Identification of Majorana Modes in Interacting Systems by Local Integrals of Motion

Andrzej Więckowski,[1] Maciej M. Maśka,[1] and Marcin Mierzejewski[2]
[1]*Institute of Physics, University of Silesia, 40-007 Katowice, Poland*
[2]*Department of Theoretical Physics, Wrocław University of Science and Technology, 50-370 Wrocław, Poland*

(Received 11 July 2017; revised manuscript received 8 September 2017; published 26 January 2018)

Recently, there has been substantial progress in methods of identifying local integrals of motion in interacting integrable models or in systems with many-body localization. We show that one of these approaches can be utilized for constructing local, conserved, Majorana fermions in systems with an arbitrary many-body interaction. As a test case, we first investigate a noninteracting Kitaev model and demonstrate that this approach perfectly reproduces the standard results. Then, we discuss how the many-body interactions influence the spatial structure and the lifetime of the Majorana modes. Finally, we determine the regime for which the information stored in the Majorana correlators is also retained for arbitrarily long times at high temperatures. We show that it is included in the regime with topologically protected soft Majorana modes, but in some cases is significantly smaller.

DOI: 10.1103/PhysRevLett.120.040504

Introduction.—Recently, a lot of hope has been pinned on Majorana zero modes as building blocks of a quantum computer [1–5]. One of the systems where these modes were proposed and observed is a semiconductor nanowire with a spin-orbit interaction coupled to an *s*-wave superconductor [6–12]. It is known that, in low-dimensional systems, Coulomb interactions are crucial and can drastically affect their properties [13–17]. Interactions are also important for practical reasons: disorder is present in any semiconductor nanowire and the Majorana states are not completely immune against it [18–20]. Moderate interactions may stabilize the Majorana states against such perturbations [21–24].

Generally, a Majorana fermion is any fermionic operator Γ that satisfies $\Gamma^2 = 1$. However, in order to perform topological quantum computing, one needs stable, non-Abelian anions [25]. They can be realized as localized Majorana zero modes (MZMs), whereby their stability follows from the commutation relation

$$[\hat{H}, \Gamma] = 0, \tag{1}$$

where $\hat{H}$ is the Hamiltonian. This equation, together with the conservation of the fermion parity lead to a non-Abelian braiding for adiabatic exchanging of Majorana quasiparticles [26]. Equation (1) can be fulfilled rigorously only in the thermodynamic limit, except for a fine-tuned symmetric point [27] where it also holds true for finite L. In general, $[\hat{H}, \Gamma] \propto e^{-L/\xi}$, where L is the system size and ξ is correlation length [28]. The nonzero value of the commutator means that, even in the absence of any external decoherence processes, the MZM will have a finite lifetime.

The question is how to find the topological order and Majorana modes in interacting systems. Several methods have been used to study MZMs in interacting nanowires [29–33], see Ref. [22] for a review. A commonly tested necessary condition [which follows from Eq. (1)] concerns degeneracy of the ground states obtained for systems with odd and even numbers of fermions. A sufficient condition for the presence of topological order is more involved. It can be formulated based on the local unitary equivalence (LUE) between the ground states of the interacting system and of the noninteracting Kitaev chain in the topological phase [34]. In order to prove LUE, it is sufficient to show that one of the ground states can be continuously deformed to the other, whereby the spectral gap above the ground state must stay open along the entire path of deformation [35,36]. But this is not equivalent to Eq. (1) and guarantees only the so-called soft mode, which is fully protected by topology only at temperatures well below the spectral gap. In other words, a soft MZM commutes with the Hamiltonian which is projected into a low-energy subspace [37]. At higher temperatures, the information encoded in this mode can be lost after some time.

In this Letter, we propose a method that allows one to find Majorana operator Γ that almost satisfies Eq. (1) within the entire Hilbert space. Our method finds the so–called strong MZM that is stable at arbitrarily high temperatures [38–40]. Perturbative construction of almost strong MZMs has recently been reported in Ref. [40] for the Ising–like model with nearest- and (integrability-breaking) next-nearest-neighbor interactions. In contradistinction, our approach is general and can be applied for arbitrary Hamiltonians, in principle, also, for spinful fermions. To this end, we derive the optimal form of a local operator Γ that guarantees the longest lifetime of the MZM. We determine the regime of existence of a strong MZM and show that it is smaller than the regime with soft modes, the latter being established from LUE.

