Nonequilibrium Steady State Generated by a Moving Defect: The Supersonic Threshold

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We consider the dynamics of a system of free fermions on a 1D lattice in the presence of a defect moving at constant velocity. The defect has the form of a localized time-dependent variation of the chemical potential and induces at long times a nonequilibrium steady state (NESS), which spreads around the defect. We present a general formulation that allows recasting the time-dependent protocol in a scattering problem on a static potential. We obtain a complete characterization of the NESS. In particular, we show a strong dependence on the defect velocity and the existence of a sharp threshold when such velocity exceeds the speed of sound. Beyond this value, the NESS is not produced and, remarkably, the defect travels without significantly perturbing the system. We present an exact solution for a δ-like defect traveling with an arbitrary velocity and we develop a semiclassical approximation that provides accurate results for smooth defects.

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Recent experimental advances in the context of cold atoms [1–16] converted the study of out-of-equilibrium closed quantum systems from an academic debate to a concrete and extremely active research topic. In this context, the simplest protocol is known as *quantum* quench [17], where the system is brought out of equilibrium by a sudden change of a coupling constant. Despite the unitary time evolution, in the thermodynamic limit, local observables reach a time-independent expectation value and the system locally equilibrates [18]. One-dimensional systems have played a special role because of the presence of special techniques, such as conformal field theory [19] and integrability [20,21]. In particular, in *homogeneous* quenches a global parameter is modified and several studies with a number of integrable models and initial conditions [22–31] (see also Ref. [32] for a review) have confirmed the validity of the generalized Gibbs ensemble (GGE) [33]. The GGE hypothesis prescribes that the steady state assumes a thermal form, but with an extended set of temperatures, conjugated to each (quasi-) local conserved quantity present in the model [33,34]. The value of such temperatures is then fixed by the initial expectation value of conserved quantities [35–49].

A more complex scenario emerges for *inhomogeneous* quenches, where, because of an asymmetry in the initial state or in the final Hamiltonian, translational invariance is explicitly broken. The simplest case is the one in which a localized defect perturbs an otherwise homogeneous system. As the spreading of correlations is bounded by a maximal speed of sound v_s, this defect cannot affect immediately the whole (thermodynamically large) system [50,51]. In the presence of *ballistic* dynamics, at late times t and large distance x from the defect, the system reaches a locally quasistationary state (LQSS) [52], whose properties depend only on the ray $\zeta = x/t$ inside the light cone $|\zeta| < v_s$. The infinite time limit at finite distance (i.e., $\zeta = 0$) corresponds instead to the nonequilibrium steady state (NESS). Our present understanding of LQSS is based on numerical studies [53–55], free models [56–68], CFT [69–74], and only recently on truly interacting integrable models [52,75–79]. In particular, the hydrodynamic description [52,78–80] has led to exact results with possible applications to several contexts [81–86].

In this Letter, we consider instead a moving defect. This setting offers an additional parameter to control the stationary state, particularly interesting in those systems whose excitations possess a maximum velocity. Moving impurities have already been experimentally probed, in particular in Ref. [15], the case of a Tonks Girardeau gas [87] (intimately linked to the free fermion case here analyzed) was considered. While several works have considered moving impurities in several contexts [88–98], the long-time out-of-equilibrium dynamics has never been addressed so far. In particular, does the system still react forming a LQSS? How the LQSS changes for different velocities of the defect? We explore these questions in the prototypical case of hopping fermions on a lattice, which is amenable to a full analytical treatment still retaining a rich phenomenology. At time $t > 0$, the dynamics is governed by the following time-dependent Hamiltonian:

$$H = \sum_j -\frac{1}{2}(d_j^\dagger d_{j+1} + d_{j+1}^\dagger d_j) + V(j - vt) d_j^\dagger d_j, \quad (1)$$

where d_j are the fermionic operators $\{d_j, d_l^\dagger\} = \delta_{j,l}$. With the current choice of couplings, the unperturbed system has a sound velocity $v_s = 1$. This lattice model can be mapped in the XX spin chain [99–101], where the defect plays the role of a traveling magnetic impurity. The system is assumed to be initially in the unperturbed ground state ($V = 0$) at fixed particle number (the finite temperature case being a trivial generalization). Here, the impurity is suddenly created and put in motion; other similar settings (e.g., the motion of a preexisting defect) would lead to the same late time physics. We show the emergence of a *moving* LQSS, whose rays refer to the instantaneous position of the defect. The amplitude of the LQSS is showed to be suppressed when the velocity of the defect is increased and the formation of a LQSS becomes *impossible* for a *supersonic* impurity. We fully determine the *exact* LQSS generated by a δ-like perturbation and provide a semiclassical expression of the LQSS, valid for smooth defects. Even though the model we are considering is free, the emergent LQSS displays absolutely nontrivial features understood thanks to its exact description, while approximated methods commonly used could lead to misleading conclusions (see the Supplemental Material [102]for discussion of the Luttinger-Liquid approximation [103,104]).

