Editors' Suggestion

Many-Body Localization Dynamics from Gauge Invariance

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We show how lattice gauge theories can display many-body localization dynamics in the absence of disorder. Our starting point is the observation that, for some generic translationally invariant states, the Gauss law effectively induces a dynamics which can be described as a disorder average over gauge superselection sectors. We carry out extensive exact simulations on the real-time dynamics of a lattice Schwinger model, describing the coupling between U(1) gauge fields and staggered fermions. Our results show how memory effects and slow, double-logarithmic entanglement growth are present in a broad regime of parameters—in particular, for sufficiently *large* interactions. These findings are immediately relevant to cold atoms and trapped ion experiments realizing dynamical gauge fields and suggest a new and universal link between confinement and entanglement dynamics in the many-body localized phase of lattice models.

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Introduction.—Over the past two decades, the impressive developments in harnessing matter at the single quantum level have paved the way to the investigation of real-time dynamics in controlled quantum systems with an unparalleled degree of accuracy [1–3]. These progresses, spanning as diverse fields as cold atoms in optical lattices, trapped ions, superconducting circuits, and more, have reinvigorated the theoretical interest in the dynamics of closed quantum systems [4]. A paradigmatic example in this direction is the quest for generic systems in which the interplay of disorder and interactions prevents thermalization, a scenario dubbed many-body localization (MBL) [5]. In this new dynamical phase, the discreteness of the local observables spectra, typical of quantum mechanics, endows the system with a set of local integrals of motion which freeze transport and localize excitations [6–13] (for reviews, see [14–16]). While this phenomenon has been predicted, and signatures observed in numerics and experiments for a variety of model Hamiltonians such as Hubbard models and spin chains [14,17,18], it is an open question to which extent such a lack of thermalization can occur in fundamental theories of matter and, in particular, if gauge invariance can play a role in the mechanism.

In this Letter, we show the emergence of MBL dynamics in lattice gauge theories (LGTs) [19–21] in the absence of any disorder. Our work is immediately motivated by recent theoretical proposals [22–27] and the experimental demonstration [28] of LGT dynamics in synthetic quantum systems and by the paradigmatic importance played by gauge theories. The latter describe a plethora of physical phenomena, from fundamental interactions in particle physics [21] to the low-energy dynamics of frustrated quantum magnets [29], and are instrumental in designing quantum computing architectures which show inherent protection against noise [30]. As such, addressing their real-time dynamics is of profound interest from a variety of perspectives, regarding both the basic understanding of lattice field theories and the possibility of safely storing quantum information via localization in quantum memories, further boosting their resilience.

Our main finding is that, starting from translational invariant states, the dynamics of LGT is profoundly influenced by the presence of superselection sectors—a key element that stems directly from gauge invariance (see Fig. 1). Even though MBL in systems without disorder has already been discussed and debated over the past years [31–39], the presence of such superselection sectors provides a pristine mechanism for localization dynamics, whose origin can be conveniently tracked by an exact integration of the

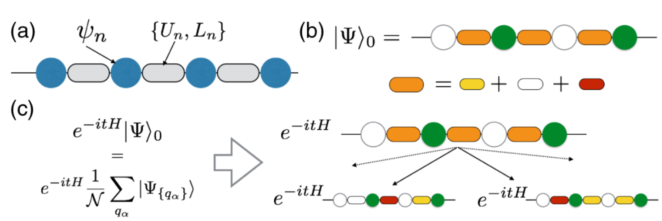

FIG. 1. (a) Schematic of a (1 + 1)-d lattice gauge theory, with matter fields ψ_n defined on the vertices and pairs of conjugate variables $\{U_n, L_n\}$ defined on bonds. (b) Typical initial states used in the simulations: Fermions are arranged in an alternate pattern of empty and occupied sites (corresponding the bare vacuum), while gauge fields are in an equal weight superposition of electric field eigenstates with eigenvalues $(0, \pm 1)$. (c) Schematics of the real-time dynamics, which, starting from $|\Psi\rangle_0$, can be decomposed in an $\mathcal{N}$ superselection sector (see the text).

gauge fields in $(1+1)$-d lattice gauge theories describing matter coupled to gauge fields.