The general method.—We consider the Hamiltonian $\hat{H} = \sum_m E_m |m\rangle\langle m|$ and assume that the relevant degrees of freedom can be expressed in terms of the standard fermionic operators a_j and $a_j^\dagger$ or, equivalently, in terms of the Majorana fermions $\gamma_{2j} = a_j + a_j^\dagger$, $\gamma_{2j+1} = i(a_j^\dagger - a_j)$. Here, j includes all quantum numbers, e.g., the spin projection. We search for particular combinations of the Majorana operators $\Gamma = \sum_i \alpha_i \gamma_i$ with real coefficients α_i such that Γ is conserved [41]. We assume normalization $\sum_i \alpha_i^2 = 1$ when $\Gamma^2 = 1$. The conservation of Γ can conveniently be studied by averaging this quantity over an infinite time window

$$\bar{\Gamma} = \lim_{\tau' \to \infty} \frac{1}{\tau'} \int_0^{\tau'} dt e^{iHt} \Gamma e^{-iHt} \tag{2}$$

$$= \lim_{\tau \to \infty} \sum_{m,n} \theta\left(\frac{1}{\tau} - |E_m - E_n|\right) \langle n|\Gamma|m\rangle |n\rangle\langle m|. \tag{3}$$

If this mode is strictly conserved, then $\bar{\Gamma} = \Gamma$. This, however, would require Eq. (1) to be satisfied, what may not be the case in finite systems. Therefore, we will usually search for an optimal choice of α_i when $\bar{\Gamma}$ is as close to Γ as possible. In order to quantify the proximity of two operators, we use the usual (Hilbert-Schmidt) inner product $\langle \hat{A}\,\hat{B} \rangle = \mathrm{Tr}(\hat{A}\,\hat{B})/\mathrm{Tr}(\hat{1})$. The optimal choice of coefficients α_i corresponds to a minimum of $\langle (\Gamma - \bar{\Gamma})^2 \rangle = 1 - \langle \bar{\Gamma}^2 \rangle$. The latter equality originates from the identity $\langle \Gamma \bar{\Gamma} \rangle = \langle \bar{\Gamma}\,\bar{\Gamma} \rangle$ (i.e., the time averaging is an orthogonal projection), as shown in the Supplemental Material [42]. Consequently, the least decaying mode can be found from the optimization problem

$$\lambda = \max_{\{\alpha_i\}} \langle \bar{\Gamma}^2 \rangle = \max_{\{\alpha_i\}} \langle \bar{\Gamma} \Gamma \rangle. \tag{4}$$

The physical meaning of λ comes from the observation that the scalar product $\langle ... \rangle$ formally represents thermal averaging carried out for infinite temperatures. Then, following Eq. (4), λ is the asymptotic value of the longest living autocorrelation function $\langle \Gamma(t) \Gamma \rangle$. If $\lambda = 1$, then Γ is a strict integral of motion. i.e., a strong MZM [37,38,43,44]. For $0 < \lambda < 1$, the information stored in the correlator $\langle \Gamma(t) \Gamma \rangle$ is partially retained for arbitrarily long times (despite Γ not being strictly conserved), while this information is completely lost when $\lambda = 0$. The optimization problem can be further simplified

$$\lambda = \max_{\{\alpha_i\}} \sum_{ij} \alpha_i \langle \bar{\gamma}_i \bar{\gamma}_j \rangle \alpha_j. \tag{5}$$

It becomes a standard eigenproblem for the (positive semidefinite) matrix $\langle \bar{\gamma}_i \bar{\gamma}_j \rangle$. Namely, λ is the largest eigenvalue of $\langle \bar{\gamma}_i \bar{\gamma}_j \rangle$, and α_j are components of the corresponding eigenvector. Essentially, all nonvanishing eigenvalues (whether degenerate or not) correspond to independent MZMs, whereby their independence follows from orthogonality of different eigenvectors and the identity $\langle \gamma_i \gamma_j \rangle = \delta_{ij}$.

The general idea behind this method is similar to another approach which has previously been used for identification of new integrals of motion in the Heisenberg model [45]. The latter approach targets operators which are conserved and local. Here, we single out Majorana operators which are conserved and, at the same time, are local. The conservation follows from the time averaging, i.e., from the identity $[\hat{H}, \bar{\Gamma}] = 0$, whereas locality originates from the fact that Γ is a linear combination of γ_i, each of them being supported on a single site only. Since we maximize the projection $\langle \bar{\Gamma} \Gamma \rangle$, the resulting operators retain the properties of both Γ and $\bar{\Gamma}$; i.e., they are local, conserved MZMs. More formal discussion concerning MZMs (including their locality [46]) can be found in the Supplemental Material [42].

