Manipulating the Flow of Thermal Noise in Quantum Devices

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There has been significant interest recently in using complex quantum systems to create effective nonreciprocal dynamics. Proposals have been put forward for the realization of artificial magnetic fields for photons and phonons; experimental progress is fast making these proposals a reality. Much work has concentrated on the use of such systems for controlling the flow of signals, e.g., to create isolators or directional amplifiers for optical signals. In this Letter, we build on this work but move in a different direction. We develop the theory of and discuss a potential realization for the controllable flow of thermal noise in quantum systems. We demonstrate theoretically that the unidirectional flow of thermal noise is possible within quantum cascaded systems. Viewing an optomechanical platform as a cascaded system we show here that one can ultimately control the direction of the flow of thermal noise. By appropriately engineering the mechanical resonator, which acts as an artificial reservoir, the flow of thermal noise can be constrained to a desired direction, yielding a thermal rectifier. The proposed quantum thermal noise rectifier could potentially be used to develop devices such as a thermal modulator, a thermal router, and a thermal amplifier for nanoelectronic devices and superconducting circuits.

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Introduction.—The control of thermal noise in complex systems has straightforward applications to the miniaturization of technology; as devices become smaller and smaller it is essential to steer thermal noise away from hot spots towards sinks where it may be disposed of (see, e.g., Ref. [1]). Recently, a significant effort has emerged that is devoted to designing a new generation of thermal-based nanoscale devices such as thermal rectifiers [2–8], thermal logic gates [9], thermal diodes [10,11], and thermal transistors [12–14]. When quantum systems are coupled together, the thermal noise associated with the reduced state of each component is affected by the coupling, leading to a flow of thermal noise [15]; controlling this thermal noise is essential in the context of quantum technologies, such as quantum computers [17] and simulators [18], especially because of the fragility of quantum states and quantum correlations [19] which is well known from the literature. Coupled quantum systems can also be used to transfer signals; a signal input to one quantum system can appear at the output of another [15]. A basic building block for controlling how such signals flow around a complex system takes the form of devices that are nonreciprocal, in which transmission of a signal from one point to another is qualitatively different [20–24], in amplitude or phase, from transmission in the reverse direction. An interesting line of research has emerged recently that aims to use complex mechanical, electromagnetic, or quantum-optical systems to create effective optical isolators [25] or other kinds of nonreciprocity [26]. Several theoretical analyses [27–39] of such systems have been published and experimental demonstrations [40–60] reported, illustrating a rich variety of mechanisms for achieving the desired nonreciprocity. In their simplest form, several such mechanisms are based on coupled quantum systems that also share a common bath [32]. These can be conceptually connected to techniques discussed several years ago under the guise of cascaded quantum systems [61].

In this Letter we will combine cascaded quantum systems, nonreciprocal devices, and controlling the flow of thermal noise to achieve a thermal rectifier. We analyze a quantum system consisting of two fields between which we set up nonreciprocal transport. Our analysis differs from what is known in the literature because we are interested not in the transport of signals, but in the transport of thermal noise between the two fields. We also use recently developed techniques [62–64] for analyzing the flow of excitations between quantum systems and their heat baths to better understand how our system manipulates the flow of thermal noise. Our work thus considers thermal noise not as a nuisance complicating our analysis, but as the object of that analysis. Our results show that the temperature of a third bath can be used to *increase or decrease* the thermal noise of one system without affecting the other, paving the way to quantum thermal transistors.

We proceed as follows. First, we describe an effective quantum optics model based on the cascaded quantum systems formalism [Fig. 1(a)]. This yields general expressions that have a transparent physical meaning. We then

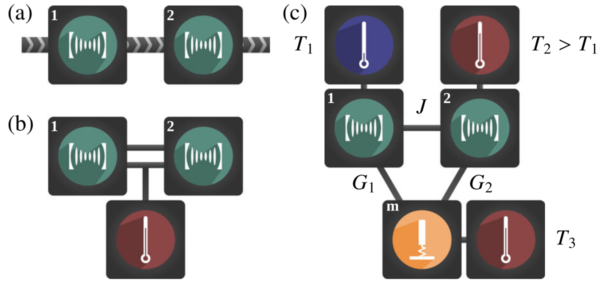

FIG. 1. In our quantum optics model (a) two harmonic oscillators, e.g., electromagnetic (optical or microwave) cavities, are arranged such that the output from system 1 is the input of system 2. (b) Equivalently, the two systems are connected via a coherent hopping term and share a common heat bath. (c) A physical realization of a thermal rectifier; a mechanical system is coupled to two electromagnetic cavities and a heat bath, is proposed as a realization. In this model, the two systems are also connected to their own heat baths.

