Microwave Properties of Superconductors Close to the Superconductor-Insulator Transition

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Strongly disordered pseudogapped superconductors are expected to display arbitrarily high values of kinetic inductance close to the superconductor-insulator transition (SIT), which make them attractive for the implementation of large dissipationless inductance. We develop the theory of the collective modes in these superconductors and discuss associated dissipation at microwave frequencies. We obtain the collective mode spectra dependence on the disorder level and conclude that collective modes become a relevant source of dissipation and noise in the outer proximity of the SIT.

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A piece of a superconductor is characterized by the phase of the order parameter φ. Because of the order parameter $\Psi = |\Psi|e^{i\varphi}$, the state of the superconductor does not change when $\varphi \to \varphi + 2\pi$ even if it is connected to other superconductors by Josephson junctions. However, for a superconductor that is also connected to others by a very long superconducting wire, the change of the phase by 2π leads to states that are distinguishable even though the energy due to the phase variations along the wire might be vanishingly small. In this system a plethora of new physical effects becomes possible such as the formation of Bloch states in the Josephson potential, current Shapiro steps, etc. All these effects require that the phase change by 2π leads to a state of the same energy but distinguishable from the original one. Quantitatively, the superconducting wire can be characterized by the energy $E = (1/2)E_L\varphi^2$, where φ is the phase difference and $E_L = \hbar^2/(e^2L)$, and L is the effective inductance. The energy E_L should be much less than all relevant energy scales; for a typical problem this translates into $L \gtrsim 1$ μH. Such a superinductor should be dissipationless, and as such it should contain no low energy modes; in particular, it should not form a low frequency resonator. This limits the geometrical size of the superinductor to a few micrometers; for a realistic thin film wire the width is limited by $w \gtrsim 20$ nm, which translates into $L_\square \gtrsim 10$ nH for the inductance per unit area. The question is whether such superinductors are physically possible.

An attractive candidate for superinductors is the superconductor close to the superconductor-insulator transition (quantum critical point). One expects that at the transition the superfluid stiffness $\rho_S = 0$ ($\rho_S = \hbar^2/e^2L_\square$), so if this transition leads to an insulating state with a large gap, in the vicinity of it the superfluid stiffness can be arbitrarily small corresponding to arbitrarily large superinductances. Generally, there are two mechanisms for the destruction of the superconductivity by disorder that lead to a quantum critical point where ρ_S is exactly zero (for recent reviews see Refs. [1,2]). The first (fermionic) mechanism attributes the suppression of the superconductivity to the increase of the Coulomb interaction, which results in the decrease of the attraction between electrons and their eventual depairing [3]. In this mechanism the state formed upon the destruction of the superconductor is essentially a poor conductor. This mechanism clearly does not lead to the formation of the superinductance. The alternative (bosonic) mechanism attributes superconductivity suppression to the localization of Cooper pairs that remain intact even when superconductivity is completely suppressed. The theory of the bosonic mechanism has a long history: this scenario of the superconductor-insulator transition was suggested long ago [4–7] but was not developed further until recently [2,8] when experimental data indicated it might indeed occur in InO [9–12].

In this Letter we show that as the bosonic superconductor-insulator transition (SIT) is approached the collective modes are pushed down to low energies. In BCS theory the critical temperature of the superconductor, or its low energy gap, does not depend on the disorder. In the simplest model of the bosonic SIT the critical temperature does not depend on the disorder until the latter exceeds some critical value. At larger values of the disorder the transition temperature decreases quickly and eventually becomes zero while the single electron gap Δ_P remains constant [2]. It is natural to associate the regime where the transition temperature depends on the disorder with the critical regime of the SIT in the bosonic model. As we show below, the collective modes are pushed to low energies

even outside the critical regime. This severely limits the possible values of the kinetic inductances that can be achieved in the strongly disordered superconductors close to the SIT.