The LQSS as a scattering problem.—The effect of the moving impurity is best understood in a pictorial representation where the initial state can be regarded as a gas of excitations. The excitations move freely in the space until the defect is met, then a scattering event takes place and the excitation continues in a free motion, with a different momentum. The nontrivial LQSS is due to the scattered particles, spreading ballistically from the defect. Thereafter, we make this argument rigorous. The initial state is a filled *Fermi sea* with Gaussian correlations. As the postquench Hamiltonian is quadratic, all the local observables at any time are fixed by the two-point correlators via the Wick theorem. Thereafter, we focus only on the case $v > 0$. Changing the reference frame to set the defect at rest would remove the explicit time dependence of the problem, but such a program is foiled by the discreteness of the lattice. This difficulty can be circumvented through a map to a continuous fermionic model $\{c_x, c_y^\dagger\} = \delta(x - y)$ with Hamiltonian

$$H_c = \int dx - \frac{1}{2}(c_x^\dagger c_{x+1} + c_{x+1}^\dagger c_x) + V(x - vt) c_x^\dagger c_x. \quad (2)$$

From this model all the discrete correlation functions are *exactly* recovered. Indeed, H_c only couples a coordinate x with $x + n$, $n \in \mathbb{Z}$. On this sublattice, continuous and discrete (normal ordered) correlation functions satisfy the same equation of motion; thus, their solution is the same provided consistent initial conditions $\langle c_j^\dagger c_l \rangle_{t=0} = \langle d_j^\dagger d_l \rangle_{t=0}$ have been chosen. We can thus employ Eq. (2) to study the dynamics of the system and later restrict ourselves to integer positions. This approach leaves us the freedom of arbitrarily choosing the correlator at noninteger values; a convenient choice is to assume the initial state in the momentum space is described by the same Fermi sea of the discrete model. As x is a continuous coordinate, we can now introduce a reference frame $c_x = \tilde{c}_{x-vt}$, where the defect is at rest. In terms of this new field, the equation of motion can be derived from the time-independent Hamiltonian

$$\tilde{H}_c = \int dx i v \tilde{c}_x^\dagger \partial_x \tilde{c}_x - \frac{1}{2}(\tilde{c}_x^\dagger \tilde{c}_{x+1} + \tilde{c}_{x+1}^\dagger \tilde{c}_x) + V(x) \tilde{c}_x^\dagger \tilde{c}_x. \quad (3)$$

Clearly, the dynamics induced by Eq. (3) depend on the details of $V(x)$. However, being that the defect is localized, we can use scattering theory. We introduce the mode operators $\eta_k = \int dx \psi_k^*(x) \tilde{c}_x$, where the $\psi_k(x)$ is the normalized wave function satisfying the Lippmann-Schwinger equation [105]. In other words, far away from the defect, it assumes the simple form of a scattering problem [106],

$$\psi_k(x) = \theta(-xv(k)) \frac{e^{ikx}}{\sqrt{2\pi}} + \sum_{k_n} S_{k\to k_n} \theta(xv(k_n)) \frac{e^{ik_n x}}{\sqrt{2\pi}}, \quad (4)$$

where the incoming wave is expanded into outgoing waves weighted with the scattering amplitudes $S_{k\to k_n}$. The wave vectors k_n are obtained via energy conservation $E(k) = E(k_n)$, with $E(k) = -\cos(k) - vk$ the single-particle energy in the defect reference frame and $v(k) = dE(k)/dk$ its group velocity. Corrections to Eq. (4) vanish exponentially in the distance and the scattering amplitude takes the form

$$S_{k\to k_n} = \delta_{k,k_n} - 2i\pi |v(k_n)|^{-1} \langle k_n | \hat{V} | \psi_k \rangle, \quad (5)$$

where we introduced a bra-ket notation with $\langle x | k \rangle = e^{ikx}/\sqrt{2\pi}$ and $\hat{V}(x')|x\rangle = V(x')\delta(x - x')|x\rangle$. The unitarity of the Lippmann-Schwinger equation permits us to derive an exact sum rule whose explicit derivation is left to Ref. [102]. The initial two point correlator is diagonal in momentum space; thus, we are ultimately led to consider the time evolution of plane waves $f_k(x,t) = \langle x | e^{-iHt} | k \rangle$. In terms of the eigenbasis $|\psi_q\rangle$ is

$$f_k(x,t) = \sqrt{2\pi} \int dq e^{-iE(q)t} \psi_q(x) \langle \psi_q | k \rangle. \quad (6)$$