develop an optomechanical model where a mechanical oscillator is coupled to two electromagnetic cavities [Figs. 1(b) and 1(c)]. We can show that these two systems behave identically with respect to quantum states that are broadband compared to coherent signals but still contained within the bandwidth of the mechanical oscillator. This allows us to use our general expressions to derive conclusions about this specific system. We then discuss how the effect we explore manifests itself in experiment, with reference to achievable parameters. Finally, we conclude with a short discussion on the implications of our model.

Effective quantum optics model.—To start off, we briefly summarize what is known about cascaded quantum systems; our aim is to build a model for the system shown in Fig. 1(a) to discuss its operation as a nonreciprocal thermal device. We start by considering two harmonic oscillators, associated with annihilation operators $\hat{c}_1$ and $\hat{c}_2$, respectively. Let $\hat{H}_{\text{sys}}$ govern the free dynamics of these systems. We *impose* cascaded dynamics (see Sec. 12.1 of Ref. [61]) onto these systems; i.e., we assume that the output of oscillator 1 forms the input of oscillator 2 via some channel, whereas the output of oscillator 2 does not feed back into oscillator 1. Define $\gamma_1 > 0$ and $\gamma_2 > 0$ as the coupling rates of the two systems, respectively, to this channel. Together with the two standard input-output relations $\hat{c}_{\text{out},i} = \hat{c}_{\text{in},i} + \sqrt{\gamma_i}\hat{c}_i$, where $i = 1, 2$, we must therefore add the restriction $\hat{c}_{\text{in},2} = \hat{c}_{\text{out},1}$. Next we can follow Ref. [61] in obtaining the Langevin equations governing the dynamics of this system, converting them into Itō stochastic differential equations, and from there deriving a master equation. In the following we denote by $\bar{N}_3$ the average occupation number of an effective common bath, and we take there to be no classical driving field associated with this bath. This master equation can be rewritten in Lindblad form to yield $\dot{\rho} = -(\imath/\hbar)[\hat{H}_{\text{sys}} + \hat{H}_{\text{hop}}, \rho] + (\bar{N}_3 + 1)\kappa_3\mathcal{D}_{\hat{c}_3}[\rho] + \bar{N}_3\kappa_3\mathcal{D}_{\hat{c}_3^\dagger}[\rho]$, where $\mathcal{D}_{\hat{c}}[\rho] = \hat{c}\rho\hat{c}^\dagger - \frac{1}{2}\{\rho, \hat{c}^\dagger\hat{c}\}$ is the standard dissipative Lindblad term, $\hat{H}_{\text{hop}} = (\imath\hbar/2)\sqrt{\gamma_1\gamma_2}(\hat{c}_1^\dagger\hat{c}_2 - \hat{c}_1\hat{c}_2^\dagger)$ is a hopping Hamiltonian, $\kappa_3 = \gamma_1 + \gamma_2$ is a collective damping rate, and $\hat{c}_3 = (\sqrt{\gamma_1}\hat{c}_1 + \sqrt{\gamma_2}\hat{c}_2)/\sqrt{\kappa_3}$ is a collective bosonic annihilation operator that obeys $[\hat{c}_3, \hat{c}_3^\dagger] = 1$. The physical content of this master equation is rather straightforward: To produce the nonreciprocal effect required of a cascaded system, the two oscillators must be coupled by a direct coherent hopping term as well as to a common bath; see Fig. 1(b). To account for an arbitrary phase ϕ in the hopping between the two oscillators, we replace $\hat{c}_2 \to e^{\imath\phi}\hat{c}_2$ throughout, yielding $\hat{H}_{\text{hop}} = (\imath\hbar/2)\sqrt{\gamma_1\gamma_2}(e^{\imath\phi}\hat{c}_1^\dagger\hat{c}_2 - e^{-\imath\phi}\hat{c}_1\hat{c}_2^\dagger)$ and $\hat{c}_3 = (\sqrt{\gamma_1}\hat{c}_1 + \sqrt{\gamma_2}e^{\imath\phi}\hat{c}_2)/\sqrt{\kappa_3}$. This master equation results in equations of motion that are maximally nonreciprocal with respect to $\hat{c}_1$ and $\hat{c}_2$, which is due to a coherent cancellation (addition) of the hopping between the direct term and through the bath in the direction $2 \to 1$ ($1 \to 2$). The phase-matching condition required to ensure this cancellation or addition is encoded in a $-$ sign in the coherent hopping Hamiltonian, compared to a $+$ sign in the dissipation-related operator $\hat{c}_3$. For further generality, we must add terms to this master equation. First, we modify the hopping Hamiltonian to $\hat{H}_{\text{hop}} = (\imath\hbar/2) \times \sqrt{\gamma_1\gamma_2}(e^{\imath\phi}\hat{c}_1^\dagger\hat{c}_2 - e^{-\imath\phi}\hat{c}_1\hat{c}_2^\dagger) + \hbar(F\hat{c}_1^\dagger\hat{c}_2 + F^*\hat{c}_1\hat{c}_2^\dagger)$, where F is an arbitrary complex constant; full nonreciprocity requires $F = 0$. Second, we add a bath for each oscillator:

