(4)$ symmetries are broken and the subgroups are identified in a nonobvious way. The basic duality of Ref. [12] is a generalization of Seiberg duality for $SU(2)_g$ theory with eight flavors. There we have 35 different dualities corresponding to a splitting of the eight chiral fields into two groups of four. However, here we naively do not have a generalization of such freedom, as the relevant representations are complex. The enhancement of symmetry is again an indication that such a generalization can be obtained as we shall now describe.

We first decompose both $SU(4)$ symmetries into $SU(2)\times SU(2)\times U(1)$ and write the matter content of the $SU(4)_g$ theory with no gauge singlet fields in terms of representations of these groups. In Table III, we detail the charges under the $SU(4)$ symmetries, as the rest are as in the previous table.

The dual theory has fields shown in Table IV.

The duality exchanges the $U(1)_L$ and $U(1)_R$ symmetries. The superpotential is

$$\sum_{l=0}^{1}(\mathfrak{M}_l^{-+}\mathfrak{Q}^+\tilde{\mathfrak{Q}}^- + \mathfrak{M}_l^{+-}\mathfrak{Q}^-\tilde{\mathfrak{Q}}^+)\mathfrak{X}^{2(1-l)} + (\mathfrak{B}^+\mathfrak{Q}^-\mathfrak{Q}^- + \tilde{\mathfrak{B}}^+\tilde{\mathfrak{Q}}^-\tilde{\mathfrak{Q}}^-)\mathfrak{X} + (\mathfrak{B}^-\mathfrak{Q}^+\mathfrak{Q}^+ + \tilde{\mathfrak{B}}^-\tilde{\mathfrak{Q}}^+\tilde{\mathfrak{Q}}^+)\mathfrak{X}. \tag{6}$$

TABLE IV. Matter and gauge content of a dual of the $SO(12)$ theory.

Field	$SU(2)_L$	$\widetilde{SU}(2)_L$	$U(1)_L$	$SU(2)_R$	$\widetilde{SU}(2)_R$	$U(1)_R$
$\mathfrak{Q}^-$	**2**	**1**	0	**1**	**1**	−1
$\mathfrak{Q}^+$	**1**	**2**	0	**1**	**1**	1
$\tilde{\mathfrak{Q}}^-$	**1**	**1**	−1	**2**	1	0
$\tilde{\mathfrak{Q}}^+$	**1**	**1**	1	**1**	**2**	0
$\mathfrak{X}$	**1**	**1**	0	**1**	**1**	0
$\mathfrak{M}_l^{-+}$	**1**	**2**	1	**2**	**1**	−1
$\mathfrak{M}_l^{+-}$	**2**	**1**	−1	**1**	**2**	1
$\mathfrak{B}^{\mp}$	**1**	**1**	0	**1**	**1**	∓2
$\tilde{\mathfrak{B}}^{\mp}$	**1**	**1**	∓2	**1**	**1**	0

We can make many checks on this duality—for example, that the indices of the two dual models are in agreement and that the 't Hooft anomalies match. The contribution of the singlet fields to all of the Abelian anomalies is vanishing. The nonobvious matching of anomalies involves the $SU(2)$ symmetries. For example,

$$\mathrm{Tr}SU(2)_L^2U(1)_L = [Q^-] = [\mathfrak{M}_1^{+-}] + [\mathfrak{M}_2^{+-}] = -2. \tag{7}$$

Let us mention that a relevant deformation of the $SO(12)$ theory with a Q^4 operator leads to symmetry enhancing to $E_7\times U(1)$.

We summarize by stating that, by using the basic observations of this Letter, one can generate many examples of conformal theories with enhanced symmetries starting from known self-dualities and can also derive new self-dualities by observing enhancements of symmetry. We plan to report in more detail on some of the plethora of examples in a forthcoming work.

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[1] K. A. Intriligator and B. Wecht, The exact superconformal R symmetry maximizes a, Nucl. Phys. **B667**, 183 (2003).
[2] C. Romelsberger, Counting chiral primaries in $N=1$, $d=4$ superconformal field theories, Nucl. Phys. **B747**, 329 (2006); J. Kinney, J. M. Maldacena, S. Minwalla, and S. Raju, An index for 4 dimensional super conformal theories, Commun. Math. Phys. **275**, 209 (2007).
[3] F. A. Dolan and H. Osborn, Applications of the superconformal index for protected operators and q-hypergeometric identities to $N=1$ dual theories, Nucl. Phys. **B818**, 137 (2009).
[4] C. Beem and A. Gadde, The $N=1$ superconformal index for class S fixed points, J. High Energy Phys. 04 (2014) 036.
[5] N. Seiberg, Electric-magnetic duality in supersymmetric non-Abelian gauge theories, Nucl. Phys. **B435**, 129 (1995).
[6] S. S. Razamat and G. Zafrir, Exceptionally simple exceptional models, J. High Energy Phys. 11 (2016) 061.
[7] V. P. Spiridonov and G. S. Vartanov, Superconformal indices for $N=1$ theories with multiple duals, Nucl. Phys. **B824**, 192 (2010); I. Gahramanov and G. Vartanov, Extended global symmetries for 4D $N=1$ SQCD theories, J. Phys. A **46**, 285403 (2013).
[8] H. C. Kim, S. S. Razamat, C. Vafa, and G. Zafrir, E-string theory on Riemann surfaces, Fortschr. Phys. **66**, 1700074 (2018).
[9] S. S. Razamat and G. Zafrir, E_8 orbits of IR dualities, J. High Energy Phys. 11 (2017) 115.

[10] T. Dimofte and D. Gaiotto, An E_7 surprise, J. High Energy Phys. 10 (2012) 129.
[11] C. Wang, A. Nahum, M. A. Metlitski, C. Xu, and T. Senthil, Deconfined Quantum Critical Points: Symmetries and Dualities, Phys. Rev. X **7**, 031051 (2017).
[12] C. Csaki, M. Schmaltz, W. Skiba, and J. Terning, Self-dual $N = 1$ SUSY gauge theories, Phys. Rev. D **56**, 1228 (1997).
[13] V. P. Spiridonov and G. S. Vartanov, Elliptic hypergeometry of supersymmetric dualities, Commun. Math. Phys. **304**, 797 (2011).