The large time behavior of f_k is readily extracted using Eq. (4) together with the aforementioned sum rule for S, as the corrections to the asymptotic approximation (4) are ineffective in the LQSS scaling limit [102]. We can then obtain the LQSS two-point correlator in the form of a ray-dependent GGE with an excitation density $\rho_\zeta(k)$, being the ensemble Gaussian and (locally) diagonal in the momentum space [107–110]

$$\begin{aligned}\langle \tilde{c}_x^\dagger \tilde{c}_y \rangle_t &= \int_{-\pi}^{\pi} \frac{dk}{2\pi} \rho_\zeta(k) e^{ik(y-x)} \\ &= \int_{-k_f}^{k_f} \frac{dk}{2\pi} e^{ik(y-x)} [1 - \theta(\zeta v(k)) \theta(|v(k)| - |\zeta|)] \\ &\quad + \sum_{k_n} |S_{k\to k_n}|^2 \theta(\zeta v(k_n)) \theta(|v(k_n)| - |\zeta|) e^{ik_n(y-x)}. \end{aligned} \tag{7}$$

As the two point correlator is a decaying function in the relative distance, in the large time limit, one must set $x/t \sim y/t \sim \zeta$. Equation (7) is easily interpreted: when a particle of momentum k collides with the defect, it can scatter in several channels k_n with amplitudes $S_{k\to k_n}$. This produces a *hole* in the old momentum and a flux of particles with the new momenta: these spread ballistically at their own velocity $v(k_n)$ leading to the ray dependence on ζ. In Fig. 1, we plot the space-time profile of the fermion density for a δ-like defect, whose detailed analysis will be considered later. With no further information on S, we can discuss the behavior of the LQSS in terms of v analyzing the scattering channels, identified by $E(k_n) = E(k)$. For generic v, several channels are open (a divergent number letting $v \to 0$) and they diminish increasing v until, for a supersonic defect, the excitations are unavoidably purely transmitted. In this case, the sum rule obeyed by S forces $|S_{k\to k}| = 1$ [102](simply interpreted as particle conservation). Therefore, in the supersonic regime, the LQSS can *never* be produced (see also Fig. 1). Note that the LQSS (7) emerges at late times and large distances from the defect, when Eq. (4) holds: despite the absence of LQSS, a supersonic defect still gives nontrivial effects localized on the impurity. We will further analyze this behavior within the semiclassical approximation.

An exactly solvable case.—We now consider an example of defect for which the LQSS can be *exactly* determined, i.e., the limit of an extremely narrow defect $V(x) = c\delta(x)$. In the discrete model (1) the Dirac-δ is ill defined when $v = 0$. However, this is not the case for a defect in motion $v \neq 0$: the δ-defect represents an impulsive kick traveling along the lattice and leads to well-defined equations of motion. The detailed calculations can be found in the Supplemental Material [102], here we simply report the result. Referring to Eq. (5), we have

$$2\pi \langle k_n | V | \psi_k \rangle = -\left[\sum_{k_m} \frac{1}{2i|v(k_m)|} - \frac{1}{2v} \cot\left(\frac{c}{2v}\right) + \mathcal{I}(k) \right]^{-1}. \tag{8}$$

Above, $\mathcal{I}(k) = \mathcal{P} \int (dq/2\pi)[E(k) - E(q)]^{-1}$, where the principal value prescription in integrating the singular points is assumed. In Fig. 2 the exact solution for the fermion density is tested against numerical simulations. Numerical data show persistent oscillations due to the interference of the various scattering channels. These oscillations decay far away from the defect and are therefore inessential in the LQSS scaling limit, but are nevertheless captured by scattering theory (see Ref. [102]). The nontrivial density profile is only one of the manifestations of the LQSS, being the complicated structure of the underlying scattering best appreciated in the excitation density propagating from the defect (Fig. 3).