$$\dot{\rho} = -\frac{\imath}{\hbar}[\hat{H}_{\text{sys}} + \hat{H}_{\text{hop}}, \rho] + \sum_{i=1}^{3}\{(\bar{N}_i + 1)\kappa_i\mathcal{D}_{\hat{c}_i}[\rho] + \bar{N}_i\kappa_i\mathcal{D}_{\hat{c}_i^\dagger}[\rho]\}. \tag{1}$$

In the following we will use this master equation to describe any system composed of two oscillators that are coupled directly to one another, to a common thermal bath, and to two individual thermal baths [Fig. 1(c)]. We will show that an effective model where the coupling between two electromagnetic cavities and their common bath are induced by a third, mechanical, mode is equivalent to the one described here.

To proceed, we convert the master equation to its equivalent quantum Langevin equations [15]: We derive the mean-field equations of motion from Eq. (1), obtain the operator equations by adding noise terms using the fluctuation-dissipation theorem, and then Fourier-transform to the frequency domain:

$$-\imath\omega\begin{pmatrix}\hat{c}_1\\ \hat{c}_2\end{pmatrix} = \begin{bmatrix}-\imath\omega_1 - \frac{\gamma_1+\kappa_1}{2} & -\imath F\\ -\imath F^* - \sqrt{\gamma_1\gamma_2}e^{\imath\phi} & -\imath\omega_2 - \frac{\gamma_2+\kappa_2}{2}\end{bmatrix}\begin{pmatrix}\hat{c}_1\\ \hat{c}_2\end{pmatrix} + \begin{pmatrix}\sqrt{\kappa_1}\hat{c}_{\text{in},1}\\ \sqrt{\kappa_2}\hat{c}_{\text{in},2}\end{pmatrix} + \begin{pmatrix}\sqrt{\gamma_1}\\ \sqrt{\gamma_2}e^{\imath\phi}\end{pmatrix}\hat{c}_{\text{in},3}. \tag{2}$$

