Steady State Entanglement beyond Thermal Limits

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Classical engines turn thermal resources into work, which is maximized for reversible operations. The quantum realm has expanded the range of useful operations beyond energy conversion, and incoherent resources beyond thermal reservoirs. This is the case of entanglement generation in a driven-dissipative protocol, which we hereby analyze as a continuous quantum machine. We show that for such machines the more irreversible the process, the larger the concurrence. Maximal concurrence and entropy production are reached for the hot reservoir being at negative effective temperature, beating the limits set by classic thermal operations on an equivalent system.

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Introduction.—Engines rely on the ability to perform useful operations by exploiting incoherent resources. Typical examples are classical thermal machines, which extract work from a "working fluid" upon transfer of heat from a hot to a cold bath: the efficiency of such machines is defined by the ratio between the work produced and the heat absorbed from the hot bath. In the quantum realm, the working medium may provide a nonclassical inner structure. New out-of-equilibrium scenarios can thus be envisioned, in which different quantized transitions are coupled to independent heat baths. This strategy can be used, e.g., to invert the population of some medium by optical pumping, and to extract work by stimulating the transition: as a matter of fact, lasers and micromasers have long been interpreted as out-of-equilibrium heat engines [1,2].

More generally, optical pumping schemes are used to selectively prepare and maintain the working medium in a given nontrivial target steady state that is different from its thermal equilibrium state. In this spirit, the potential of achieving steady state entanglement of pairs of qubits through quantum optical bath engineering has started to be explored [3–16], most of the attention being focused on thermal baths as purely incoherent sources of nonclassical correlations [17–22]; however, for the latter protocols the amount of entanglement that can be generated without any additional feedback or filtering operation is typically rather modest [21,22]. By using reservoirs acting on collective degrees of freedom of the qubits, the upper theoretical limit for the concurrence can be increased to $C = 1/3$, which is asymptotically reached only under unrealistically large temperature gradients between the two reservoirs [17,19].

While no work is effectively extracted, these operations are still typical of a machine, for they reach a useful goal (i.e., the preparation of some out-of-equilibrium desired steady state) by exploiting incoherent resources. This calls for the definition of new criteria to assess the performance of such devices operating in the continuous regime [23–25]. On a parallel route, looking at the inner structure of a quantum system as a thermodynamic resource can be exploited, e.g., to increase the efficiency of miniaturized engines [26–28]. In this Letter, we study and thermodynamically characterize an optical pumping-based quantum machine, which allows for the generation of steady state entanglement in a bipartite system coupled to two incoherent reservoirs at different temperatures. We show that the machine performs all the better as its lead to larger amounts of steady state entropy production, consistently with the irreversible character of the protocol. Our study is based on a general definition of entropy production taken from stochastic thermodynamics [29–33], which interestingly allows us to extend the theoretical analysis to the case of reservoirs with negative effective temperatures. This generalized definition of baths allows us to increase the amount of steady state entanglement beyond the known limits imposed by classical heat baths at thermal equilibrium [17,19]. Finally, we propose a practical realization of the quantum thermal machine based on two independent and incoherently pumped qubits that are coupled to a leaky cavity mode [34,35].

Optical pumping and steady state entanglement.—As an elementary model of a driven-dissipative quantum machine producing steady state entanglement, we consider a generic bipartite quantum system consisting of two independent qubits of ground and excited levels respectively denoted $|0\rangle_i$ and $|1\rangle_i$ ($i = 1, 2$). The internal level structure of the composite system is then characterized by a diamondlike scheme, as represented in Fig. 1(a), with degenerate transitions energies $\omega_0 = \omega_A - \omega_G = \omega_S - \omega_G$, where $\sqrt{2}|S\rangle = (|0\rangle_1|1\rangle_2 + |1\rangle_1|0\rangle_2)$ and $\sqrt{2}|A\rangle = (|0\rangle_1|1\rangle_2 - |1\rangle_1|0\rangle_2)$ are the two maximally entangled Bell states,