Before we give the details of the model of the bosonic SIT and its low energy properties we discuss its main physical assumptions and materials in which such physics might be realized. The main assumption of the bosonic model is that Coulomb repulsion does not lead to electron depairing. This might occur if it is screened by the electrons far from the Fermi surface. In other words, the Coulomb interaction between superconducting electrons is small due to a large effective dielectric constant of the material. Empirically, in this case one expects that superconductivity occurs against the background of the insulating $R(T)$. This is the situation in InO, which displays strong insulating temperature behavior that is followed by superconductivity at very low T [9–11]. Large dielectric constants, $\kappa \gtrsim 10^3$, are expected in superconductors derived from a high-κ host [13], $SrTiO_3$, such as $SrTiO_3$-$LaAlO_3$ interfaces [14,15] or Nb-doped $SrTi_{1-x}Nb_xO_3$ [16]. In the material where the Coulomb interaction is completely suppressed by a large κ one expects that T_c and Δ_P will initially increase with disorder due to the electron wave function localization before the effects of the suppression of Cooper pair tunneling suppress T_c and ρ_S leading to the SIT while the single electron gap Δ_P remains large everywhere. Such unusual behavior (with the maximum of T_c) was indeed observed in the $SrTiO_3$-$LaAlO_3$ system [15,17,18]. The increase of T_c followed by an abrupt transition to the insulating state was also observed in Li_xZrNCl crystals [19] as well as in slightly oxidized aluminum wires (also known as granular aluminum); in the latter the suppression of the superfluid density is not accompanied by a significant dissipation at high frequencies [20], pointing towards the bosonic mechanism. Finally, a likely candidate for this physics is superconducting semiconductors with a low density of carriers, such as In-doped $Pb_zSn_{1-z}Te$ [21,22]. The distinguishing feature of the bosonic SIT is the different behavior of the tunneling and conductivity gaps, which allows their experimental identification [23–26].

An excellent probe for the absence of the low energy modes is provided by the appearance of the coherent phase slips that are expected in the wires made from thin films with large Δ_P and small ρ_S. This was indeed observed [27] in InO wires and other strongly disordered superconductors that retain a significant single electron gap: NbN and TiN [28]. However, in all these materials the quality factor remains low indicating a significant intrinsic dissipation. While expected for the fermionic suppression mechanism in NbN [29–31] and TiN [32,33] that leads to the formation of the subgap states, the reason for the dissipation in InO remains unclear.

Model.—We consider a simplified model of a pseudogapped superconductor where single-particle excitations are totally absent so that all electronic degrees of freedom can be represented in terms of Anderson pseudospins [34] that describe the population and hopping of localized electron pairs. In other words, we assume that Δ_p is larger than all relevant energy scales of the problem. The low energy physics is described by

$$H = \sum_i 2\xi_i s_i^z - \sum_{(ij)} (J_{ij} s_i^+ s_j^- + \text{H.c.}), \tag{1}$$

where the indices i, j enumerate localized single-electron states, the notation (i, j) indicates a pair of connected sites, the ξ_i represent their energies, and the spin-$\frac{1}{2}$ operators $\mathbf{s}_i$ are related to the electron creation or annihilation operators $a_{i,\sigma}^+, a_{i,\sigma}^-$ by $2s_i^z = a_{i,\uparrow}^+ a_{i,\uparrow} + a_{i,\downarrow}^+ a_{i,\downarrow} - 1$, $s_i^+ = a_{i,\uparrow}^+ a_{i,\downarrow}^+$ and $s_i^- = a_{i,\downarrow} a_{i,\uparrow}$. The matrix elements J_{ij} that describe the hopping of localized Cooper pairs are determined by single-electron wave functions $\psi_i^2(\mathbf{r})$, which are supposed to be localized at a relatively long spatial scale: $J_{ij} = \tilde{g} \int d^3\mathbf{r} \psi_i^2(\mathbf{r}) \psi_j^2(\mathbf{r})$. In a 3D pseudogapped superconductor a typical value of the matrix element J_{ij} depends in a nontrivial way on the energy difference between the participating states: $\epsilon_{ij} = |\xi_i - \xi_j|$, see Ref. [2]; this dependence is due to the fractal nature of the nearly critical (in terms of Anderson localization) electron eigenfunctions. An effective number Z of localized electron states $j(i)$ coupled to a given state i by hopping matrix elements J_{ij} depend on the difference between the Fermi energy E_F and the localization threshold E_c; an increase of disorder moves E_F further into the localized part of the spectrum, decreasing Z. The model (1) neglects the effect of the long range Coulomb interaction that is inconsistent with the bosonic mechanism (see the Supplemental Material [35]).