Following this analytical understanding, we provide extensive numerical evidence for MBL dynamics in the lattice Schwinger model—a $(1+1)$-d version of quantum electrodynamics, with U(1) gauge fields coupled to Kogut-Susskind fermions [20,21]—based on both strong memory effects and entanglement dynamics. The latter is characterized by a sublogarithmic growth of the bipartite entanglement entropy, which we interpret as a transport-inhibiting mechanism due to confinement. Our results have immediate experimental relevance in trapped ion systems, where the Schwinger model has already been experimentally realized [28], and in cold atom gases in optical lattices, where various implementations schemes for U(1) LGT [22–27] have been put forward.

Model Hamiltonian.—We are interested here in the dynamics of Abelian lattice gauge theories on a one-dimensional lattice. While our approach is general and can be applied to arbitrary Abelian (and continuous non-Abelian) gauge theories in both Wilson [20,21] and quantum link formulations [40], for the sake of simplicity, we focus in the following on the U(1) Wilson LGT, the lattice Schwinger model (SM). Despite its simplicity, the SM still displays many paradigmatic features also found in more complex gauge theories, such as confinement and string-breaking dynamics. The system is described by a Kogut-Susskind Hamiltonian [20] of the form

$$H = -iw\sum_{n=1}^{N-1}[\psi_n^\dagger U_n \psi_{n+1} - \text{H.c.}] + J\sum_{n=1}^{N-1} L_n^2 + m\sum_{n=1}^{N}(-1)^n \psi_n^\dagger \psi_n, \tag{1}$$

where ψ_n are fermionic annihilation operators defined on vertices, $U_n = e^{i\varphi_n}$ are U(1) parallel transporters defined on bond $(n, n+1)$, whose corresponding electric field operator is defined as $L_n = -i\partial/\partial\varphi_n$, so that $[L_n, U_n] = U_n$, which ensures gauge covariance. The first term describes the gauge-matter coupling; the second is a model-dependent electric field contribution [27,41] and indicates the strength of the interparticle interaction mediated by the gauge field; finally, the last term describes the staggered mass of the fermions (which we will set to 0 in the following). While we are not interested in the continuum limit of the theory here, it is possible to safely take it using the lattice formulation and properly scaling the coupling parameters [27,41]. As in conventional lattice gauge theories, gauge invariance is manifest after defining a set of generators, $G_n = L_n - L_{n-1} - \psi_n^\dagger\psi_n + \frac{1}{2}[1-(-1)^n]$, which satisfy $[H, G_n] = 0$. States in the Hilbert space as defined by the Gauss law, which reads

$$G_n|\Psi_{\{q_\alpha\}}\rangle = q_n|\Psi_{\{q_\alpha\}}\rangle, \tag{2}$$

where $\{q_\alpha\}$ represents the distribution of background charges q_n on each state in the Hilbert space, defining a superselection sector. In a U(1) LGT, $q_n \in [-\infty, \infty]$. We emphasize that the presence of the Gauss law and superselection sectors has nothing to do with integrability but stems solely from gauge invariance.

Superselection sectors as a mechanism for disorder-free localization.—The presence of these superselection sectors drastically affects the system dynamics. In $(1+1)$-d, this can be elegantly seen by integrating out the gauge fields and then studying the resulting Hamiltonian acting on the matter degrees of freedom. Below, we show that this procedure indicates that MBL is a rather generic scenario for LGT in the absence of any underlying disorder—e.g., systems with homogeneous couplings and homogeneous initial states show robust memory effects and slow (sublogarithmic) growth of entanglement entropy as a function of time.