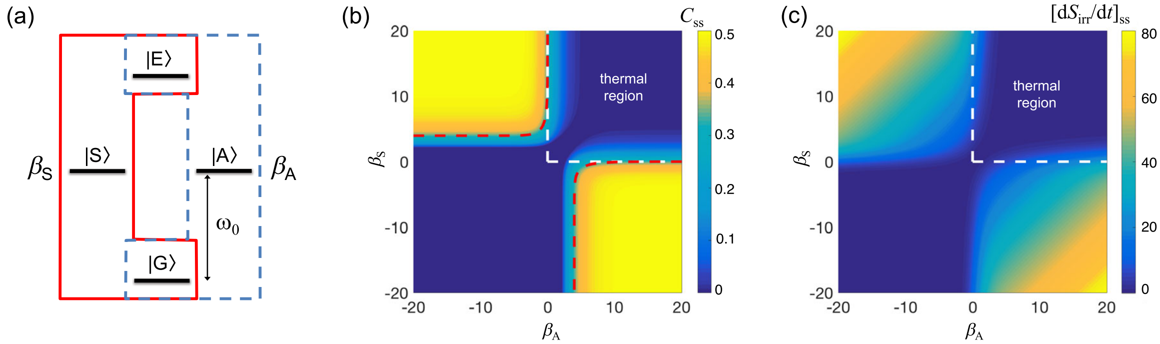

FIG. 1. (a) Elementary model of a driven-dissipative quantum thermal machine with diamondlike internal level structure and degenerate symmetric (S) and antisymmetric (A) states; the collective eigenstates are assumed to be connected to two independent reservoirs, defined through their effective temperatures $\mathcal{T}_S = \omega_0/\beta_S$ and $\mathcal{T}_A = \omega_0/\beta_A$, respectively. (b) Steady state concurrence, Eq. (4), plotted against β_A and β_S, respectively. White dashed lines mark the thermal region ($\beta_{A,S} \geq 0$); the red dashed curves show the contour line for the limiting value $C = 1/3$. (c) Rate of entropy production in steady state, Eq. (6), plotted in the same (β_A, β_S) plane as (b) and normalized to ω_0 and Γ^+.

respectively. The goal is to generate steady state entanglement by optically pumping the system in one of these states with high probability and by using incoherent resources, which can practically be realized by engineering some unbalance between the steady state population of $|S\rangle$ and $|A\rangle$.

We assume the two qubits to be coupled to two independent effective baths, each one acting on a collective degree of freedom. The inverse temperatures of these baths are denoted as β_A and β_S, which are allowed to assume negative values [36]. The dynamics of the open quantum system is completely described by the master equation [37] for the density matrix ($\hbar = 1$ and $k_B = 1$ in the following)

$$\partial_t \rho = i[\rho, \hat{H}_0] + \mathcal{L}(\rho), \tag{1}$$

where $\hat{H}_0 = \omega_0(\hat{c}_1^\dagger \hat{c}_1 + \hat{c}_2^\dagger \hat{c}_2)$ is the Hamiltonian, and

$$\mathcal{L}(\rho) = \sum_{i=A,S} \left(\frac{\Gamma_i^+}{2} \mathcal{D}_{\hat{J}_i^\dagger}(\rho) + \frac{\Gamma_i^-}{2} \mathcal{D}_{\hat{J}_i}(\rho) \right) \tag{2}$$

is the Liouvillian operator in Lindblad form, with $\mathcal{D}_{\hat{o}}(\rho) = 2\hat{o}\rho\hat{o}^\dagger - \{\hat{o}^\dagger \hat{o}, \rho\}$. We have introduced the collective operators $\hat{J}_S = \hat{c}_1 + \hat{c}_2$ and $\hat{J}_A = \hat{c}_1 - \hat{c}_2$, respectively, where $\hat{c}_i$ ($\hat{c}_i^\dagger$, $i = 1, 2$) are destruction (creation) operators obeying anticommutation rules $\{\hat{c}_i, \hat{c}_j^\dagger\} = \delta_{ij}$. We assume a common pumping rate for the two collective modes, i.e., $\Gamma_i^+ = \Gamma^+$ for $i = A, S$, while we will allow for independent dissipation rates, Γ_S^- and Γ_A^-, verifying

$$\frac{\Gamma^+}{\Gamma_S^-} = e^{-\beta_S}, \qquad \frac{\Gamma^+}{\Gamma_A^-} = e^{-\beta_A}, \tag{3}$$

in which the effective temperatures of the baths are given in units of ω_0.