Solution.—In order to obtain the analytical solution we simplify further the model (1). Namely, we assume that all the sites i, j where the spins $\mathbf{s}_i$ are located belong to a Bethe lattice with coordination number $Z = K + 1$ and all nonzero couplings J_{ij} are equal and connect each spin with its Z nearest neighbors: $J_{ij} = 2g/K$; such a normalization is used to allow for a well-defined limit of $K \to \infty$. Random variables ξ_i are distributed independently over sites i with the flat density $P(\xi) = \frac{1}{2}\theta(1 - |\xi|)$. Within this model, an increase of disorder corresponds to the decrease of K. We have shown previously [36] that within such a model a standard BCS-type phase transition takes place at very large K, $K \geq g\exp(1/g)$, while at lower (but still large) values of K spatial fluctuations of the superconducting order parameter become large and eventually lead to an unusual kind of a quantum $T = 0$ phase transition from the superconducting to the insulating state.

In the present Letter we concentrate upon the low-temperature properties of a superconducting state at moderately large values of K in the range $K_c < K_1 < K \leq K_2$, where $g \ll 1$ and

$$K_c = ge^{1/(eg)}, \qquad K_1 = ge^{1/2g}, \qquad K_2 = \frac{g}{4}e^{1/g}. \quad (2)$$

The region $K > K_1$ is known [36] to possess a usual BCS-like temperature-controlled superconducting transition with $T_c = T_{c0}(g) = [4e^C/\pi]e^{-1/g}$ and a low-temperature amplitude of the order parameter $\Delta(T = 0, g) = 2e^{-1/g}$. At smaller K the superconducting transition temperature $T_c(g, K)$ is suppressed with respect to $T_{c0}(g)$ and eventually vanishes at $K = K_c$. In the range $K_c < K < K_1$ local values Δ_i of the order parameter fluctuate strongly [36], with a "fat tail" extending to the range of Δ_i much larger than its typical value $\Delta_{\text{typ}} = \exp(\langle \ln|\Delta_i|\rangle)$, which also vanishes at $K \to K_c + 0$. At larger $K > K_1$ the order parameter follows the BCS relation and its spatial fluctuations are relatively weak.

Contrary to expectations at $K > K_1$ there is a whole band of delocalized low-lying collective excitation modes with a lower cutoff of their energies $\omega_1(K)$ growing upon the increase of K. Moreover, we find a band of localized collective modes with $\omega < \omega_1(K)$ that extends down to zero energy as long as $K \le K_2$.

We start the derivation of our results by writing the action for low-ω transverse fluctuations $b_i(\omega)$ of the order parameter. These fluctuations are parametrized via phase rotation of the mean-field solution: $\Delta_i(\omega) = \Delta_i e^{i\varphi(\omega)} \equiv \Delta_i + b_i(\omega)$ with action

$$\mathcal{A} = -\sum_{i,j} b_i(\omega)\hat{J}_{ij}^{-1} b_j(\omega) + \sum_i \frac{b_i^2(\omega)\sqrt{\xi_i^2 + \Delta_i^2}}{\xi_i^2 + \Delta_i^2 - \bar{\omega}^2}, \quad (3)$$

where $\bar{\omega} \equiv \omega/2$. At $K > K_1$, $\Delta_i \approx \Delta = 2e^{-1/g}$. The action (3) is directly applicable for $\omega \ll \Delta$; at energies comparable to Δ antisymmetric coupling [neglected in Eq. (3)] between the transverse mode the and longitudinal (gap full) mode might become relevant. The equation for the collective mode can be obtained as an extremum of the action (3) with respect to $b_i(\omega)$:

$$b_i(\omega) = \sum_j J_{ij} b_j(\omega)\eta_j(\omega),$$

$$\text{where } \eta(\omega) \equiv \frac{\sqrt{\xi_j^2 + \Delta^2}}{\xi_j^2 + \Delta^2 - \omega^2}. \quad (4)$$

At $\omega = 0$ it is satisfied automatically for $b_i = \text{const} \times \Delta$ due to the self-consistency equations for the local order parameters Δ.