We investigate the time evolution of initial states of the form

$$|\Psi\rangle_0 = |0101\ldots\rangle_\psi \otimes |\bar{L}_n\ldots\rangle_\sigma, \tag{3}$$

where the fermions are in a Néel state (corresponding to the bare vacuum of staggered fermions) and the gauge fields L_n are in an equal weight superposition of $\{-1, 0, 1\}$ at each site. This state is translational invariant up to translations of two lattice spacings and can be decomposed into U(1) superselection sectors as

$$|\Psi\rangle_0 = \frac{1}{\mathcal{N}^{1/2}}\sum_{\bar{q}_n=0,\pm1}|0101\ldots\rangle_\psi \otimes |\bar{q}_n\ldots\rangle_\sigma = \frac{1}{\mathcal{N}^{1/2}}\sum_{q_\alpha}|\Psi_{\{q_\alpha\}}\rangle \tag{4}$$

with $\mathcal{N}$ denoting the total number of different superselection sectors. In order to derive the dynamics within each sector, we analytically integrate out the gauge fields [41]. We assume $\bar{L}_0 = 0$ to minimize boundary effects and apply a Jordan-Wigner transformation to the fermionic fields in order to recast the dynamics as a spin model: $\psi_n^\dagger\psi_n = (\sigma_n^z + 1)/2$. The gauge fields can be sequentially integrated out by noting that

$$L_\ell = L_{\ell-1} + [\sigma_\ell^z + (-1)^\ell]/2 + q_\ell, \tag{5}$$

which simply describes the fact that, given the value of the ingoing electric field on the left side of a site and given the values of the dynamical and static charge, the value of the outgoing electric field is unambiguously fixed.

After integration, the resulting Hamiltonian dynamics crucially depends on $\{q_\alpha\}$—that is, states in different superselection sectors will evolve according to different Hamiltonians $H^{\{q_\alpha\}}$. This is a direct consequence of the fact that, because of the Gauss law, different superselection sectors describe dynamics subject to different static charge configurations. In each sector, the corresponding

Hamiltonian is made of two contributions, $H^{\{q_\alpha\}} = H_\pm + H_{In}^{\{q_\alpha\}}$. The first one describes electron-gauge coupling, which is not sensitive to background charges, and is given by $H_\pm = w\sum_{n=1}^{N-1}[\sigma_n^+\sigma_{n+1}^- + \text{H.c.}]$. The second term originates from the electric field potential term, which is now a function of the fermionic populations only. It contains an interaction part:

$$H_{ZZ} = \frac{J}{2}\sum_{n=1}^{N-2}\sum_{l=n+1}^{N-1}(N-l)\sigma_n^z\sigma_l^z, \tag{6}$$

which is related to the linear growth of Coulomb interactions in one-dimensional systems, and single spin terms:

$$H_Z^{\{q_\alpha\}} = \frac{J}{2}\sum_{n=1}^{N-1}\left(\sum_{\ell=1}^{n}\sigma_\ell^z\right)\left[\left(\sum_{j=1}^{n}q_j\right) - n \mod 2\right]. \tag{7}$$

Crucially, this last part of the Hamiltonian depends explicitly on the superselection sector via $\{q_\ell\}$. As such, starting from initial states of the form in Eq. (4), the system dynamics is dictated by a *charge distribution average*, that is,

$$|\Psi(t)\rangle = e^{-itH}|\Psi\rangle_0 = \frac{1}{\mathcal{N}^{1/2}}\sum_{\{q_\alpha\}}e^{-itH^{\{q_\alpha\}}}|\Psi_{\{q_\alpha\}}\rangle, \tag{8}$$