Under the white-noise assumption, these zero-mean noise operators are such that $\langle \hat{c}_{\mathrm{in},i}(t)\hat{c}^\dagger_{\mathrm{in},j}(t')\rangle = (\bar{N}_i+1)\delta_{i,j}\delta(t-t')$, $\langle \hat{c}^\dagger_{\mathrm{in},i}(t)\hat{c}_{\mathrm{in},j}(t')\rangle = \bar{N}_i\delta_{i,j}\delta(t-t')$, and $\langle \hat{c}_{\mathrm{in},i}(t)\hat{c}_{\mathrm{in},j}(t')\rangle = 0$ $(i,j=1,2,3)$. Since Eq. (2) is a linear system of equations, a full description of the state at any point in time requires only the first and second moments of the quadrature operators $\hat{x}_i = (\hat{c}_i+\hat{c}_i^\dagger)/\sqrt{2}$ and $\hat{p}_i = -\imath(\hat{c}_i-\hat{c}_i^\dagger)/\sqrt{2}$ $(i=1,\ 2)$. It can be shown that the covariance matrix V of this system obeys the Lyapunov equation $\dot{V} = AV + VA^{\mathrm{T}} + N$, where the drift matrix A is related to the matrix in the first term of Eq. (2) and the noise matrix N is related to the second and third terms of this same equation. When the eigenvalues of A all have negative real parts, a unique solution to V exists. In our case, we define $\bar{n}_i = \langle \hat{c}_i^\dagger\hat{c}_i\rangle$ and $\Delta \coloneqq \omega_2-\omega_1$, and simplify our expressions by taking $\kappa_1=\kappa_2=\gamma_1=\gamma_2 \eqqcolon \kappa$. We want to compare our system to one in which the two oscillators lack any direct coupling or common bath. Simply removing the common bath and the link between the oscillators fundamentally alters the nature of the system, as it changes the number of baths each oscillator is connected to. For a physically meaningful comparison we must modify the bath parameters appropriately. In this *disconnected* scenario, which is physically equivalent to taking $|\Delta|\to\infty$ in the above expressions while keeping F, κ, and $\bar{N}_i$ $(i=1,2,3)$ fixed, the steady-state occupation numbers are $\bar{m}_i = \frac{1}{2}(\bar{N}_i+\bar{N}_3)$ $(i=1,2,3)$; note that the $\bar{m}_i$ are independent of F and that $\bar{m}_3=\bar{N}_3$. Define $\Delta n_i \coloneqq \bar{n}_i-\bar{m}_i$ $(i=1,2)$ to quantify the difference between the two scenarios, whose explicit expressions we reproduce elsewhere [15]. For simplicity let us look at the maximally nonreciprocal case $(F=0)$, whereby

$$\Delta n_1 = 0 \quad \text{and} \quad \Delta n_2 = \frac{2\kappa^2}{4\kappa^2+\Delta^2}(\bar{m}_1-\bar{m}_3). \tag{3}$$

This very clearly shows that, whatever the value of $\bar{m}_1-\bar{m}_2$, we find an *increase* (*decrease*) in $\bar{n}_2$ over the disconnected case for $\bar{m}_1>\bar{m}_3$ $(\bar{m}_1<\bar{m}_3)$, whereas $\bar{n}_1$ is unaffected by the presence of the other oscillator. It is interesting to note that this conclusion remains unchanged if we have $\bar{m}_2=\bar{m}_1$. In other words, even if the two oscillators equilibrate to the same temperature in the disconnected case, the channel will cause an excess or depleted flow of thermal noise towards oscillator 2 that depends only on the temperature difference between oscillators 1 and 3. Figure 2 shows that the temperature of the common bath can be used as a control knob to modulate the flow of thermal noise into or out of the second oscillator. Note that, for $F=0$ the temperature of the first oscillator is unaltered. The temperature of the second oscillator can be lower (blue), the same (green), or higher (red) in comparison to the disconnected scenario depending on the temperature of the common bath and Δ

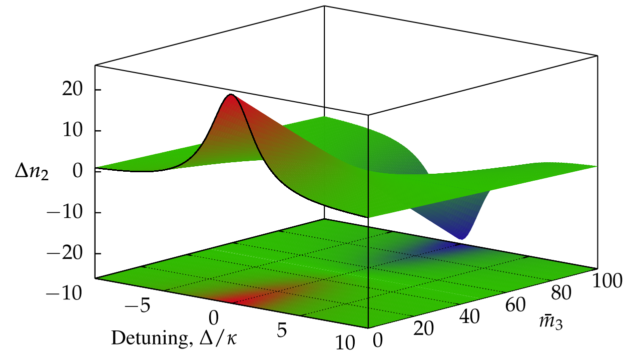

FIG. 2. Change in occupation number of the second oscillator, Δn_2, as a function of the detuning Δ between the two oscillators and the occupation number $\bar{m}_3$ of the common bath. Red (blue) regions correspond to increased (decreased) thermal noise. Note that $\Delta n_1 = 0$ throughout. $(\phi=0,\ \bar{m}_1=50,\ \bar{m}_2=100)$.

which, e.g., can be chosen to reduce the flow of thermal noise into oscillator 2 even when all coherent signals flow from oscillator 1 to 2. The case for $F\neq 0$ is shown in Fig. S.1 of Ref. [15]. Regardless of the temperature difference between the two oscillators *and the direction of signal flow*, the thermal noise flowing into the second oscillator can be increased or decreased.