Equations (3) and (4) are general; below, we study the eigenfunctions of Eq. (4) defined on the Bethe lattice and employ the method developed in the seminal paper [37]. To use this method we need to introduce the self-adjoint linear operator $\hat{L}$ related to Eq. (4); its matrix elements are $C_{ij} = J_{ij}[\eta_i(\omega)\eta_j(\omega)]^{1/2}$. Equations (4) possess delocalized solutions if the expansion for the imaginary part of the Green function $\hat{G} = (\hat{1} - \hat{C} + i\delta)^{-1}$ in powers of $\hat{C}$ is singular. This singularity is indicated by the nonzero value of the typical imaginary part $(\text{Im}G_{ii})_{\text{typ}}$ of the local Green function in the limit $\delta \to 0$. We look for the singularity threshold within the "forward path" approximation [36,38] equivalent to the "Anderson upper limit" condition [37]; i.e., we neglect self-energy corrections for the Green function $G_{ii}(\omega)$. Each path over the Bethe lattice that contributes to $(\text{Im}G_{ii})$ is traversed twice (forward and backward). Therefore, the summation over the paths is equivalent to the calculation of the partition function $Z_{\text{DP}}(N)$ for the N-link directed polymer (DP) model with weights $w_{ij} = J_{ij}^2\eta_i(\omega)\eta_j(\omega)$ defined on nearest-neighbor links: $Z_{\text{DP}}(N) = \sum_P \prod_{\{l(P)\}} w_{ij}$.

We need to find an extensive part of the DP free energy $F_{\text{DP}}(N) = \ln Z_{\text{DP}}(N) \approx Nf$ at $N \to \infty$; the localization threshold is determined by the condition $\langle f\rangle = 0$, where the averaging is over the distribution of random ξ_i. An equivalent way to calculate f is to use the modified weights $\tilde{w}_{ij} = J_{ij}^2\eta_j^2$; the difference between the corresponding partition functions Z_{DP} and $\tilde{Z}_{\text{DP}}$ is concentrated at the end points of each contributing path and thus does not contribute to $f = \lim_{N\to\infty}(1/N)F_{\text{DP}}(N)$.

The shortest method to calculate f is to use the replica trick as described in Refs. [36,38]. It gives

$$e^{xf(x)} \equiv K\int_0^1 d\xi \left(\frac{g}{K}\frac{\sqrt{\xi^2 + \Delta^2}}{\xi^2 + \Delta^2 - \bar{\omega}^2}\right)^{2x} = 1, \qquad \frac{\partial f}{\partial x} = 0. \quad (5)$$

Here, $0 < x < 1$ is an anomalous exponent that measures the degree of replica symmetry breaking (RSB) for the DP problem [within usual mean-field theory $x = 1$ and the second equation in Eqs. (5) is absent]. The condition $\partial f/\partial x|_{x_0} = 0$ selects typical Green functions of the operator $\hat{C}$ introduced above; the first equation in Eqs. (5) then leads to $f(x_0) = 0$, which indicates a critical point between the localized domain for $f(x_0) < 0$, where the typical Green function decays upon iterations, and the extended domain, which corresponds then to $f(x_0) > 0$, where linear iterations diverge and nonlinear terms should be taken into account to get a stable distribution.

