Longitudinal Negative Magnetoresistance and Magnetotransport Phenomena in Conventional and Topological Conductors

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Recently, a large negative longitudinal (parallel to the magnetic field) magnetoresistance was observed in Weyl and Dirac semimetals. It is believed to be related to the chiral anomaly associated with topological electron band structure of these materials. We show that in a certain range of parameters such a phenomenon can also exist in conventional centrosymmetric and time-reversal invariant conductors, lacking topological protection of the electron spectrum and the chiral anomaly. We also discuss the magnetic field enhancement of the longitudinal components of the thermal conductivity and thermoelectric tensors.

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One can distinguish two types of magnetoresistance depending on the mutual orientation of the current and the magnetic field: transverse and longitudinal. If the magnetic field is sufficiently small, the magnetoresistance can be described by the quasiclassical Boltzmann kinetic equation (see, for example, [1–5]). A change in the transverse resistance due to a magnetic field can be related to the fact that electrons experience the Lorentz force in that direction. Since there is no Lorentz force in the direction parallel to the magnetic field, the origin of the longitudinal magnetoresistance is more complicated. Moreover, the longitudinal magnetoresistance is absent in the approximation of a spherical Fermi surface and in the relaxation time approximation [1]. Although no theorem was proven, so far all results based on the conventional Boltzmann kinetic equation correspond to positive longitudinal magnetoresistance (see, for example, Refs. [5,6], and references therein). Nielsen and Ninomiya [7] suggested a chiral anomaly-related [8,9] mechanism of negative longitudinal magnetoresistance (NLMR) in materials with massless Dirac and Weyl electronic spectra, which recently attracted great theoretical interest [10–13]. The calculations of Ref. [7] were done in the ultraquantum limit at zero temperature and in the case where the chemical potential is at the Dirac point. However, in most existing Dirac and Weyl semimetals, the chemical potential is located away from the Dirac points. In this case, a quasiclassical description of the chiral anomaly-related NLMR was developed in Refs. [14,15]. It was shown that the existence of strong NLMR requires a large ratio between the chirality and transport relaxation times. Recently, large NLMR was observed both in Weyl and in Dirac materials (see, for example, Refs. [13,16–20]).

In Weyl semimetals, the gapless character of the electron spectrum is protected by topology. In Dirac metals, the massless Dirac points are protected only by the crystalline symmetry. Therefore, a small lattice distortion of a Dirac semimetal can open a gap in the electronic spectrum, making it nontopological. Below, we consider magnetoresistance in Dirac-type materials in which the electron spectrum is either massless or has a small gap. The existence of a small gap in a Dirac semimetal was reported already in the first observation of NLMR in these materials [16]. Furthermore, NLMR was observed in Weyl materials in which the Weyl valleys merge into a single electron pocket with zero net topological charge [21]. This implies that the existence of massless Dirac points in the spectrum, their topological protection, and the chiral anomaly are not necessary ingredients of large NLMR.

In this Letter, we show that a negative contribution to the longitudinal magnetoresistance and other longitudinal magnetotransport phenomena exists even in conventional centrosymmetric and time-reversal symmetric semiconductors and metals. However, for this contribution to dominate the effect, a certain hierarchy of relaxation times should take place.

To illustrate the origin of the effect, we consider a model [22] where the energy gap E_g between the conduction and the valence bands is significantly smaller than the energy separation from other bands and the external potential $V(\boldsymbol{r})$ is smooth on the interatomic scale. In this case, the electron dynamics may be described by the Dirac Hamiltonian (for a recent review, see Ref. [23])

$$\hat{H} = u\boldsymbol{p}\cdot\boldsymbol{\sigma}\tau_3 + E_g\tau_1 + V(\boldsymbol{r}). \tag{1}$$

Here, $\boldsymbol{p} = -i\hbar\boldsymbol{\nabla} - (e/c)\boldsymbol{A}(\boldsymbol{r})$ [with $\boldsymbol{A}(\boldsymbol{r})$ being the vector potential] is the kinematic momentum, E_g is half the band gap, and σ_i and τ_i denote the Pauli matrices that act in the spin and chirality subspaces, respectively.