When studying systems with fixed boundary conditions, it is utterly important, that the limit for the size of the system $L \to \infty$ precedes the limit for time $\tau \to \infty$, [47,48]. Since numerical calculations can be carried out for finite systems only, τ in Eq. (3) should be kept large but finite until the finite–size scaling is accomplished. All the discussed properties of the correlation functions also hold true for finite τ [49,50], even though it is not the case for finite τ' in Eq. (2).

Example.—As an example, we study a one–dimensional system of interacting, spinless fermions with hard-wall boundary conditions. The system is described by the Kitaev Hamiltonian [27] extended by the many-body interactions

$$\hat{H} = -t_0 \sum_{i=1}^{L-1} (a_{i+1}^\dagger a_i + \mathrm{H.c.}) + \Delta \sum_{i=1}^{L-1} (a_{i+1}^\dagger a_i^\dagger + \mathrm{H.c.}) - \mu \sum_{i=1}^{L} \tilde{n}_i + V \sum_{i=1}^{L-1} \tilde{n}_i \tilde{n}_{i+1} + W \sum_{i=1}^{L-1} \tilde{n}_i \tilde{n}_{i+2}. \tag{6}$$

Here, t_0 refers to hopping amplitude, μ is a chemical potential, Δ is the superconducting gap and $\tilde{n}_i = a_i^\dagger a_i - \frac{1}{2}$. V and W are potentials of the first and second nearest-neighbor interactions. For simplicity, we use dimensionless units by putting $\hbar = 1$ and $t_0 = 1$.

Test for noninteracting systems.—Numerical implementation of our approach consists of three consecutive steps: (i) exact diagonalization of the Hamiltonian (6); (ii) numerical construction of time–averaged Majorana operators $\bar{\gamma}_i$ as defined by Eq. (3) but for finite τ; (iii) construction and diagonalization of the matrix $K_{ij} = \langle \bar{\gamma}_i \bar{\gamma}_j \rangle$. Because of the orthogonality relation $\langle \bar{\gamma}_{2i} \bar{\gamma}_{2j+1} \rangle = 0$, one may separately study two cases $\Gamma^+ = \sum_i \alpha_i^+ \gamma_{2i}$ and $\Gamma^- = \sum_i \alpha_i^- \gamma_{2i+1}$, whereby, now, the index i enumerates the lattice sites. In the rest of this Letter, we discuss the two most stable modes (one in each sector Γ^+ and Γ^-). All other eigenvalues of the matrix K are much smaller and vanish in the thermodynamic limit (not shown). It remains in agreement with a common knowledge that the homogeneous chain described by the Hamiltonian (6) may host at most two MZMs exponentially localized at the boundaries [27,28,36].

The complexity of our approach is independent of whether or not the many-body interactions are present; hence, the method can be tested by investigating a noninteracting system with $V = W = 0$. Figure 1(a) shows τ dependence of λ [see Eq. (4)] for the most stable MZM Γ^+. Results for Γ^- are exactly the same. One may introduce the lifetime of the MZMs, τ_M, corresponding to the vertical sections of curves shown in the latter plot. Figure 1(c) shows that, for the finite system, τ_M is finite as well, despite the absence of the many-body scattering. The only exception concerns $|\Delta| = 1$ when $\tau_M \to \infty$ for arbitrary L. Otherwise, τ_M increases exponentially with L, as follows from the equal spacing of the vertical sections in Fig. 1(a). The latter result clearly illustrates the importance of the correct order of limits: $\lim_{\tau\to\infty}\lim_{L\to\infty}\lambda = 1$; i.e., the MZMs are strictly conserved in the thermodynamic limit, while $\lim_{L\to\infty}\lim_{\tau\to\infty}\lambda = 0$. All the obtained results remain in agreement with the well established properties of the MZMs in a noninteracting case, see, e.g., [28].

We have also calculated the local density of states at zero energy for the noninteracting Hamiltonian

$$\rho_i(E=0) = -\frac{1}{\pi}\mathrm{Im}G_{ii}(E=0),$$
$$G(E) = (E - \hat{H} + i\eta)^{-1}, \qquad (7)$$

where $\hat{H}$ is given by Eq. (6) but with $V = W = 0$. In Figs. 2(a) and 2(b), rescaled $\rho_i(E=0)$ is compared with the spatial density of the Majorana fermions contributing to both Majorana modes, $|\alpha_i^+|^2 + |\alpha_i^-|^2$. Perfect agreement between both methods illustrates accuracy of the approach proposed in this Letter.