At $K = K_1 = ge^{1/2g}$ and $\omega = 0$ the system of equations (5) can be solved exactly (up to relative corrections $\sim e^{-1/g} \ll 1$), with $x = 1/2$. At slightly large $K > K_1$ and low energies $\bar{\omega} = E\Delta$ we look for the solution assuming $2x - 1 \equiv \epsilon \ll 1$ and $E \ll 1$. Expanding the integral in Eqs. (5) up to second order in ϵ and up to first order in $\delta K = K - K_1$, we find [the term $\propto E^2$ can be omitted in the second of Eqs. (5)]:

$$E^2 = \epsilon\frac{\delta K}{K_1} - \frac{\epsilon^2}{24g^2}, \qquad \epsilon = 12g^2\frac{\delta K}{K_1}, \quad (6)$$

leading to the result for the threshold energy in the main order expansion over $\delta K/K_1 \ll 1$:

$$\frac{\omega_1}{2\Delta} \equiv E(K) = \sqrt{6}g\frac{K-K_1}{K_1}. \quad (7)$$

Eigenfunctions with $\omega > \omega_1$ are extended, while those with lower energies are localized. The numerically obtained delocalization line for $\omega_1(K)$ is shown in green in Fig. 1 for the specific choice of $\Delta = 10^{-3}$, which corresponds to $g = 0.129$ and $K_1 = 5.85$.

To find the domain of the existence of localized eigenfunctions with low energies $\omega \ll \Delta$, we use another criterion based upon Eqs. (5). Namely, we look for solutions of the equation $\partial f/\partial x|_{x_0} = 0$ such that $x_0 < 1$ and $f(x_0) < 0$. The condition $x_0 < 1$ guarantees RSB, which implies the different behavior of typical and average Green functions. Namely, in the limit $\delta \to 0$ the average imaginary part of the Green function has a finite value, which implies that the density of states is nonzero in this regime. The condition $f(x_0) < 0$ implies that the wave function decreases, so this regime corresponds to the localized states. This band of localized states ends when x_0 coincides with unity: at this point the typical average of the imaginary part of the Green function $\langle \mathrm{Im}G(\omega)\rangle_{\mathrm{typ}}$ becomes equal to the simple average, $\langle \mathrm{Im}G(\omega)\rangle = \pi\rho(\omega)$. Because at the same time $f(x_0 = 1) < 0$, $\langle \mathrm{Im}G(\omega)\rangle$ decays upon iterations over the Bethe lattice, and $\rho(\omega) = 0$ at the stationary point of these iterations. Therefore, the boundary of the parameter region with $\rho(\omega) > 0$ is given by the solution of the equation $\partial f/\partial x|_{x_0=1} = 0$, where $f(x) \equiv f(x, \omega, K)$ is defined in Eqs. (5). At $\omega = 0$ a straightforward calculation leads to the result (2); in deriving it we used the equality $\int_0^\infty (dt/\cosh t)\ln\cosh t = (\pi/2)\ln 2$. At $\omega > 0$ the same procedure provides the dependence of the spectrum boundary ω_2 on K in the region $\omega \ll \Delta$:

$$K_2(\omega) = K_2\frac{\Delta}{\sqrt{\Delta^2 - \bar{\omega}^2}} \approx K_2\left[1 + \frac{1}{2}\left(\frac{\omega}{2\Delta}\right)^2\right]. \quad (8)$$

The numerical solution of the equation $\partial f/\partial x|_{x_0=1} = 0$ gives the red line in Fig. 1(b). Qualitatively, the appearance of K_2 as one of characteristic values for the coordination number $Z = K + 1$ in our model can be understood by noticing that at $K \gg K_2$ the total number of neighbors in which the local energies $\xi_i \sim \Delta$ becomes large, so at these K the system becomes similar to conventional Ginzburg-Landau superconductor.