The amount of steady state entanglement that can be generated is quantified from the degree of nonseparability of the given steady state ρ_{SS}, i.e., the solution of the linear equation $[\rho_{SS}, \hat{H}_0] = i\mathcal{L}(\rho_{SS})$. As an entanglement measure we hereby use the concurrence [38], an entanglement monotone defined for bipartite quantum systems as $C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, where λ_i^2 are the eigenvalues of the Hermitian matrix $\rho\tilde{\rho}$ ordered as $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$, and $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$. For maximally entangled pure states such as, e.g., Bell states, the concurrence is bound to $C[\rho] = 1$. For the model above, the steady state concurrence can be analytically solved as [39]

$$C_{SS} \equiv C(\rho_{SS}) = \max\{0, (N_1 - N_2)/d\}, \tag{4}$$

with

$$\begin{aligned} N_1 &= |\mathcal{A}(\mathcal{S}/2 - 1)|, \\ N_2 &= \{(\mathcal{S}/2 + 1)[2\mathcal{S}^2 + 2\mathcal{P}(\mathcal{S} - 2)]\}^{1/2}, \\ d &= 1 + \mathcal{S}^2 + \mathcal{S}(\mathcal{P} + 3)/2 - \mathcal{P}, \end{aligned} \tag{5}$$

in which we defined $\mathcal{S} = \exp(\beta_A) + \exp(\beta_S)$, $\mathcal{A} = \exp(\beta_A) - \exp(\beta_S)$, and $\mathcal{P} = \exp(\beta_A + \beta_S)$.

A plot of Eq. (4) is given in Fig. 1(b) as a function of the two inverse temperatures. As expected, the steady state is fully separable ($C_{SS} = 0$) for balanced reservoirs, i.e., when $\beta_S \simeq \beta_A$. In this case, most of the stationary population is either in $|G\rangle$ or $|E\rangle$ on average, while the rest is in an equal mixture of the two Bell states $|A\rangle$ and $|S\rangle$. On the other hand, an unbalance in the two thermal reservoirs allows for driving the system in a nonseparable steady state, with the population of either $|A\rangle$ or $|S\rangle$ dominating over the other. The amount of entanglement is limited to the value $C = 1/3$ when classical thermal reservoirs at positive temperatures are assumed (see dashed lines superimposed to the color scale plot), as also inferred from the analytic expression above [39]. This limiting value is reached when the cold bath is at zero temperature while the hot one goes to infinite temperature. In such a case, the population of the

Bell states is unbalanced, such that $1/3$ of the weight is in the entangled state coupled to the hot bath, while the rest of the, i.e., $2/3$, is in the ground state $|G\rangle$. A similar result was found in alternative models of bipartite quantum systems coupled to thermal reservoirs [17,19].

Going beyond previous studies, we see that the optical pumping can be improved by relaxing the conditions of real thermal reservoirs with positive temperatures. If negative temperatures are authorized, maximal steady state entanglement is reached when the hot bath is at effective negative temperature, while the cold bath is at a positive and small one [Fig. 1(b)]. Then, the system is pumped into the maximally entangled state coupled to the negative effective temperature bath with $1/2$ stationary probability, with zero probability in the other, giving the limiting value $C_{\mathrm{SS}} = 1/2$. This is a key result of this work: a bipartite quantum system can be optically pumped into a maximally entangled steady state by exploiting purely incoherent resources, with the largest concurrence reaching the limiting value of 0.5 if one of the two Bell states is coupled to a bath at negative effective temperature. In the absence of feedback or further purification of the steady state [22,42,43], this is the theoretical limiting value. Notice that the regions with the highest concurrence are all outside the thermal region, which could in principle be reached with classic thermal baths. Notice also that the lower left region, corresponding to both reservoirs being at negative effective temperature, gives $C_{\mathrm{SS}} = 0$ due to the largest occupancy of the fully separable $|E\rangle$ state, i.e., corresponding to the population inversion of the diamond at large pumping.

Entropy production and irreversibility.—The optically pumped bipartite system is now analyzed in terms of its thermodynamic properties. The whole protocol aims at driving and maintaining a quantum system out of equilibrium and, therefore, is irreversible by nature. The degree of irreversibility is quantified by the rate of steady state entropy production, $\dot{S}_{\mathrm{irr}} = [dS_{\mathrm{irr}}/dt]_{\mathrm{SS}}$. If the reservoirs are real thermal baths, this rate is classically given by

$$\dot{S}_{\mathrm{irr}}[\rho_{\mathrm{SS}}] = -\beta_A \dot{Q}_A[\rho_{\mathrm{SS}}] - \beta_S \dot{Q}_S[\rho_{\mathrm{SS}}] \geq 0, \tag{6}$$