Systems with many-body interactions.—All results in the main text will be shown for $W = V/2$, whereas the commonly studied case $W = 0$ (which contains some peculiar features) is discussed in the Supplemental Material [42]. Results in Figs. 1(b) and 1(d) show the most stable Majorana autocorrelation function [Eq. (4)] in the presence of weak to moderate interactions. Similar to the noninteracting case [Fig. 1(a)], the position of the steep sections of $\lambda(\tau)$ increases exponentially with the system size indicating that $\lim_{\tau\to\infty}\lim_{L\to\infty}\lambda \simeq 1$ but, in contrast to noninteracting systems, $\lambda < 1$. In the Supplemental Material [42], we show that the latter inequality seems to be generic for systems with many-body interactions. It implies that the strictly local operator Γ is not a strict integral of motion. Our approach singles out Γ which contains the largest possible conserved part represented by $\lim_{\tau\to\infty}\bar{\Gamma}$.

For finite systems, the many-body interactions may extend the time scale in which the correlator $\langle\Gamma(t)\Gamma\rangle$ is large. Interestingly, this extension can exceed 1 order of magnitude, as shown in Fig. 1(d). Figures 2(c) and 2(d) explain the origin of this extension. They show how the many-body interactions modify the spatial structure of the MZMs. There are two modes which vanish exponentially outside of the edges of the system. Note that this property is not built into our algorithm but appears as a result which doesn't need to hold true for other geometry of the system. Despite the exponential decay, these two modes still do overlap, and this overlap is responsible for a finite-lifetime of the MZMs in a noninteracting system with $L < \infty$. Then, the many-body interactions push these modes further towards the edges of the system [see Figs. 2(c) and 2(d)], reducing the overlap between them and, in this way, increasing their lifetime. This mechanism holds true as

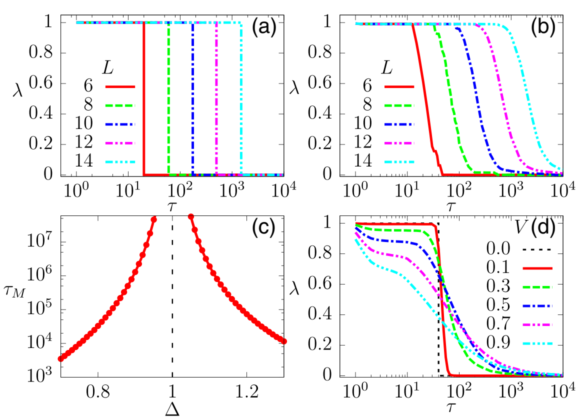

FIG. 1. Results for systems without (a),(c) and with (b),(d) many-body interactions and $\mu = 0$. (a), (b), and (d) The Majorana autocorrelation function λ [see Eq. (4)] for: (a) $V = 0$, $\Delta = 0.5$; (b) $V = 0.2$, $\Delta = 0.5$; (d) $L = 12$, $\Delta = 0.3$. (c) Lifetime of MZMs for a finite noninteracting system of $L = 10$ sites.

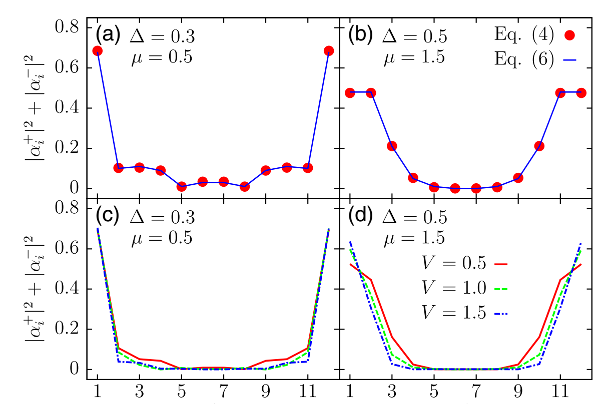

FIG. 2. Spatial structure of MZMs, $\Gamma^+ = \sum_i \alpha_i^+ \gamma_{2i}$ and $\Gamma^- = \sum_i \alpha_i^- \gamma_{2i+1}$. (a) and (b) Rescaled local density of states at energy $E = 0$ for noninteracting system [$V = 0$, Eq. (7)] (solid line) compared with solution of Eq. (5) (points). (c) and (d) Results for $V \neq 0$ from Eq. (5).

