Editors' Suggestion

Noise of a Chargeless Fermi Liquid

Cătălin Paşcu Moca,[1,2] Christophe Mora,[3] Ireneusz Weymann,[4] and Gergely Zaránd[1]
[1]*BME-MTA Exotic Quantum Phase Group, Institute of Physics, Budapest University of Technology and Economics, H-1521 Budapest, Hungary*
[2]*Department of Physics, University of Oradea, 410087 Oradea, Romania*
[3]*Laboratoire Pierre Aigrain, École Normale Supérieure, Université Paris 7 Diderot, CNRS, 24 rue Lhomond, 75005 Paris, France*
[4]*Faculty of Physics, Adam Mickiewicz University, 61-614 Poznań, Poland*

(Received 18 April 2017; revised manuscript received 17 October 2017; published 5 January 2018)

We construct a Fermi liquid theory to describe transport in a superconductor-quantum dot-normal metal junction close to the singlet-doublet (parity changing) transition of the dot. Though quasiparticles do not have a definite charge in this chargeless Fermi liquid, in the case of particle-hole symmetry, a mapping to the Anderson model unveils a hidden U(1) symmetry and a corresponding pseudocharge. In contrast to other correlated Fermi liquids, the back scattering noise reveals an effective charge equal to the charge of Cooper pairs, $e^* = 2e$. In addition, we find a strong suppression of noise when the linear conductance is unitary, even for its nonlinear part.

DOI: 10.1103/PhysRevLett.120.016803

Introduction.—Apart from disrupting our communication, noise contains interesting and abundant information. Advances in experimental techniques gradually allowed us to enter the quantum regime and access this information through noise measurements in various setups, ranging from nanocircuits [1–7] or quantum optics devices [8] to bosonic and superfluid systems in cold atomic settings [9]. Noise can reveal the quantum statistics and the charge of quasiparticles as well as the nature of their interactions. The bunching of photons in quantum optics, e.g., reflects the bosonic nature of light [10], while the complete suppression of noise, in the case of a perfectly transmitting conductance channel in a nanocircuit [11], is a consequence of electrons being fermions.

Low temperature noise has been used to extract the transmission amplitudes of a point contact [5,12,13] and also to gain insight into the structure of quantum fluctuations [1,14]. In such interacting nanocontacts, the shot noise carries information on the structure of residual interactions and elementary excitations. For example, the noise of a back-scattered current in quantum-Hall devices has been used, e.g., to extract the fractional charge e^* of excitations at fillings $\nu = 1/3$ [15,16] or $\nu = 2/3$ [17]. Furthermore, in a strongly-interacting Fermi liquid, realized, e.g., in a quantum dot (QD) attached to normal electrodes at very low temperatures, the noise of the back-scattered current is induced by interactions, and the corresponding effective charge, $e^* = 5e/3$ turns out to reflect the structure of local interactions rather than that of elementary excitations [4,18–20].

Superconductors, from this perspective, are of particular interest; while they obviously carry current, elementary excitations in a superconductor do not have a definite charge and possess only spin. In particular, attaching a superconductor to normal electrodes destroys the charge of electrons in its neighborhood by the proximity effect [21], and this too makes the charge ill-defined. Here we wish to understand the structure of the low temperature electric noise of these *chargeless* excitations in the presence of strong interactions. For this purpose, we propose to study the superconductor-quantum dot-normal metal (S-QD-N) system depicted in Fig. 1, and which was investigated experimentally and theoretically by several groups [22–27]. The

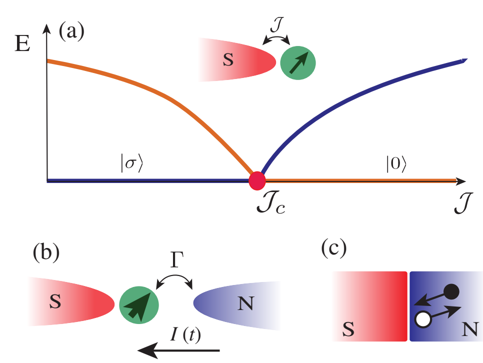

FIG. 1. (a) Subgap states of the S-QD system depicted in the inset. At $\mathcal{J} = \mathcal{J}_c$, a parity-changing transition occurs. For $\mathcal{J} > \mathcal{J}_c$, the ground state is a many-body singlet $|0\rangle$, while for $\mathcal{J} < \mathcal{J}_c$, it is a doublet $|\sigma\rangle$. (b) We consider a localized spin strongly coupled to a superconductor (S) and weakly to a normal (N) lead. (c) Andreev scattering processes in the Fermi liquid state: an electron is reflected as a hole, while a Cooper pair is transmitted to the S-QD device.

setup consists of an artificial atom attached more strongly to a superconductor, and probed by a weakly coupled normal electrode or a scanning tunneling microscope tip.