The semiclassical approximation.—For general potential, determining the scattering amplitudes requires some approximation schemes. Here, we develop a semiclassical analysis in which the scattering interpretation is most easily displayed. Our derivation is based on the Wigner distribution [111–113]. Semiclassical approaches are commonly found in literature [114–118] (see also Refs. [119–122] for quantum corrections) even though, to the best of our knowledge, the problem at hand has never been addressed. Consider the two-point correlator $\mathcal{C}_t(x, s) = \langle \tilde{c}^\dagger_{x+s/2} \tilde{c}_{x-s/2} \rangle_t$, under the assumption of (i) weak dependence with respect to the x coordinate, and (ii) fast decay of $\mathcal{C}_t(x, s)$ as a

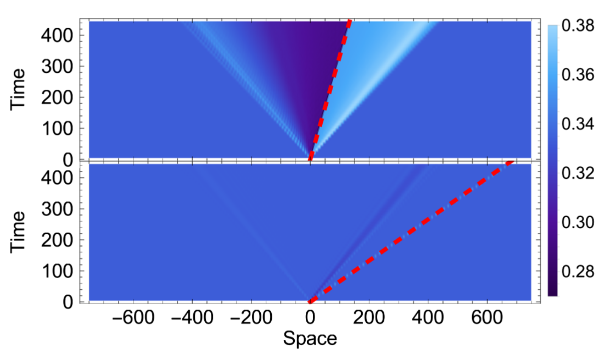

FIG. 1. Fermion density generated by a δ-like defect [$V(x) = c\delta(x)$, $c = 0.5$] moving at $v = 0.3$ (top) and $v = 1.5$ (bottom). The defect is initially placed at zero and it moves along the line dashed in red. In the subsonic case the constant values along the rays indicate the realization of a LQSS, which is instead absent in the supersonic regime.

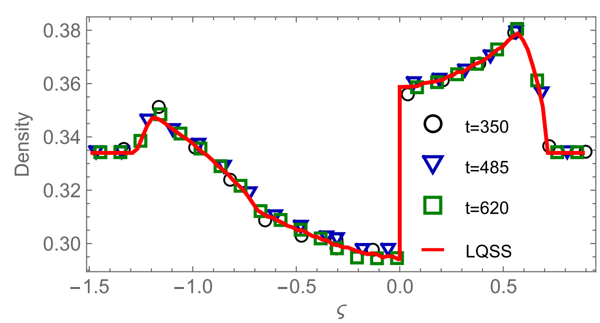

FIG. 2. The numeric fermionic density $\langle d_j^\dagger d_j \rangle$ with $j = (v + \zeta)t$ as a function of ζ at three different times is tested against the analytic LQSS for a δ-like defect $V(x) = c\delta(x)$ with $c = 0.5$ and velocity $v = 0.3$. The density is fixed by $k_f = \pi/3$. The oscillations mentioned in the main text are smeared out by averaging on neighboring sites.

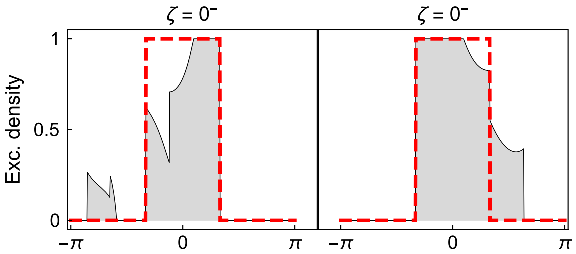

FIG. 3. Analytic prediction for the excitation density profile propagating on the left (left panel) and on the right (right panel) of a δ-like defect. The dashed red line is the initial Fermi sea, the black line the spreading excitation whose integral (shaded area) equals the spatial fermionic density, which jumps around the defect (see Fig. 2). The same parameters of Fig. 2 are used.