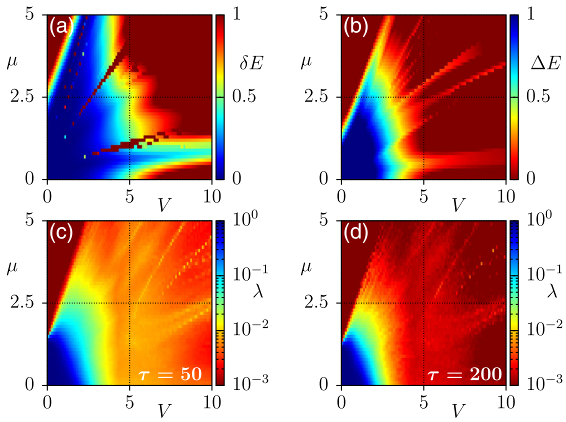

FIG. 3. Results for $\Delta = 1$. (a) Degeneracy of the ground states. (b) The spectral gap. (c) and (d) show the Majorana autocorrelation function λ for various times τ and $L = 12$. Note different color schemes in (a) and (b).

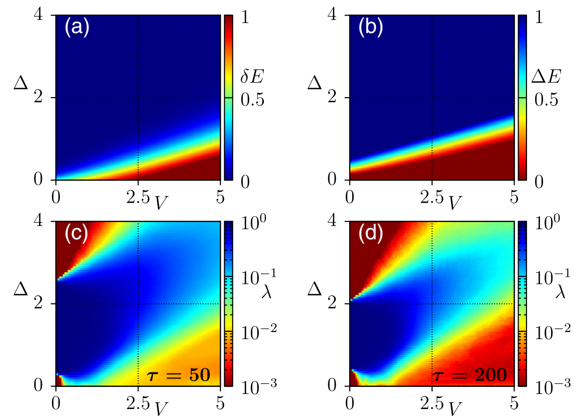

FIG. 4. The same as in Fig. 3, but as a function of V and Δ for $\mu = 0$.

long as the interactions are not too strong, when the MZMs eventually disappear.

Next, we compare our results for strong MZMs with the presence of the topological order. We check the degeneracy of the ground state (necessary condition) as well as LUE to the topological regime in the noninteracting Kitaev model (sufficient condition). To this end, we study chains of $L = 8, 10, \ldots, 20$ and find the two lowest energies in the subspaces with odd and even particle numbers, denoted, respectively, as $E_{0,o}(L)$, $E_{1,o}(L)$ and $E_{0,e}(L)$, $E_{1,e}(L)$. We introduce the measure of the ground–state degeneracy $\delta E(L) = E_{0,o}(L) - E_{0,e}(L)$ and two spectral gaps, $\Delta E_{o(e)}(L) = E_{1,o(e)}(L) - E_{0,o(e)}(L)$. Typically, the gaps between the low–energy levels decay algebraically with L; hence, we carry this out linearly in $1/L$ extrapolations of $\Delta E_{o(e)}(L)$. However, $\delta E(L)$ should decay exponentially in the topological regime; thus, we use the fitting function $\delta E(L) = A\exp(-BL) + \delta E(\infty)$. These extrapolations break down when V and μ are large [42], what shows up as large errors for the extrapolated quantities, $\sigma_{\delta E}$ and $\sigma_{\Delta E}$. We identify the degenerate ground states as a regime where both $|\delta E(\infty)|$ and $\sigma_{\delta E}$ are small, defining $\delta E \equiv |\delta E(\infty)| + \sigma_{\delta E} \ll 1$ as the lower bound on the degenerate region. The LUE implies that the gap $\min\{\Delta E_o(\infty), \Delta E_e(\infty)\}$ doesn't vanish along a path that reaches the topological regime for $V = 0$, while $\sigma_{\Delta E}$ remains small. Then, we define the lower bound on the corresponding region by $\Delta E \equiv \min\{\Delta E_o(\infty), \Delta E_e(\infty)\} - \sigma_{\Delta E} > 0$. Results for δE and ΔE are shown in Figs. 3(a), 4(a) and 3(b), 4(b), respectively. The actual topological region may be larger than it follows from lower bounds shown in Figs. 3(b) and 4(b).

Results in Figs. 3(c) and 3(d) show that the strong MZMs, indeed, exist for very long times ($\tau > 200$) not only in the ground state but, essentially, in the entire energy spectrum. We also confirm that a moderate many-body interaction extends the range of μ where soft and strong MZMs are present [22].