which is effectively describing a disorder average, since the terms in $H_Z^{\{q_\alpha\}}$ effectively act as a (correlated) disorder for the spin dynamics. The observation in Eq. (8) is applicable to arbitrary Abelian Wilson theories, and even non-Abelian quantum link model, in one-dimensional systems. For the specific case of $\mathbb{Z}_2$ quantum link models, our theory (see Ref. [42]) recovers the results of Ref. [43], which reported Anderson localization in disorder-free models. The same reasoning can be applied to two-dimensional systems, in particular, to the evolution of quenched gauge theories. It explicitly shows that, contrary to conventional spin models, the dynamics of systems endowed by a gauge symmetry can naturally lead to phenomena related to disordered systems, even in the absence of any disorder (both on the initial state and in the dynamics). This happens for generic initial states which are in product form of the matter and gauge fields, as in those cases the weight of superselection sectors with effectively no disorder decreases exponentially with the system size. Finally, we emphasize that the mechanism discussed here is very different from the disorder-free localization dynamics discussed in the context of fractons [38,39], which relies on slow dynamics of topological excitations and separation of energy scales [48,49].

Many-body localization dynamics in U(1) lattice gauge theories.—While our theory predicts a generic mechanism for effectively disordered dynamics in LGTs, the question if this finally leads to localization behavior has to be addressed by nonperturbative methods. We thus turn to a numerical investigation of the system dynamics described by Eq. (8). We address this by employing a computationally optimized approach to the method of Krylov subspaces—for details, see Ref. [42]. As a first figure of merit, we focus on the staggered occupation of the fermions:

$$\nu(t) = \frac{1}{2N}\sum_{n=1}^{N}\langle(-1)^n\psi_n^\dagger(t)\psi_n(t) + 1\rangle,$$

which, by a Jordan-Wigner transformation, can be expressed as $\nu(t) = (1/2N)\sum_{n=1}^{N}\langle(-1)^n\sigma_n^z + 1\rangle$ by transforming fermionic fields to spin operators. Since we employ staggered fermions, this quantity corresponds to the total number of particles created in the system. In addition, we also monitor the staggered population for the central two sites of the system:

$$\mu(t) = \langle\hat{n}_{N/2}(t) - \hat{n}_{(N/2)+1}\rangle,$$

where $\hat{n}_i(t) \equiv \psi_i^\dagger(t)\psi_i(t)$ is the fermion counting operator on site i.

In Figs. 2(a) and 2(b), we plot typical results of our simulations for $N = 26$. In the weak coupling limit, both quantities reach their average thermodynamic value: 0.5 and 0, respectively. In contrast, for the strong coupling phase, the system does not relax for both observables (we have carried out simulations up to times 10^{20} until $L = 14$ to check this). This behavior is analyzed using finite-size scaling in Figs. 2(c) and 2(d): For $J = 0.1$, both quantities approach their thermodynamic value as N is increased.

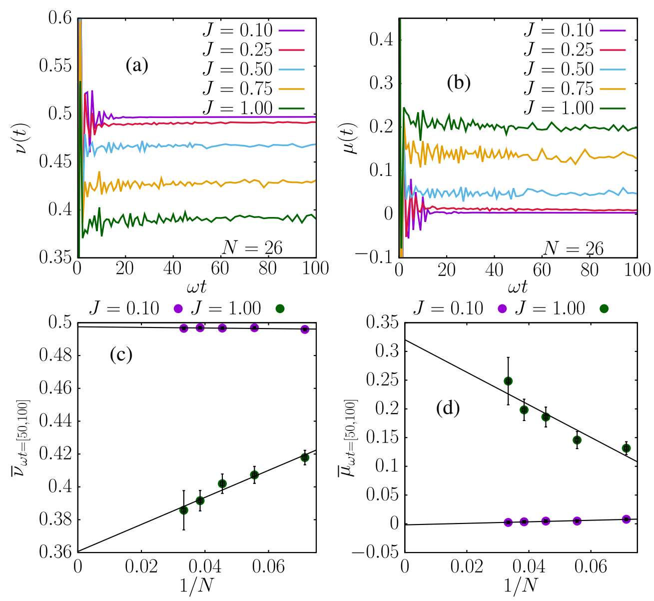

FIG. 2. Dynamics of the overall staggered fermion occupation (a) and central staggered fermion occupation (b) from a bare vacuum initial state for different values of coupling strength parameter J, and $N = 26$. (c),(d) Final time value of $\nu(t)$ and $\mu(t)$, respectively, averaged from $t = 50$ to $t = 100$ for different system sizes up to $N = 30$ and two limiting values of J. The absolute error due to averaging realizations is shown as vertical bars for each point.