We focus on the typical situation in which the electron chemical potential μ is larger than the gap E_g. In this regime, electron transport may be described by two equivalent approaches. The first one is based on the quasiclassical kinetic equation, while in the second one the free electron motion is described in terms of the Landau

levels. Here we will use the latter approach. In a uniform magnetic field $\mathbf{B}$ directed along the z direction, the energy spectrum of Eq. (1) has the form (see, for example, Sec. 32 of Ref. [24])

$$\epsilon_n^2(p_z) = E_g^2 + u^2 p_z^2 + \frac{u^2\hbar^2}{l_B^2}(2n + 1 + \sigma). \tag{2}$$

Here p_z is the electron momentum along the magnetic field, $l_B = \sqrt{\hbar c/|eB|}$ is the magnetic length, $n = 0, 1, 2, \ldots$ labels Landau levels, and $\sigma = \pm 1$ is a spin index.

At $E_g = 0$, the Hamiltonian (1) decouples into a sum of Weyl Hamiltonians describing right- and left-handed chiral fermions. As a result, the electronic states can be classified by chirality (R and L), $\tau_3\Psi_R = \Psi_R$ and $\tau_3\Psi_L = -\Psi_L$. All Landau levels except the lowest one ($n = 0$, $\sigma = -1$) are doubly degenerate. The electron states in these levels consist of opposite chirality pairs. The spectrum of the lowest Landau level consists of two nondegenerate linear branches, $\epsilon_0 = \pm u p_z$, formed by the states with opposite chirality. As a result, in the presence of an electric field the system exhibits the chiral anomaly [7–9]. The acceleration of the electrons by the electric field creates a population imbalance of electrons with different chirality. Since the Hamiltonian Eq. (1) decouples into a pair of chiral (L and R) Weyl Hamiltonians in the presence of an arbitrary potential $V(\mathbf{r})$, scattering by disorder does not relax the chirality imbalance. Therefore, even at full momentum relaxation of the electron distribution with a given chirality, there is a finite electric current proportional to the chirality imbalance (chiral magnetic effect) [25,26]. In this approximation, the electrical conductivity is infinite.

At $E_g \neq 0$, chirality is no longer conserved, as the second term in Eq. (1), $E_g\tau_1$, couples the Weyl fermions with opposite chirality. However, the helicity operator $\hat{\alpha} = \mathbf{p}\cdot\boldsymbol{\sigma}/p$ still commutes with the Hamiltonian of the free electron motion described by the first two terms in Eq. (1). Thus, the states of the free electron motion may be classified by the helicity eigenvalues, $\alpha = \pm 1$ (at $E_g = 0$, the helicity of free electron states coincides with the chirality up to the sign of the electron energy). Note that this classification applies even in the presence of a magnetic field, since the operator $\boldsymbol{p}\cdot\boldsymbol{\sigma}$ is still diagonal in the basis of energy eigenstates. The quantity p in the definition of helicity should be understood as the modulus of the eigenvalue of this operator. The helicity content of Landau level states is shown in Fig. 1. The states in the doubly degenerate Landau levels come in opposite helicity pairs, while the helicity of states in the nondegenerate lowest Landau level is given by $\alpha = \mathrm{sgn}(p_z)$.

Although at $E_g \neq 0$ there is no chiral anomaly, the mechanism of longitudinal magnetoresistance is quite similar to that due to the chiral anomaly. Namely, the acceleration of electrons by the electric field directed along $\mathbf{B}$ produces a helicity imbalance. The helicity imbalance in turn produces an electric current even at full momentum relaxation within a population of electrons with the same helicity. In contrast to chirality, helicity is not conserved by disorder scattering. Nevertheless, if the Fermi energy E_F strongly exceeds the gap E_g, the helicity relaxation rate is parametrically small. As is shown in Supplemental Material [27], in this regime the helicity relaxation time $\tau_h(\varepsilon)$ may be expressed in terms of the transport mean free time $\tau_{\mathrm{tr}}(\varepsilon)$ as

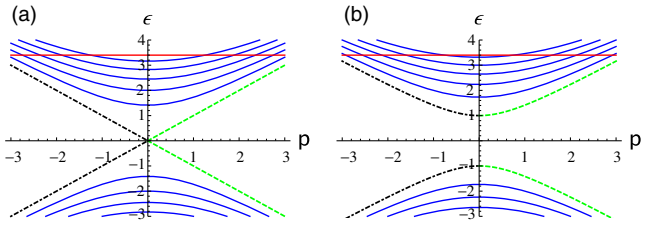

FIG. 1. Landau level spectrum of the Dirac equation in the gapless case $E_g = 0$ [panel (a)] and the gapped case [panel (b)]. All Landau levels except the lowest one are degenerate in helicity and are shown by solid blue lines. The lowest Landau level is nondegenerate. The helicity of electronic states in it is indicated by the line style: Positive helicity states are shown by the green dashed line, and negative helicity states by the black dash-dotted line. The red horizontal line indicates the location of the chemical potential.