We next turn our attention to an experimentally feasible optomechanical platform that can realize this model. We shall use terminology related to platforms operating in the optical domain, but all of our results hold identically for microwave-based systems. Our results are important for interfacing with such systems, since the thermal occupation of the electromagnetic field at microwave frequencies is often non-negligible.

Optomechanical realization.—Our aim in this section is to employ a mechanical degree of freedom interacting with two optical fields, acting as a controllable reservoir. The result is an optomechanical system that works as a thermal rectifier, with the temperature of the mechanical oscillator bath controlling the steady-state temperature of the second optical field. A schematic realization of this optomechanical system is sketched in Fig. 1(c). Here we consider an optomechanical platform consisting of two optical cavities with resonance frequencies ω_i $(i=1,\ 2)$, which interact simultaneously with a mechanical resonator with frequency ω_m, and where the single-photon optomechanical coupling strength between the oscillator and the ith cavity is g_i $(i=1,\ 2)$. The direct photon hopping rate between the cavities is denoted by J, which is assumed real for simplicity. The Hamiltonian governing the unitary evolution of this system is given by [60,65,66]

$$\hat{H} = \hbar\omega_m\hat{b}^\dagger\hat{b} + \sum_{i=1,2}\hbar[\omega_i\hat{a}_i^\dagger\hat{a}_i + g_i(\hat{b}+\hat{b}^\dagger)\hat{a}_i^\dagger\hat{a}_i] + \hbar J(\hat{a}_1\hat{a}_2^\dagger + \hat{a}_1^\dagger\hat{a}_2) + \sum_{i=1,2}\hbar\mathcal{E}_i(\hat{a}_ie^{-\imath\omega_d t} + \mathrm{H.c.}), \tag{4}$$

where $\hat{a}_i$ (with $[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}$) are the annihilation operators of the cavity fields and $\hat{b}$ is the mechanical annihilation operator. The first and second terms of Eq. (4) describe the free Hamiltonians of the mechanical and cavity fields, respectively; the third term indicates the optomechanical coupling between the cavities and the mechanical resonator; and the fourth term shows the cavity-cavity photon hopping. The last term represents the driving of each cavity i by a coherent electromagnetic field with frequency ω_d, which we assume to be the same for both cavities, and amplitude $\mathcal{E}_i$. We note that our analysis also applies to systems where two mechanical modes are used to generate nonreciprocal coupling between two electromagnetic cavities. Recent realizations of such systems [52,55,58,60] illustrate the feasibility of implementing nonreciprocal transport of thermal noise and signals.

In a rotating frame with respect to ω_d, and after adding losses by means of dissipative Lindblad terms as in the preceding section, the dynamics of the system can be fully characterized by the quantum Langevin equations of motion ($i = 1, 2$)

$$\dot{\hat{a}}_i = -\left(\imath\Delta_i + \frac{\kappa_i}{2}\right)\hat{a}_i - \imath J\hat{a}_{\bar{i}} - \imath g_i\hat{a}_i(\hat{b} + \hat{b}^\dagger) + \mathcal{E}_i + \sqrt{\kappa_i}\hat{a}_{\mathrm{in},i}, \tag{5a}$$

$$\dot{\hat{b}} = -\left(\imath\omega_m + \frac{\gamma_m}{2}\right)\hat{b} - \imath\sum_{i=1,2} g_i\hat{a}_i^\dagger\hat{a}_i + \sqrt{\gamma_m}b_{\mathrm{in},m}, \tag{5b}$$