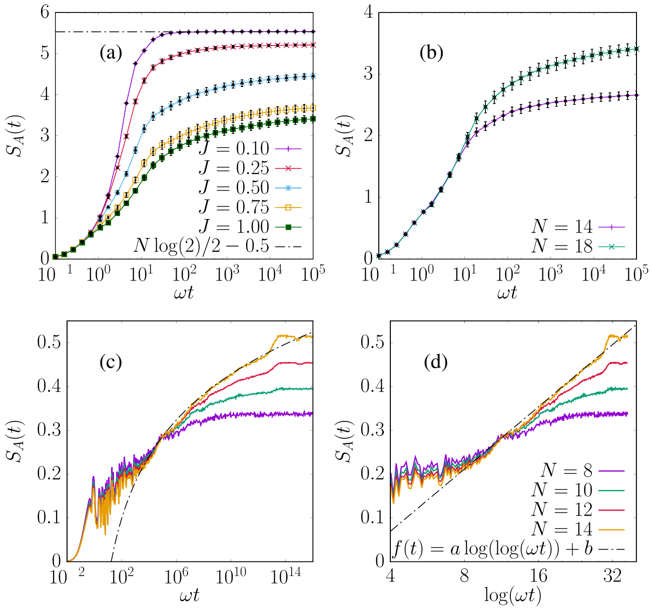

FIG. 3. Time evolution of the bipartite von Neumann entanglement entropy $S_A(t)$ from a bare vacuum initial state. (a) $S_A(t)$ for different values of coupling strength parameter J and $N = 18$. The dashed line refers to the value $N\log(2)/2 - 1/2$ of the entanglement entropy as expected for a random state. (b) $S_A(t)$ on intermediate time scales for $J = 1$ and different system sizes N. (c) Long-time growth of $S_A(t)$ for $J = 10$ including a fit (solid line) $f(t) = a\log[\log(wt)] + b$ to the $N = 14$ data. (d) The same data are plotted on a double logarithmic scale.

Instead, for $J = 1.0$, for both $\nu(t)$ and $\mu(t)$ the deviation from thermodynamic values actually *increases* as a function of the system size, signaling strong memory effects.

Another key aspect of MBL is a very slow propagation of quantum information [6–9], which can be studied by monitoring the bipartite von Neumann entanglement entropy after the quench, defined as

$$S_A = -\mathrm{Tr}_A \rho_A \log \rho_A, \tag{9}$$

where ρ_A denotes the reduced density matrix of the state in the region A, which in the following will be half system. For systems whose dynamics is captured by the eigenstate thermalization hypothesis, one expects that, after a quantum quench, the bipartite entanglement entropy grows linearly as a function of time; instead, MBL systems are characterized by slow entanglement spreading, which is typically logarithmic in time [6,8,51].

In Figs. 3(a) and 3(b), we plot S_A as a function of time for the same parameter regimes as in Fig. 2. While for small $J = 0.1$ the entropy grows faster than logarithmically [and approaches a saturation value of the order of $N/2\log(2) - 0.5$, as expected for a random state [52]], in the localized phase the growth is extremely slow: In particular, it is slower than $\log(t)$. Using full diagonalization of the Hamiltonian, we further analyze this behavior in Figs. 3(c) and 3(d) for the strongly interacting regime, $J = 10$: In this case, the decay is consistent with a double-logarithmic behavior, as it is definitely slower than $[\log(t)]^\alpha$. This type of growth has never been reported in MBL systems, where the typical $\log(t)$ behavior can be inferred in the local integral of motion picture and considering short-ranged interactions.