$$\frac{\tau_h(\varepsilon)}{\tau_{\mathrm{tr}}(\varepsilon)} = \xi\frac{4\varepsilon^2}{E_g^2} \gg 1. \tag{3}$$

Here ξ is a numerical coefficient of the order of unity which depends on the angular dependence of the impurity scattering cross section. In the Born approximation, it is given by Eq. (A7) in Supplemental Material [27]. Below, we develop a theory of electron magnetotransport phenomena in the leading approximation in $\tau_{\mathrm{tr}}/\tau_h$.

In the regime $\tau_{\mathrm{tr}}/\tau_h \ll 1$, during a short time τ_{tr} the electron distribution becomes isotropic in momentum and becomes dependent only on the electron energy ε and helicity $\alpha = \pm 1$, i.e., takes the form $n_\alpha(\varepsilon)$. Then the total electron density n is given by

$$n = \sum_\alpha \int d\epsilon \nu(\epsilon) n_\alpha(\epsilon), \tag{4}$$

where $\nu_\alpha(\epsilon)$ is the density of states with helicity α. In the leading approximation in $\tau_{\mathrm{tr}}/\tau_h$, the equations describing electronic transport have the form (see Appendix B in Supplemental Material [27])

$$\partial_t n_\alpha(\varepsilon) = -\frac{\boldsymbol{\nabla}\cdot\boldsymbol{j}_\alpha(\varepsilon)}{\nu_\alpha(\varepsilon)} - \frac{k_\alpha}{\nu_\alpha(\varepsilon)}\frac{e^2\boldsymbol{E}\cdot\boldsymbol{B}}{h^2 c}\partial_\varepsilon n_\alpha^{(0)}(\varepsilon) - \frac{n_\alpha(\varepsilon) - n_{-\alpha}(\varepsilon)}{\tau_h(\varepsilon)} + I_\alpha^{\mathrm{in}}\{n_\alpha(\varepsilon)\}, \tag{5}$$

where $h = 2\pi\hbar$ and $k_\alpha = \alpha$. The collision integral due to inelastic electron-electron and electron-phonon scattering processes is denoted by $I_\alpha^{\rm in}\{n_\alpha(\varepsilon)\}$, and we expressed the collision integral due to impurity scattering in terms of the helicity relaxation time; see Eq. (A4) in Ref. [27]. Finally,

$$\mathbf{j}_\alpha(\varepsilon) = \frac{ek_\alpha n_\alpha(\varepsilon)}{h^2 c}\boldsymbol{B} \tag{6}$$

denotes the density of a particle current with helicity α per unit energy [25,26]. The electric current $\boldsymbol{j}$ and the heat flux $\boldsymbol{j}_q$ may be expressed, respectively, as

$$\mathbf{j} = e\sum_\alpha \int d\varepsilon \mathbf{j}_\alpha(\varepsilon), \qquad \mathbf{j}_q = \sum_\alpha \int d\varepsilon(\varepsilon-\mu)\mathbf{j}_\alpha(\varepsilon), \tag{7}$$

where μ is the chemical potential.

Note that in the limit $\tau_{\rm tr}(\varepsilon)/\tau_h(\varepsilon) \to 0$ both the current $\mathbf{j}_\alpha(\varepsilon)$ and the helicity pumping are associated only with the lowest Landau level. Accordingly, the second term in the right-hand side of Eq. (5) can be written as $(dp_z/dt)(\nu_L(\varepsilon)/\nu(\epsilon))(d\epsilon_0(p_z)/dp_z)(\partial n(\epsilon)/\partial\epsilon)$, where $(dp_z/dt) = eE_z$ and $\nu_L(\varepsilon) = (1/2\pi h l_B^2)(dp_z/d\epsilon_0(p_z))$ is the density of states of the chiral Landau level [here the derivative $(d\epsilon_0(p_z)/dp_z)$ is evaluated at $\epsilon_0(p_z) = \varepsilon$]. Then elastic scattering redistributes the helicity imbalance created by the electric field between the electron states with a given energy in all Landau levels. This is the reason why there is a density of states in the denominator in the first and the second terms in the right-hand side of Eq. (5).