In Fig. 4, we show similar results but for $\mu = 0$ and various magnitudes of the superconducting gap Δ. In this case, an exact solution is known but only for $\Delta = 1$ and $W = 0$ [36,51]. For large τ and $\Delta \gg 1$, the strong MZMs seem to be absent even for a very weak many-body interaction. However, it is a finite-size effect that, again, shows how important is the correct order of limits for time and the system size. Therefore, in Fig. 5, we set $\tau = 100$ and show the Majorana autocorrelation function for various values of L together with results extrapolated to $L \to \infty$. The details of extrapolation and results for $\lim_{\tau\to\infty}\lim_{L\to\infty}\lambda(\tau)$ are shown in the Supplemental material [42]. The regime with $\lambda > 0$ covers roughly the entire topological regime determined via LUE to the single-particle Kitaev model [compare Figs. 4(b) and 5(d)]. However, λ gradually decreases with increasing interactions, and a strong MZM with large λ exists within a much smaller regime, as shown, e.g., by the contour in Fig. 5(d).

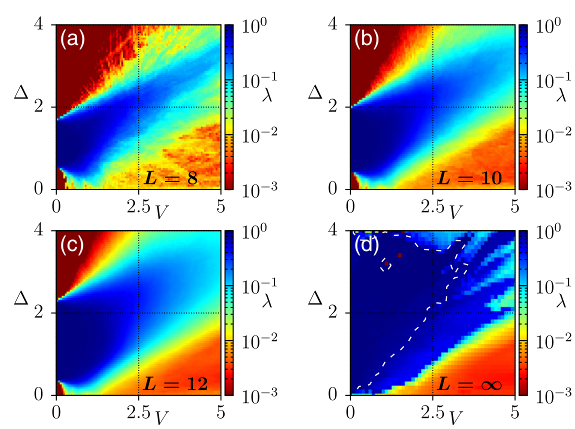

FIG. 5. The autocorrelation function λ as a function of V and Δ for $\tau = 100$ and system sizes $L = 8$ (a), 10 (b), 12 (c), and 14 (d). Contour in (d) marks $\lambda = 0.8$.

Conclusions.—We have proposed an approach for finding local (strong or almost strong) MZMs which can be implemented for an arbitrary many-body interaction. We have found that, even at elevated temperatures, the lifetime of these modes is long enough so that they may be used effectively to store information. The regime where the strong MZMs exist (as quantified by large λ in our approach) is included, but is smaller than the regime which is unitarily equivalent to the topological regime in the single-particle Kitaev model. It means that not all topological states are equally protected to be useful in, e.q., quantum computing. At finite temperatures, the systems with weak many-body interactions are preferable; however, these interactions may still be significant, when compared to other energy scales in the system. Our results also suggest that, in systems with many-body interactions, the strictly local Majorana operators are not strict integrals of motion; however, their autocorrelation function remains large for arbitrarily long times.

This work is supported by the National Science Centre, Poland via Projects No. 2016/23/B/ST3/00647 (A. W. and M. M.) and No. DEC–2013/11/B/ST3/00824 (M. M. M.).

[1] David Aasen, M. Hell, R. V. Mishmash, A. Higginbotham, J. Danon, M. Leijnse, T. S. Jespersen, J. A. Folk, C. M. Marcus, K. Flensberg, and J. Alicea, Milestones Toward Majorana-Based Quantum Computing, Phys. Rev. X **6**, 031016 (2016).

[2] S. D. Sarma, M. Freedman, and C. Nayak, Majorana zero modes and topological quantum computation, npj Quantum Inf. **1**, 15001 (2015).

[3] T. Karzig, C. Knapp, R. M. Lutchyn, P. Bonderson, M. B. Hastings, C. Nayak, J. Alicea, K. Flensberg, S. Plugge, Y. Oreg, C. M. Marcus, and M. H. Freedman, Scalable designs for quasiparticle-poisoning-protected topological quantum computation with Majorana zero modes, Phys. Rev. B **95**, 235305 (2017).

[4] S. Plugge, L. A. Landau, E. Sela, A. Altland, K. Flensberg, and R. Egger, Roadmap to Majorana surface codes, Phys. Rev. B **94**, 174514 (2016).

[5] A. R. Akhmerov, Topological quantum computation away from the ground state using Majorana fermions, Phys. Rev. B **82**, 020509 (2010).