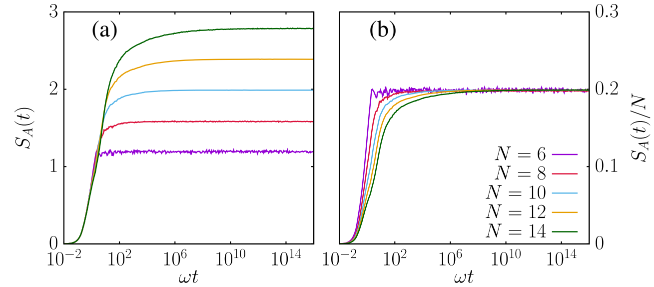

FIG. 4. (a) Asymptotic long-time dynamics of the entanglement entropy density $S_A(t)$ for different system sizes N at $J = 1$ suggesting saturation to an extensive value by plotting the entanglement entropy density $S_A(t)/N$ (b).

In LGT, however, we argue that confinement plays an important role in determining entanglement dynamics. For this purpose, we compare the present case, which is confining, with a deconfined regime. The latter can be achieved in the presence of a finite θ angle with $\theta = \pi$ [53]; in this regime, including a four-Fermi coupling interaction Vn_jn_{j+1}, the dynamics of the states $|\Psi\rangle_0$ is mapped exactly to an XXZ chain with (correlated) disorder in σ^z terms, a model whose transport and entanglement properties have been extensively studied in the context of MBL [54–59].

The comparison between the two cases reveals that the fact that excitations are confined drastically changes entanglement spreading, further decreasing its growth from logarithmic to sublogarithmic. The fact that confinement affects even this "high-energy" behavior is not surprising, as it describes the behavior of large interparticle separations, very far from ground state physics.

Finally, in Fig. 4, we show the long-time dynamics of S_A and, in particular, its saturation value. As illustrated in Fig. 4(b), the latter scales linearly as a function of the system size, another characteristic feature of MBL dynamics.

Conclusions and outlook.—We have shown how many-body localization dynamics can naturally emerge in lattice gauge theories in the absence of any disorder. The physical interpretation is that, due to the presence of superselection sectors, the dynamics of translational invariant states is effectively described by an average over random charge configurations, with electrons interacting via long-range Coulomb potential. While our results are derived from a fully local field theory, after gauge field integration they signal that certain types of long-range interactions *per se* are not sufficient to delocalize the charges. This is surprising in view of earlier results [60–62] but in line with recent treatments of the long-range interactions in a nonperturbative way [63]. We remark that there is a key difference between gauge theories and, say, generic long-range XY models: Only the former can be exactly recast in a local fashion. It would be interesting to see whether matching this key requirement leads to a strict criterion for MBL in long-range systems.

Moreover, we provide numerical evidence that shows how confinement substantially favors localization, in particular, by inhibiting entanglement growth in a qualitative stronger manner with respect to the local integrals of motion in statistical mechanics models of MBL. Our results point toward intriguing possibilities to study MBL dynamics in 2D LGT: In particular, due to the modest Hilbert space dimension growth in simple quantum link models, numerical simulations are expected to be comparatively easier with respect to spin models [especially for U(1) theories], and gauge invariant tensor network methods [64–68] could be used thanks to the considerably slower entanglement growth in LGT.

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Note added.—Recently, a preprint appeared [44] where a $\mathbb{Z}_2$ quantum link model with four-Fermi coupling was considered. Our theoretical analysis predicts disorder-free MBL and logarithmic entanglement growth there due to the absence of confinement and is in perfect qualitative agreement with the conclusion of Ref. [44].

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