where $\Delta_i \coloneqq \omega_i - \omega_d$, $\bar{1} = 2$, and $\bar{2} = 1$. Here, $\kappa_i \coloneqq \kappa_{\mathrm{int},i} + \kappa_{\mathrm{ext},i}$ are the linewidths of the cavities in which $\kappa_{\mathrm{int},i}$ and $\kappa_{\mathrm{ext},i}$ are the intrinsic and extrinsic linewidths, respectively. Intrinsic losses and input quantum noise are associated with the zero-mean noise operators $\hat{a}_{\mathrm{int},i}$ and $\hat{a}_{\mathrm{ext},i}$, respectively; we can conveniently define $\hat{a}_{\mathrm{in},i} \coloneqq (\sqrt{\kappa_{\mathrm{ext},i}}\hat{a}_{\mathrm{ext},i} + \sqrt{\kappa_{\mathrm{int},i}}\hat{a}_{\mathrm{int},i})/\sqrt{\kappa_i}$. The damping of the mechanical resonator is given by γ_m. The zero-mean quantum fluctuations $\hat{a}_{\mathrm{in},i}$ and $\hat{b}_{\mathrm{in},m}$ satisfy the usual white noise correlations [15]. Equations (5) can be solved by linearization around the classical steady state of the system. We define the zero-mean cavity field fluctuation operators $\delta\hat{a}_i \coloneqq \hat{a}_i - \alpha_i$ where $\alpha_i = 2\mathcal{E}_i e^{\imath\phi_i}/\sqrt{4\Delta_i^2 + \kappa_i^2}$ are the steady-state solutions, ignoring a small change in Δ_i due to a static shift in the position of the mechanical oscillator, and assuming $|\alpha_i| \gg 1$.

If the driving frequencies are chosen such that $\Delta_i \approx \omega_m$ and the system is in the sideband-resolved regime, i.e., $\omega_m \gg \kappa_i$, it is possible to use the rotating-wave approximation to drop the rapidly rotating terms oscillating at $\pm\omega_m$. This allows us to eliminate the mechanical degree of freedom, whereby the equations can be approximated in the frequency domain by

$$-\imath\omega\begin{pmatrix}\delta\hat{a}_1\\ \delta\hat{a}_2\end{pmatrix} = \begin{bmatrix}-\imath\Delta_1 - \frac{\kappa_1}{2} - G_1^2\chi_m(\omega) & -\imath J - \chi_m(\omega)G_1G_2e^{-\imath\phi}\\ -\imath J - \chi_m(\omega)G_1G_2e^{\imath\phi} & -\imath\Delta_2 - \frac{\kappa_2}{2} - G_2^2\chi_m(\omega)\end{bmatrix}\begin{pmatrix}\delta\hat{a}_1\\ \delta\hat{a}_2\end{pmatrix} + \begin{pmatrix}\sqrt{\kappa_1}\hat{a}_{\mathrm{in},1}\\ \sqrt{\kappa_2}\hat{a}_{\mathrm{in},2}\end{pmatrix} + \begin{pmatrix}G_1\sqrt{\gamma_m}\tilde{\chi}_m(\omega)\\ G_2\sqrt{\gamma_m}\tilde{\chi}_m(\omega)e^{\imath\phi}\end{pmatrix}\hat{b}_{\mathrm{in},m} \tag{6}$$

where $G_i = g_i\alpha_i$ is the effective optomechanical coupling rate and the mechanical susceptibility is defined as $\chi_m(\omega) = 1/[\gamma_m/2 - \imath(\omega - \omega_m)]$. To simplify matters, we chose the phase reference such that G_1 is real and set $G_2 \to G_2e^{\imath\phi}$ (where the G_2 on the right-hand side is real). We also defined $\tilde{\chi}_m(\omega) \coloneqq \chi_{\mathrm{m}}(\omega)|\chi_m(\Omega)|/\chi_m(\Omega)$, where Ω is some frequency of interest. This procedure is detailed elsewhere [15].

Equation (6) reveals that in general the photon hopping between cavities is not symmetric—note that the off-diagonal terms of the drift matrix on the right-hand side of the equation are *not* complex conjugates of one another. This means that by properly choosing the system parameters one can break the reversibility of the thermal photon hopping between the cavities and set up a preferred direction for the flow of thermal noise. For example, a situation of full nonreciprocity at frequency Ω, where the photon hopping is entirely suppressed in the direction $2 \to 1$, may be obtained by choosing the parameters such that $J = \imath\chi_m(\Omega)G_1G_2e^{-\imath\phi}$.