where the steady state heat currents are defined as $\dot{Q}_i[\rho_{\mathrm{SS}}] = \mathrm{Tr}\{H_0 \mathcal{L}_i(\rho_{\mathrm{SS}})\}$, with $\mathcal{L}_i(\rho_{\mathrm{SS}})$ $(i = A,\ S)$ as in Eq. (2), which verify $\dot{Q}_A[\rho_{\mathrm{SS}}] + \dot{Q}_S[\rho_{\mathrm{SS}}] = 0$. In this classical case, Eq. (6) simply corresponds to the increase of entropy of the isolated system consisting of the two qubits and the two baths, consistently with the second law. Stochastic thermodynamics allows us to extend the concept of entropy production to new regimes where incoherent resources do not reduce to thermal baths [29–33]. Here, entropy production is defined at the single realization level, by comparing the respective probabilities of the realization in the direct and in some fictitious, reversed protocol. Such definition verifies the second law (the rate of entropy production is positive on average), and matches Eq. (6) if the reservoirs are thermal. Remarkably, based on stochastic thermodynamics it can be shown that the validity of Eq. (6) still holds in the case of reservoirs at negative effective temperature [39]. The results are shown in Fig. 1(c) as a function of β_A and β_S, displaying a striking correlation with the concurrence plot: our protocol can be seen as a machine operating in the steady state regime, whose ability to generate entanglement is maximized with the entropy production rate.

Cavity QED-based implementation.—A natural question is whether the theoretical model in Eq. (2) can be practically realized in a physical system that is amenable to experimental implementation. We show here that this is the case for a quite straightforward cavity QED situation in which two independent and incoherently pumped qubits are coupled to a single radiation mode of an electromagnetic resonator, as schematically represented in Fig. 2(a). Specifically, we consider a pair of pointlike two-level

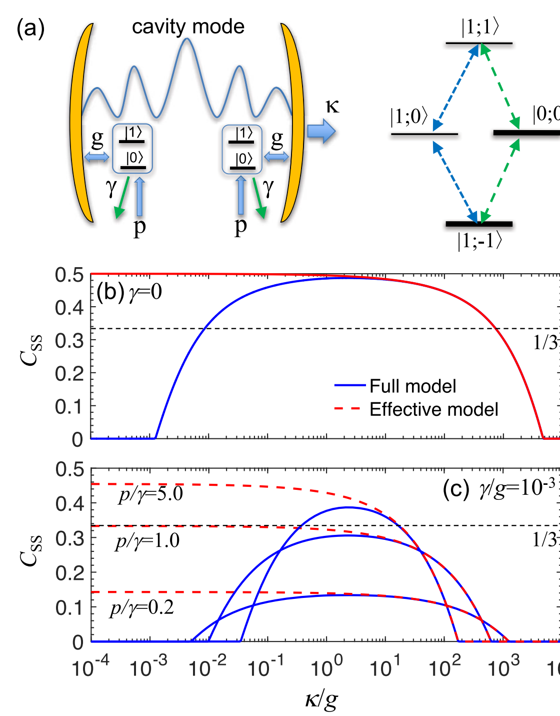

FIG. 2. (a) Quantum optical scheme of the full cavity QED model representing a pair of two-level emitters coupled to the same lossy cavity mode and incoherently pumped by an external drive, and the corresponding level scheme of the effective model after adiabatic elimination of the cavity degree of freedom. (b) Comparison between the steady state concurrence calculated numerically for the full quantum optical model, Eq. (8), and analytically for the effective model, Eq. (2) with parameters as in Eq. (9), as a function of the cavity dissipation rate for the ideal case of negligible qubits relaxation rate ($\gamma = 0$, at pumping strength $p = 0.0002g$). (c) Same comparison for finite γ and varying pumping strength: $p = 0.0002g$, $p = 0.001g$, $p = 0.005g$. Here, the qubit-cavity coupling rate is chosen to be $g/\omega_0 = 0.001$.

systems that are resonantly ($\omega_{\rm cav} = \omega_0$) coupled to a single-mode resonator at the same rate $g \ll \omega_0$, such that rotating wave approximation is justified and their Hamiltonian is a two-emitters Tavis-Cummings model,

$$\hat{H}_{\rm TC} = \sum_{i=1}^{2} \omega_0 \hat{c}_i^\dagger \hat{c}_i + \omega_{\rm cav} \hat{a}^\dagger \hat{a} + \sum_{i=1}^{2} g(\hat{c}_i^\dagger \hat{a} + \hat{c}_i \hat{a}^\dagger), \quad (7)$$

where $\hat{a}$ ($\hat{a}^\dagger$) is the destruction (creation) operator of the single-mode cavity photons. The master equation describing the driven-dissipative system of Fig. 1(b) is thus $\partial_t \rho = i[\rho, \hat{H}_{\rm TC}] + \mathcal{L}(\rho)$, where the full Liouvillian explicitly reads