[6] R. M. Lutchyn, J. D. Sau, and S. D. Sarma, Majorana Fermions and a Topological Phase Transition in Semiconductor-Superconductor Heterostructures, Phys. Rev. Lett. **105**, 077001 (2010).

[7] Y. Oreg, G. Refael, and F. von Oppen, Helical Liquids and Majorana Bound States in Quantum Wires, Phys. Rev. Lett. **105**, 177002 (2010).

[8] M. Franz, Majorana's wires, Nat. Nanotechnol. **8**, 149 (2013).

[9] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, Signatures of Majorana fermions in hybrid superconductor-semiconductor nanowire devices, Science **336**, 1003 (2012).

[10] S. Nadj-Perge, I. K. Drozdov, J. Li, H. Chen, S. Jeon, J. Seo, A. H. MacDonald, B. A. Bernevig, and A. Yazdani, Observation of Majorana fermions in ferromagnetic atomic chains on a superconductor, Science **346**, 602 (2014).

[11] R. Pawlak, M. Kisiel, J. Klinovaja, T. Meier, S. Kawai, T. Glatzel, D. Loss, and E. Meyer, Probing atomic structure and Majorana wavefunctions in mono-atomic Fe chains on superconducting Pb surface, npj Quantum Inf. **2**, 16035 (2016).

[12] M. Ruby, F. Pientka, Y. Peng, F. von Oppen, B. W. Heinrich, and K. J. Franke, End States and Subgap Structure in Proximity-Coupled Chains of Magnetic Adatoms, Phys. Rev. Lett. **115**, 197204 (2015).

[13] F. D. M. Haldane, 'Luttinger liquid theory' of one-dimensional quantum fluids. I. Properties of the Luttinger model and their extension to the general 1D interacting spinless Fermi gas, J. Phys. C **14**, 2585 (1981).

[14] S. Gangadharaiah, B. Braunecker, P. Simon, and D. Loss, Majorana Edge States in Interacting One-Dimensional Systems, Phys. Rev. Lett. **107**, 036801 (2011).

[15] A. Manolescu, D. C. Marinescu, and T. D. Stanescu, Coulomb interaction effects on the Majorana states in quantum wires, J. Phys. Condens. Matter **26**, 172203 (2014).

[16] A. Vuik, D. Eeltink, A. R. Akhmerov, and M. Wimmer, Effects of the electrostatic environment on the Majorana nanowire devices, New J. Phys. **18**, 033013 (2016).

[17] F. Domínguez, J. Cayao, P. San-Jose, R. Aguado, A. L. Yeyati, and E. Prada, Zero-energy pinning from interactions in Majorana nanowires, npj Quantum Mater. **2**, 13 (2017).

[18] M. M. Maśka, A. Gorczyca-Goraj, J. Tworzydło, and T. Domański, Majorana quasiparticles of an inhomogeneous Rashba chain, Phys. Rev. B **95**, 045429 (2017).

[19] R. M. Lutchyn, T. D. Stanescu, and S. D. Sarma, Search for Majorana Fermions in Multiband Semiconducting Nanowires, Phys. Rev. Lett. **106**, 127001 (2011).

[20] A. R. Akhmerov, J. P. Dahlhaus, F. Hassler, M. Wimmer, and C. W. J. Beenakker, Quantized Conductance at the Majorana Phase Transition in a Disordered Superconducting Wire, Phys. Rev. Lett. **106**, 057001 (2011).

[21] F. Domínguez, J. Cayao, P. San-Jose, R. Aguado, A. L. Yeyati, and E. Prada, Zero-energy pinning from interactions in Majorana nanowires, npj Quantum Mater. **2**, 13 (2017).

[22] E. M. Stoudenmire, J. Alicea, O. A. Starykh, and M. P. A. Fisher, Interaction effects in topological superconducting wires supporting Majorana fermions, Phys. Rev. B **84**, 014503 (2011).

[23] N. M. Gergs, L. Fritz, and D. Schuricht, Topological order in the Kitaev/Majorana chain in the presence of disorder and interactions, Phys. Rev. B **93**, 075129 (2016).

[24] F. Hassler and D. Schuricht, Strongly interacting Majorana modes in an array of Josephson junctions, New J. Phys. **14**, 125018 (2012).

[25] C. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. D. Sarma, Non-Abelian anyons and topological quantum computation, Rev. Mod. Phys. **80**, 1083 (2008).

[26] D. J. Clarke, J. D. Sau, and S. Tewari, Majorana fermion exchange in quasi-one-dimensional networks, Phys. Rev. B **84**, 035120 (2011).