Equations (5)–(7) coincide with those obtained in Refs. [14,15] for Weyl semimetals with a topologically protected gapless electron spectrum. In Weyl semimetals, $k_\alpha = \pm 1$ is given by the quantized monopole charge of the Berry curvature flux, and Eq. (5) describes the chiral anomaly. The above consideration shows that both the generation of a helicity imbalance due to the acceleration of electrons by the electric field, described by Eq. (5), and the current proportional to helicity imbalance, Eq. (6), exist in generic conductors with no topological protection of the electron spectrum.

Below, we discuss longitudinal magnetotransport phenomena: NLMR, enhancement of thermal conductivity, and the thermoelectric effect by a magnetic field. Generally speaking, linear response phenomena are characterized by tensor transport coefficients. Equations (5)–(7), on the other hand, describe only the "anomalous" contributions to the transport coefficients which affect only the zz components of the tensors. Here $\hat{z}$ is the direction of the magnetic field.

Using Eq. (5) and assuming that $n_\alpha(\varepsilon) = n_F(\varepsilon) + \delta n_\alpha(\varepsilon)$, where $n_F(\varepsilon) = [e^{(\varepsilon-\mu)/T(r)} + 1]^{-1}$ is the locally equilibrium Fermi distribution function, we get

$$I_\alpha^{\rm in}\{n_\alpha(\varepsilon)\} = \frac{\delta n_\alpha(\varepsilon) - \delta n_{-\alpha}(\varepsilon)}{\tau_h(\varepsilon)} + \frac{ek_\alpha(e\boldsymbol{E} - \frac{\varepsilon-\mu}{T}\boldsymbol{\nabla}T)\cdot\boldsymbol{B}}{\nu_\alpha(\varepsilon)h^2 c}\partial_\varepsilon n_F(\varepsilon). \tag{8}$$

Note that, although both terms in the right-hand side are odd in k_α, their effect on the nonequilibrium distribution function is drastically different. Only the first term creates the helicity imbalance, whereas the second term creates an energy imbalance between the electron populations with different helicities. The inelastic collisions relax this energy imbalance but not the helicity imbalance. As a result, the nonequilibrium distribution function may be written in the form

$$\delta n_\alpha(\varepsilon) = \frac{ek_\alpha[\tau_{\rm eff}\frac{\varepsilon-\mu}{T}\boldsymbol{\nabla}T - \tau_h(\varepsilon)e\boldsymbol{E}]\cdot\boldsymbol{B}}{2\nu_\alpha(\varepsilon)h^2 c}\frac{dn_F(\varepsilon)}{d\varepsilon}. \tag{9}$$

Here $1/\tau_{\rm eff}$ is the effective rate of energy transfer between the electron populations with opposite helicity. Treating the inelastic collision integral in the relaxation time approximation, we may express it as

$$1/\tau_{\rm eff} = 1/\tau_h + 1/\tau_\epsilon, \tag{10}$$

where $1/\tau_\epsilon$ is the inelastic relaxation rate.

Substituting Eq. (9) into Eqs. (6) and (7) and expressing the electric current and energy flux densities in the form

$$\begin{pmatrix}\mathbf{j}\\ \mathbf{j}_q\end{pmatrix} = \begin{pmatrix}\hat{\sigma} & \hat{\beta}\\ \hat{\gamma} & \hat{\zeta}\end{pmatrix}\begin{pmatrix}\mathbf{E}\\ \boldsymbol{\nabla}T\end{pmatrix}, \tag{11}$$

we obtain the following expressions for the zz components of the transport tensors:

$$\sigma_{zz} = \left(\frac{e^2B}{h^2c}\right)^2\int d\varepsilon\left(-\frac{dn_F(\varepsilon)}{d\varepsilon}\right)\frac{\tau_h(\varepsilon)}{\nu_\alpha(\varepsilon)}, \tag{12a}$$

$$\beta_{zz} = \left(\frac{eB}{h^2c}\right)^2\int d\varepsilon\frac{e(\varepsilon-\mu)}{T}\frac{dn_F(\varepsilon)}{d\varepsilon}\frac{\tau_{\rm eff}(\varepsilon)}{\nu_\alpha(\varepsilon)}, \tag{12b}$$

$$\zeta_{zz} = \left(\frac{eB}{h^2c}\right)^2\int d\varepsilon\frac{(\varepsilon-\mu)^2}{T}\frac{dn_F(\varepsilon)}{d\varepsilon}\frac{\tau_{\rm eff}(\varepsilon)}{\nu_\alpha(\varepsilon)}. \tag{12c}$$