Consider, now, a quantum state centered around frequency Ω in the rotating frame and whose bandwidth Γ is much smaller than γ_m, such that $\chi_m(\omega) = \tilde{\chi}_m(\omega) \approx \chi_m(\Omega)$, constant over the bandwidth of the signal. Under these "large bandwidth" conditions, when $\gamma_m \gg \Gamma$, all the parameters entering Eq. (6) can be held constant, and this equation therefore becomes identical to Eq. (2), with the following replacements: $\omega_i \to \Delta_i + G_i^2\Im\{\chi_m(\Omega)\}$, $\gamma_i \to 2G_i^2\Re\{\chi_m(\Omega)\}$, and $F \to J - \imath\chi_m(\Omega)G_1G_2e^{-\imath\phi}$. For example, perfect nonreciprocity requires $J = G_1G_2[((\gamma_m/2))^2 + (\Omega - \omega_m)^2]^{-1/2}$, with ϕ chosen such that $F = 0$. A detailed discussion of the equivalence between the two systems is presented elsewhere [15]. We can therefore apply the formalism developed previously to conclude that any thermal noise in the signal will be suppressed *in one direction* only. By manipulating the properties of the mechanical oscillator, e.g., using an auxiliary optical field, one may control the flow of thermal energy in the electromagnetic signal transmitted between the two cavities. An in-depth analysis [15] may be performed to derive the flow of excitations between the system and the three baths it is connected to. Figure S.2 in Ref. [15] shows that changing the temperature of *either* resonator does not affect the flow of excitations between the other resonator and its own bath. Any excess flow between the resonators is therefore borne exclusively by their common bath and the link between them. The net flow, given by the sum of the flows to all baths, is shown to be equal to zero, as required for physical consistency.

This proposed thermal rectifier can be implemented using an on-chip microwave electromechanical system based on a lumped-element superconducting circuit with a drumhead capacitor [52,58] or a dielectric nanostring mechanical resonator [60]. We assume the following experimentally feasible parameters: Optomechanical coupling rates of $G_1 = G_2 = 2\pi \times 7$ kHz, cavities resonant at $2\pi \times 5$ GHz and having damping rates of $\kappa_1 = \kappa_2 = 2\pi \times 2$ MHz, mechanical resonance frequency $\omega_m = 2\pi \times 6$ MHz, and damping rate $\gamma_m = 2\pi \times 100$ Hz. Inductive or capacitive coupling between microwave resonators can yield $J = 2\pi \times 1$ MHz. An auxiliary cavity can be used to change the isolation bandwidth γ_m. The ambient temperature of the microwave and mechanical resonators can be kept below 10 mK by using cryogen-free dilution refrigerators. Optomechanical cooling can be used to cool the mechanical resonator down to ∼0.5 phonons (260 μK). For these parameters, the temperature of resonator 2 is lower with respect to the disconnected case, and depends linearly on that of resonator 1. Furthermore, the temperature of resonator 1 is independent of that of resonator 2.

Conclusions.—We have investigated a generic framework to describe nonreciprocal transport in compound quantum systems. In contrast to several previous studies, we chose to concentrate on the transport of thermal states rather than coherent signals. Our framework can easily be mapped to a prototypical optomechanical realization, which we discussed explicitly in the text. We have also shown how, with parameters typical of present-day microwave optomechanical experiments, the effects we describe should be visible in a proof-of-concept experiment. In the context of quantum measurements and emerging quantum technologies, these techniques and ideas will find use in the manipulation of flow of thermal noise inside quantum devices for phonon-based signal processing and computation, as well as in the construction of quantum-limited amplification systems that perform measurements on sensitive quantum devices without adding thermal noise. Our system can be realized with state-of-the-art technology both in optical [55] and microwave [60] domains, and is potentially suited to control the flow of thermal noise in nanoscale devices and to design a new generation of thermal rectifiers, thermal diodes, and transistors. Our work could facilitate noise control and remote cooling of nanoelectronic devices and superconducting circuits using *in situ*-engineerable thermal sinks with possible applications in emerging quantum technologies such as quantum computers and simulators.

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