[27] A. Yu Kitaev, Unpaired Majorana fermions in quantum wires, Phys. Usp. **44**, 131 (2001).

[28] G. Kells, Many-body Majorana operators and the equivalence of parity sectors, Phys. Rev. B **92**, 081401 (2015).

[29] H. T. Ng, Decoherence of interacting Majorana modes, Sci. Rep. **5**, 12530 (2015).

[30] W. Su, M. N. Chen, L. B. Shao, L. Sheng, and D. Y. Xing, Electron-electron interaction effects in Floquet topological superconducting chains: Suppression of topological edge states and crossover from weak to strong chaos, Phys. Rev. B **94**, 075145 (2016).

[31] Y.-H. Chan, C.-K. Chiu, and K. Sun, Multiple signatures of topological transitions for interacting fermions in chain lattices, Phys. Rev. B **92**, 104514 (2015).

[32] J. S. Hofmann, F. F. Assaad, and A. P. Schnyder, Edge instabilities of topological superconductors, Phys. Rev. B **93**, 201116 (2016).

[33] R. Thomale, S. Rachel, and P. Schmitteckert, Tunneling spectra simulation of interacting Majorana wires, Phys. Rev. B **88**, 161103 (2013).

[34] X. Chen, Z.-C. Gu, and X.-G. Wen, Classification of gapped symmetric phases in one-dimensional spin systems, Phys. Rev. B **83**, 035107 (2011).

[35] L. Fidkowski and A. Kitaev, Effects of interactions on the topological classification of free fermion systems, Phys. Rev. B **81**, 134509 (2010).

[36] H. Katsura, D. Schuricht, and M. Takahashi, Exact ground states and topological order in interacting Kitaev/Majorana chains, Phys. Rev. B **92**, 115137 (2015).

[37] J. Alicea and P. Fendley, Topological phases with parafermions: Theory and blueprints, Annu. Rev. Condens. Matter Phys. **7**, 119 (2016).

[38] D. V. Else, P. Fendley, J. Kemp, and Ch. Nayak, Prethermal Strong Zero Modes and Topological Qubits, Phys. Rev. X **7**, 041062 (2017).

[39] G. Goldstein and C. Chamon, Exact zero modes in closed systems of interacting fermions, Phys. Rev. B **86**, 115122 (2012).

[40] J. Kemp, N. Y. Yao, C. R. Laumann, and P. Fendley, Long coherence times for edge spins, J. Stat. Mech. (2017) P063105.

[41] Also, within the proposed approach, terms including three or more γ's can (and in a general case should) be taken into account. We have checked for at least a few parameter sets that the contribution from terms with three γ's is negligible.

[42] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.120.040504 for the details of the finite size scaling, results for nearest-neighbor interaction, and more formal discussion of the Majorana zero modes.

[43] P. Fendley, Strong zero modes and eigenstate phase transitions in the XYZ/interacting Majorana chain, J. Phys. A **49**, 30LT01 (2016).

[44] A. S. Jermyn, R. S. K. Mong, J. Alicea, and P. Fendley, Stability of zero modes in parafermion chains, Phys. Rev. B **90**, 165106 (2014).

[45] M. Mierzejewski, P. Prelovšek, and T. Prosen, Identifying Local and Quasilocal Conserved Quantities in Integrable Systems, Phys. Rev. Lett. **114**, 140601 (2015).

[46] T. E. O'Brien and A. R. Wright, A many-body interpretation of Majorana bound states, and conditions for their localisation, arXiv:1508.06638.

[47] M. Rigol and B. S. Shastry, Drude weight in systems with open boundary conditions, Phys. Rev. B **77**, 161101 (2008).

[48] J. Sirker, N. P. Konstantinidis, F. Andraschko, and N. Sedlmayr, Locality and thermalization in closed quantum systems, Phys. Rev. A **89**, 042104 (2014).

[49] M. Mierzejewski, T. Prosen, and P. Prelovšek, Approximate conservation laws in perturbed integrable lattice models, Phys. Rev. B **92**, 195121 (2015).

[50] P. Prelovšek, M. Mierzejewski, O. Barišič, and J. Herbrych, Density correlations and transport in models of many-body localization, Ann. Phys. (Amsterdam) **529**, 1600362 (2017).

[51] J.-J. Miao, H.-K. Jin, F.-C. Zhang, and Y. Zhou, Exact Solution for the Interacting Kitaev Chain at the Symmetric Point, Phys. Rev. Lett. **118**, 267701 (2017).