By the Onsager symmetry principle, $\gamma_{zz} = -\beta_{zz}T$. The electronic contribution to thermal conductivity κ_{zz} may be expressed in terms of the electrical conductivity σ_{zz} and other transport coefficients in Eq. (12) as [1] $\kappa_{zz} = -\zeta_{zz} - T\beta_{zz}^2/\sigma_{zz}$. Since at high temperatures the considered effects are small, we concentrate on the low-temperature regime $T \ll \mu$. In this case, Eqs. (12) simplify to

$$\sigma_{zz}(\mu) = \left(\frac{e^2 B}{h^2 c}\right)^2 \frac{\tau_h(\mu)}{\nu(\mu)}, \tag{13a}$$

$$\zeta_{zz}(\mu) = -\frac{\pi^2 T}{3e^2}\frac{\tau_{\rm eff}(\mu)}{\tau_h(\mu)}\sigma_{zz}(\mu), \tag{13b}$$

$$\beta_{zz}(\mu) = e\frac{d\zeta_{zz}(\mu)}{d\mu}. \tag{13c}$$

Under the conditions specified above, the results in Eq. (13) are valid not only for Weyl and Dirac materials but also for conventional conductors. In the case of Weyl and Dirac semimetals, these equations reproduce results obtained in Refs. [14,15]. The difference between the conventional time-reversal invariant and centrosymmetric materials and Weyl semimetals is in value of the helicity relaxation time τ_h. In noncentrosymmetric Weyl semimetals with a spin-nondegenerate electron spectrum, the large value of $\tau_h/\tau_{\rm tr}$ may be associated with the fact that for a smooth disorder potential their intervalley transitions associated with a large momentum transfer are suppressed. In conventional conductors, the large value of $\tau_h/\tau_{\rm tr}$ arises from the large ratio of the Fermi energy to the band gap $E_g \ll \mu$, as described by Eq. (3). Taking $\nu(\mu) = \mu^2/\hbar^3 u^3$, we get

$$\frac{\sigma_{zz}}{\sigma_D} \sim \left(\frac{\hbar u e B}{c\mu E_g}\right)^2 \sim \left(\frac{\hbar\omega_c}{E_g}\right)^2. \tag{14}$$

Here $\sigma_D = 2e^2\nu D$, with $D = u^2\tau_{\rm tr}/3$ being the intravalley diffusion coefficient, is the Drude conductivity, and $\omega_c \sim eBu/c\mu$ is the cyclotron frequency. Equation (14) may be considered as an upper bound estimate for the magnitude of NLMR. The presence in the material of short-range impurities, which cannot be described by Eq. (1), decreases the magnitude of the effect.

Of course there are other, "conventional" contributions to the longitudinal magnetoresistance associated with the Fermi surface anisotropy (see, for example, Ref. [6] and references therein). Typically, at a small magnetic field these contributions to magnetoconductivity scale as $[\sigma_{zz}(B) - \sigma(0)] \sim \chi\sigma(0)(\omega_c\tau_{\rm tr})^2$ and saturate at $\omega_c\tau_{\rm tr} \sim 1$. Here $\chi < 1$ is a parameter characterizing the Fermi surface anisotropy. Thus, the condition for Eq. (14) to dominate the LMR is

$$\chi(E_g\tau_{\rm tr}/\hbar)^2 < 1. \tag{15}$$

Even if this condition is not satisfied, the negative contribution to the magnetoresistance, Eq. (14), can dominate at high magnetic fields where the conventional contribution saturates. In this case, the longitudinal magnetoresistance is a nonmonotonic function of the magnetic field. We note that a nonmonotonic B dependence of σ_{zz} at a low magnetic field was observed in most experiments on Dirac and Weyl metals.

In experiments on Dirac semimetals, the observed magnetoconductance was a few times greater than the Drude value of the conductivity at $\mathbf{B} = 0$. According to Eq. (14), this may happen if $\hbar\omega_c/E_g > 1$. Note that in the quasiclassical limit $\hbar\omega_c \ll \mu$. Then to have a big effect one should have $\mu \gg E_g$.