$$\mathcal{L}(\rho) = \frac{p}{2} \sum_{i=1,2} \mathcal{D}_{\hat{c}_i^\dagger}(\rho) + \frac{\gamma}{2} \sum_{i=1,2} \mathcal{D}_{\hat{c}_i}(\rho) + \frac{\kappa}{2} \mathcal{D}_{\hat{a}}(\rho), \quad (8)$$

in which p and γ are the incoherent pumping and relaxation rates of the two (identical) qubits, and κ describes the photon emission rate from the cavity. Notice that an incoherent pumping scheme is realized whenever high-energy excitations relax to a well-defined ground state transition at a certain rate, even if the original source of excitation can be a coherent one (e.g., an off-resonant laser). The steady state of the full model can be solved numerically [39]. This model has been previously analyzed, e.g., in Ref. [35], when it was evidenced that a steady state subradiant regime exists over a broad range of values κ/g, under weak pumping conditions $p \ll g$. This system can be effectively described in the collective spin basis by adiabatically eliminating the cavity mode [34], which is fully justified for $\kappa/g > 1$. In fact, under such conditions the cavity only acts as an additional dissipation channel in the reduced two-qubits subspace [35] in which each qubit is further relaxed at a rate $\Gamma = 4g^2/\kappa$ in addition to the intrinsic spontaneous emission at rate γ. Hence, Eq. (8) can be recast exactly as Eqs. (1) and (2), after straightforward algebra with the following relations:

$$\Gamma^+ = p/2, \qquad \Gamma_A^- = \gamma/2, \qquad \Gamma_S^- = \Gamma + \gamma/2. \quad (9)$$

The effective temperatures result now from combinations of the physical parameters of the model: $\beta_A = \log[\gamma/p_x]$ and $\beta_S = \log[(\gamma + 2\Gamma)/p_x]$.

In the subradiant regime the system is optically pumped in the dark $|A\rangle = |0;0\rangle$ state (i.e., the singlet in the $|J;M_J\rangle$ notation for eigenstates of the total angular momentum), thus creating an imbalanced population with respect to the $|S\rangle = |1;0\rangle$ (triplet) state [35], as schematically represented in the diamondlike level structure of Fig. 2(a). This is confirmed by plotting the steady state concurrence of the two qubits in Figs. 2(b) and 2(c), which is evidently different from zero only when κ/g falls in the subradiant sector of the model. There exists an optical pumping range for which the system reaches its maximal concurrence, which also depends on γ/g. In particular, the maximal value is $C_{\rm SS} \simeq 0.4$ around $p/\gamma \simeq 5$ for the case shown in Fig. 2, but it can be even larger and approaching the $C_{\rm SS} = 0.5$ limit for smaller values of γ/g (see, e.g., full numerical results in Ref. [39]). First, in Fig. 2(b) we show the ideal result for $\gamma = 0$, corresponding to the negative effective temperature reservoir coupled to the dark state, which gives the limiting value $C_{\rm SS} \to 0.5$ when $p/\Gamma \to 0$ (in agreement with the results in Fig. 1); the full model only follows the effective model for $\kappa/g > 1$, i.e., until the adiabatic elimination of the cavity mode holds. At difference with the general model of the previous section, here the A-S symmetry is broken since only the antisymmetric state is dark and, from Eq. (9), $\beta_S > \beta_A$. Hence, with reference to Fig. 1, only the part above the $\beta_S = \beta_A$ diagonal should be considered when dealing with this cavity QED implementation. In Fig. 2(c) we show the behavior of the steady state concurrence for $\gamma = 10^{-3}g$, which is usually the case in most practical realizations of this quantum optical model, e.g., in solid-state cavity QED. While it is evident that the regime of nonseparability narrows in κ as p increases, it should also be noted that for the proper values of κ the thermodynamic limit is overcome (i.e., $C_{\rm SS} > 1/3$) as soon as $p > \gamma$. The latter condition corresponds to the onset of negative effective temperature for the dark state reservoir [39].

Discussion.—We propose and thermodynamically analyze a new protocol to generate steady state entanglement of a bipartite quantum system from incoherent resources. We show that this scheme can be interpreted as a continuous quantum engine whose performances are optimized when the entropy production rate is maximal. This effect makes this class of engines very different from classical engines, whose yield is usually maximized in the reversible regime. We finally highlight the potential interest of this result from an experimental point of view: there are different platforms where these results at the forefront between quantum optics, quantum information, and quantum thermodynamics could be tested, ranging from semiconductor quantum dots spatially and spectrally matched to photonic nanoresonators [44–46] to superconducting circuit quantum electrodynamics devices [47–49]. More quantitative information is provided in Ref. [39].

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