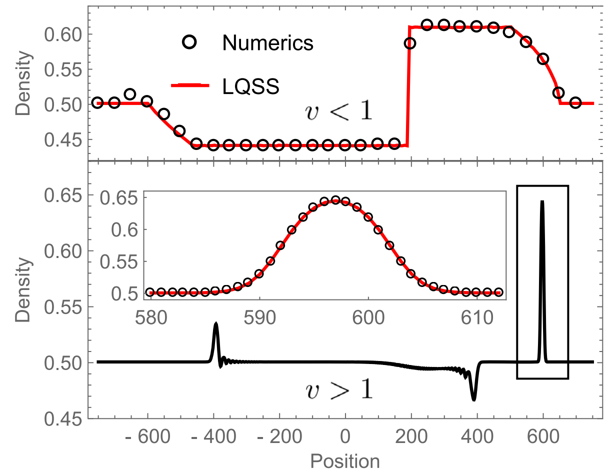

FIG. 4. The numerical fermionic density is tested against the semiclassical LQSS for a Gaussian shaped repulsive potential $V(x) = e^{-\sigma x^2}$ with $\sigma = 0.04$, $k_f = \pi/2$. Top: subsonic defect ($v = 0.3$) and $t = 636$. The defect is placed where the discontinuity occurs and a LQSS is produced. Bottom: supersonic defect ($v = 1.5$, $t = 388$) and placed in correspondence with the rightmost peak: the LQSS is indeed absent and the semiclassical prediction captures the density profile on the defect (inset). The corrections to the analytic prediction are a combined effect of (i) the particles initially sat on the defect that have not yet managed to spread and (ii) the delay time experienced by the particles scattering on the defect. Both these effects are negligible at late time and in the scaling limit. Quantum effects can be recognized in the propagating ripples [119–121], more evident in the supersonic case.

function of s, on a length scale much smaller than the length on which the defect varies, we can approximate the equation of motion of the correlator as [123]

$$\begin{aligned} i\partial_t \mathcal{C}_t(x,s) = & \frac{1}{2}\partial_x[\mathcal{C}_t(x,s+1) - \mathcal{C}_t(x,s-1)] \\ & - s\partial_x V(x)\mathcal{C}_t(x,s) + iv\partial_x \mathcal{C}_t(x,s) + \cdots \end{aligned} \tag{9}$$

Higher derivatives of the correlators and of the defect are neglected. Being that the initial state is homogeneous, this approximation is verified for a smooth potential $V(x)$. In terms of the Wigner function $\rho_t(x,k)$ defined as

$$\rho_t(x,k) = \int_{-\infty}^{\infty} ds e^{iks} \mathcal{C}_t(x,s), \tag{10}$$

these apparently complicated equations reduce to a classical Liouville equation

$$\partial_t \rho_t + \{\mathcal{H}_{\mathrm{cl}}, \rho_t\}_{k,x} = 0, \quad \mathcal{H}_{\mathrm{cl}} = -\cos(k) - vk + V(x). \tag{11}$$

Above, $\{\ldots\}_{k,x}$ indicates the standard Poisson bracket and $\mathcal{H}_{\mathrm{cl}}$ is the classical Hamiltonian. In the semiclassical limit, ρ_t evolves as the density distribution of particles subjected to the classical equations of motion $\dot{x} = v(k)$, $k = -\partial_x V(x)$. The precise solution $x(t)$, $k(t)$ of these equations depends on the details of $V(x)$, but the trajectories $k(x)$ of the particles can be easily computed combining the conservation of the classical energy and the determination of the turning points, i.e., those points where $dx/dt = 0$ (see Ref. [102]). In the semiclassical language, computing the LQSS is reduced to simple classical scattering processes: whether a particle is reflected or transmitted by the defect is simply determined reconstructing the trajectories. The case of a supersonic defect is remarkably simple, since the equation of motion does not have turning points. In this case we readily obtain

$$\lim_{t\to\infty} \mathcal{C}_t(x,s) = \int_{-k_f}^{k_f} \frac{dk}{2\pi} \left| \frac{v(k)}{v(q_{k,x})} \right| e^{-iq_{k,x}s}, \tag{12}$$

valid also in proximity of the defect (within the semiclassical approximation). Above, $q_{k,x}$ is determined by energy conservation

$$-\cos(q_{k,x}) - vq_{k,x} + V(x) = -\cos(k) - vk, \tag{13}$$

which has a unique solution for a supersonic v. The semiclassical results and the numerics are compared in Fig. 4, further details about the semiclassical approximation are left to the Supplemental Material [102].

Conclusions and outlook.—We analyzed a local quench where the defect is moving at constant velocity in a system of hopping fermions on a lattice. In particular, we focused on the emergence of a locally quasistationary state and we studied the dependence of the latter in terms of the velocity of the defect. With general arguments, we showed the impossibility of a LQSS formation for supersonic defects. We provided *exact* results for a δ-like defect and a semiclassical analysis for general shapes. We are confident that our framework can be studied in forthcoming cold-atoms experiments, as in Ref. [15], a very similar setting was realized. The moving defect would then be an intriguing way to induce nonequilibrium dynamics and to probe the scattering properties of quasiparticle excitations.

Being that the hopping fermions is a free model, its simplicity allowed for exact computations: an intriguing question concerns the same problem in a truly interacting model like integrable spin chains. We expect that a promising investigative line could be the recently introduced generalized hydrodynamics [52,78–86], which for a free model reduces precisely to the semiclassical approach we used. This will be the subject of a forthcoming publication. Another natural approach would be the use of the recently introduced curved-CFT formalism [124].

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