We would like to point out an important physical difference between the expressions for the magnetoconductivity σ_{zz} in Eq. (12a), on the one hand, and κ_{zz} and the thermoelectric α_{zz} in Eqs. (12b) and (12c), on the other hand. The magnetoconductance in Eq. (12a) is controlled by the helicity relaxation time τ_h, while the magnetic field dependence of the thermoelectric coefficient and thermal conductivity are controlled by $\tau_{\rm eff}$, which is a combination of the helicity relaxation time τ_h and the inelastic relaxation time τ_ϵ. Thus, according to Eq. (13b), the Wiedemann-Franz law is violated at high temperatures where $\tau_{\rm eff} \ll \tau_h$. Furthermore, despite the conventional form of Eq. (13c), the Mott relation also does not hold, $\beta_{zz}(\mu) \neq -\pi^2 T/(3e)\partial_\mu\sigma_{zz}(\mu)$. The aforementioned difference, and, consequently, the violation of the Wiedemann-Franz and Mott relations, can be traced to the difference in the physical processes which determine magnetoconductance $\sigma_{zz}(B)$ and the magnetic field dependence of $\zeta_{zz}(B)$ and $\beta_{zz}(B)$. The magnetoconductance is controlled by the long relaxation time τ_h of helicity imbalance at the Fermi level, which is created by the acceleration of the electrons in the lowest Landau level in the presence of the electric field. This is similar to the chiral anomaly. As long as $T \ll \mu$ and τ_h slowly depends of the electron energy, the temperature dependence of the negative longitudinal magnetoresistance is weak. This explains why NLMR was observed up to relatively high temperatures. In contrast, the temperature gradient does not create helicity imbalance but only produces an energy imbalance between the electron populations with opposite helicity. The relaxation of the energy imbalance is governed by the time $\tau_{\rm eff}$, which at $\tau_h > \tau_\epsilon$ coincides with the inelastic relaxation time $\tau_{\rm eff} \approx \tau_\epsilon$. As a result, the thermal conductivity and the thermoelectric coefficient exhibit a strong temperature dependence. In the "hydrodynamic" regime where $\tau_{\rm tr} \gg \tau_\epsilon$, the described above contributions $\kappa_{zz}(B)$ and $\alpha_{zz}(B)$ become negligible compared to the conventional contributions. Thus, the dependence of the thermal conductivity and the thermoelectric coefficient on the magnetic field is unrelated to the chiral anomaly.

In conclusion, we have shown that positive contributions to the parallel magnetoconductance $\sigma_{zz}(B)$, the magnetic field-dependent parallel thermal conductivity, and the thermoelectric coefficient $\beta_{zz}(B)$ can exist not only in Weyl and Dirac semimetals but also in conventional centrosymmetric conductors as well. We also would like to mention that the magnetic field dependence of the sound absorption coefficient exhibits similar properties [15]. We also expect that, similarly to the negative magnetoresistance of pn junctions in Weyl semimetals [28], the

magnetoresistance of pn junctions in Dirac semimetals with a sufficiently small gap E_g will also be negative.

Our consideration focused on the quasiclassical regime $\hbar u/l_B \ll \mu$. In the ultraquantum limit $\hbar u/l_B \gg \mu$, when only the zeroth Landau level is occupied, the situation is more complicated. In Weyl semimetals in the single particle approximation, an expression for conductivity in this regime was obtained in Ref. [7], $\sigma_{zz} \propto (e^2 u/4\pi\hbar l_B^2)\tau_h$. A similar result can be obtained for degenerate Dirac metals in the ultraquantum regime. The magnetic field dependence of the longitudinal magnetoresistance in this regime is controlled by the corresponding magnetic field dependence of the helicity relaxation rate. The latter depends on the type of impurities. Its evaluation is not essentially different from the calculation of the backscattering rate in conventional semiconductors in the ultraquantum limit. In the context of conventional semiconductors in a quantized magnetic field, there is also a strongly anisotropic contribution to magnetoresistance, unrelated to the chiral anomaly, which may become negative in the longitudinal direction (see, for example, Refs. [29–33]). It is related to the fact that in the presence of a smooth potential in a quantized magnetic field the small angle scattering is suppressed. An additional difficulty in interpreting magnetotransport measurements in the ultraquantum regime is associated with the instability of the electron liquid with respect to charge density wave formation, which drives the system to the insulating state. In contrast, in the semiclassical limit, the theoretical consideration of electron transport is free of the aforementioned complications.