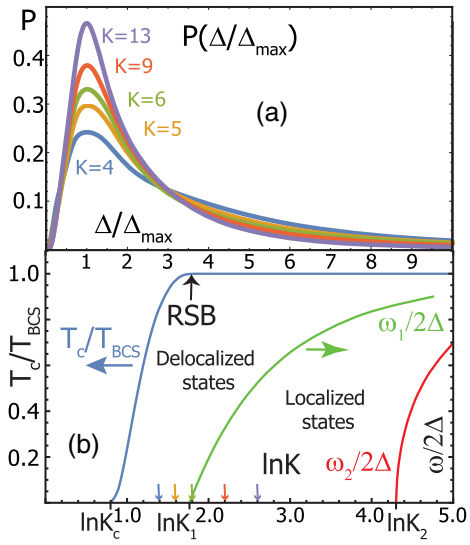

FIG. 1. Schematics of the phase diagram, order parameter distribution function, and collective mode spectra at low T of strongly disordered superconductors obtained from the solution of the model (1) in the cavity approximation. At large disorder, $K < K_1$, the distribution of the order parameter becomes anomalously broad (upper panel) and T_c is rapidly suppressed and becomes $T_c = 0$ at $K < K_c$ (lower panel). In the regime of the critical suppression of T_c, $K < K_1$, delocalized collective modes exist for all frequencies. For smaller disorder, $K_1 < K < K_2$, very low frequency modes are localized. The modes $\omega = 0$ disappear completely only for $K_2 < K$. The numerical values of K shown here correspond to the interaction constant $g = 0.129$, which gives $T \approx 10^{-3}E_F$. Arrows indicate the values of K for which the distribution is shown in the upper plot.

Experimentally observable properties.—The spectrum shown in Fig. 1(b) translates into the microwave properties of the superconductors. In the vicinity of the transition the spectrum of delocalized collective modes extends to zero frequency. Even for $K > K_1$, at which the critical temperature of the superconductor does not experience the suppression due to the quantum critical point, the low energy modes are delocalized at relatively low frequencies $\Delta > \omega > \omega_1(K)$ resulting in a relatively large intrinsic dissipation of the superconductor at these frequencies. The resonators made from such superconductors exhibits low quality factors. As the disorder is decreased the delocalized modes are shifted to higher frequencies. At $\omega < \omega_1(K)$ the oscillation with frequency ω excites only long-living localized states, so that the dissipation in the superconductor is suppressed. However, the localized modes extend down to zero frequencies for $K < K_2$. At any nonzero temperatures these low frequency bosonic modes are excited. Because the relaxation of these modes is slow, their occupation numbers fluctuate slowly with time. This, together with the mode-mode interaction, implies that the frequency of the high energy modes experiences significant jitter in the range $K_1 < K < K_2$. The microwave properties described above can be compared with the other predictions of the model (1). Namely, one expects a broadening of the distribution function at $K < K_1$ sketched in Fig. 1(a) that was observed in Ref. [12]. Another experimentally measurable characteristic is the behavior of superfluid stiffness that is proportional to Δ^2 in the whole range of K considered here [39]. Finally, we note that the fluctuational conductivity is given by a slightly modified [40] Aslamazov-Larkin formula above T_c for $K > K_2$, which can serve as yet

another verification of the applicability of the theory; similarly, one can estimate the value of K_2 from ultrasound attenuation measurements that are expected [41] to become exponentially low only at $K > K_2$. Notice that these different regimes happen within the pseudogapped regime where localization of the single electron function leads to the formation of preformed Cooper pairs [2]. Such materials are expected to have normal-state resistivity R_n only several times below the critical value R_c. Experimentally, for moderately thin films the value of $R_c \sim 10$ kΩ. Assuming that ρ_s for the film is suppressed by a factor of 2–5 compared to the BCS formula $\rho_{\mathrm{BCS}} = \pi\Delta/R_\square$ [27,28,39] we conclude that for the films with $\Delta \sim 1$–2 K and $R_\square \sim 1$–2 kΩ one should be able to reach $L_\square \sim 10$ nH as required for the superinductor. However, to achieve this goal the material should be tuned into the regime where the resistance is already large but not too large so that the effective $K > K_2$. Notice that a very small gap in the microwave experiment was reported recently in strongly disordered NbN films, see Fig. 3(d) in Ref. [25]. We also mention a recent complementary approach clarifying the classical Mattis-Bardeen theory of microwave conductivity for strongly disordered superconductors [42,43].

Conclusion.—We demonstrated theoretically the presence of low-lying collective modes in disordered superconductors in the outer proximity of the SIT, and formulated the conditions for the realization of dissipationless superinductors.

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