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[1] A. A. Abrikosov, *Fundamentals of the Theory of Metals* (North-Holland, Amsterdam, 1988).
[2] I. M. Lifshits, M. Y. Azbel, and M. I. Kaganov, *Electron Theory of Metals* (Consultants Bureau, New York, 1973).
[3] M. Y. Azbel, M. Kaganov, and I. Lifshitz, Sci. Am. **228**, 88 (1973).
[4] A. B. Pippard, *Magnetoresistance in Metals* (Cambridge University Press, Cambridge, England, 1989), Vol. 2.
[5] M. I. Kaganov and I. Lifshits, Sov. Phys. Usp. **22**, 904 (1979).
[6] H. K. Pal and D. L. Maslov, Phys. Rev. B **81**, 214438 (2010).
[7] H. Nielsen and M. Ninomiya, Phys. Lett. **130B**, 389 (1983).
[8] S. L. Adler, Phys. Rev. **177**, 2426 (1969).
[9] J. S. Bell and R. Jackiw, Nuovo Cimento A **60**, 47 (1969).
[10] X. Wan, A. M. Turner, A. Vishwanath, and S. Y. Savrasov, Phys. Rev. B **83**, 205101 (2011).
[11] A. A. Burkov and L. Balents, Phys. Rev. Lett. **107**, 127205 (2011).
[12] O. Vafek and A. Vishwanath, Annu. Rev. Condens. Matter Phys. **5**, 83 (2014).
[13] M. Hirschberger, S. Kushwaha, Z. Wang, Q. Gibson, S. Liang, C. A. Belvin, B. Bernevig, R. Cava, and N. Ong, Nat. Mater. **15**, 1161 (2016).
[14] D. T. Son and B. Z. Spivak, Phys. Rev. B **88**, 104412 (2013).
[15] B. Z. Spivak and A. V. Andreev, Phys. Rev. B **93**, 085107 (2016).
[16] Q. Li, D. E. Kharzeev, C. Zhang, Y. Huang, I. Pletikosić, A. Fedorov, R. Zhong, J. Schneeloch, G. Gu, and T. Valla, Nat. Phys. **12**, 550 (2016).
[17] J. Xiong, S. K. Kushwaha, T. Liang, J. W. Krizan, M. Hirschberger, W. Wang, R. Cava, and N. Ong, Science **350**, 413 (2015).
[18] X. Yang, Y. Liu, Z. Wang, Y. Zheng, and Z.-a. Xu, arXiv: 1506.03190.
[19] C.-Z. Li, L.-X. Wang, H. Liu, J. Wang, Z.-M. Liao, and D.-P. Yu, Nat. Commun. **6**, 10137 (2015).
[20] J. Cano, B. Bradlyn, Z. Wang, M. Hirschberger, N. P. Ong, and B. A. Bernevig, Phys. Rev. B **95**, 161306 (2017).
[21] F. Arnold *et al.*, Nat. Commun. **7**, 11615 (2016).
[22] E. O. Kane, J. Phys. Chem. Solids **1**, 249 (1957).
[23] W. Zawadzki, arXiv:1701.07067.
[24] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Quantum Electrodynamics* (Butterworth-Heinemann, London, 1982), Vol. 4.
[25] A. Vilenkin, Phys. Rev. D **22**, 3080 (1980).
[26] K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D **78**, 074033 (2008).
[27] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.120.026601 for the derivation of the helicity relaxation rate, Eq. (3), and of the evolution equation, Eq. (5).
[28] S. Li, A. V. Andreev, and B. Z. Spivak, Phys. Rev. B **94**, 081408(R) (2016).
[29] P. Argyres and E. Adams, Phys. Rev. **104**, 900 (1956).
[30] L. S. Dubinskaya, Sov. Phys. JETP **29**, 436 (1969).
[31] R. Mansfield and T. Ellis, J. Phys. C **9**, 3781 (1976).
[32] S. S. Murzin, Phys. Usp. **43**, 349 (2000).
[33] P. Goswami, J. H. Pixley, and S. Das Sarma, Phys. Rev. B **92**, 075205 (2015).