As we discuss below, at very low temperatures, this system becomes a Fermi liquid of chargeless quasiparticles. Nevertheless, we can still describe it in terms of the Noziéres' Fermi liquid theory [28], generalized for the Anderson model [29]. Surprisingly, in the limit of maximal conductance, the current through this device turns out to be almost "noiseless," up to (and including) $\mathcal{O}(V^3)$ order. Moving slightly away from this sweet point, the shot noise S is found to be generated by the back-scattering current δI [18,30,31], yielding an effective charge

$$e^* = \frac{S}{\delta I} = 2e + \cdots. \tag{1}$$

In stark contrast to regular Fermi liquids [18,32], this charge appears to be related only to the fact that electrons enter the superconductor as Cooper pairs and, unexpectedly, is not influenced by the otherwise strong interactions—a prediction that could easily be verified experimentally [33,34].

Model.—Throughout this work, we shall focus on the local moment regime, where there is just a single electron on the QD, which can be described as a single spin $S = 1/2$, coupled antiferromagnetically to the two electrodes by an exchange coupling (see Fig. 1).

In the absence of the normal electrode [Fig. 1(a)], the spin on the dot binds a quasiparticle to itself antiferromagnetically, appearing as an excited singlet state $|0\rangle$ inside the gap [35] for small exchange couplings, $\mathcal{J}$ [36]. With increasing $\mathcal{J}$, the quasiparticle's binding energy becomes larger, and the energy of this so-called Shiba state drifts towards zero until at a critical value, $\mathcal{J} = \mathcal{J}_c$, the exchange energy gain becomes larger than the quasiparticle gap, the energy of $|0\rangle$ becomes smaller than that of the doubly degenerate spin states $|\pm\rangle$ (dressed by the superconductor) [37], and a parity changing, quantum phase transition occurs [23,26,38–42]. Coupling the QD weakly to a normal electrode at $\mathcal{J} \approx \mathcal{J}_c$ induces strong quantum fluctuations between the states $|\pm\rangle$ and $|0\rangle$, and this leads to a strongly correlated local Fermi liquid state [28,50,51] with a close to perfect transmission and conductance [52] $G \approx 4e^2/h$. Though our conclusions turn out to be quite general, throughout this Letter, we focus on the vicinity of this transition and further assume that the tunneling rate Γ between the QD and the normal electrode is much smaller than the superconducting gap, $\Gamma \ll \Delta_S$. Then tunneling can be described just in terms of the three states, $|0\rangle$ and $|\sigma\rangle$, and a simple Hamiltonian [42]

$$H = \Delta E\left(\sum_\sigma |\sigma\rangle\langle\sigma| - |0\rangle\langle 0|\right) + H_v + H_\psi. \tag{2}$$

Here, the first term describes the level crossing in Fig. 1, with $\Delta E = E_\sigma - E_0 \propto \mathcal{J} - \mathcal{J}_c$ vanishing at the transition point, while the last term H_ψ accounts for electrons in the normal electrode. The second term

$$H_v = \sum_\sigma [|\sigma\rangle\langle 0|(v_\sigma^+ \psi_\sigma - v_{\bar\sigma}^- \psi_{\bar\sigma}^\dagger) + \text{H.c.}], \tag{3}$$

generates tunneling between the normal electrode and the QD, with the operator ψ_σ removing a conduction electron of spin σ from the normal electrode [53]. This tunneling induces quantum fluctuations between the states $|\sigma\rangle$ and $|0\rangle$, and this turns the parity changing transition in Fig. 1(a) into a crossover with resonant features. The structure of H_v follows simply from symmetry considerations: the states $|\sigma\rangle$ have a well-defined spin and a well-defined charge *parity*, but they do not have a well-defined charge, and possess both holelike and electronlike components. Therefore, both spin $\uparrow$ electrons and spin $\downarrow$ holes can tunnel into the state $|\uparrow\rangle$ from the normal electrode. Furthermore, electron-hole symmetry and time-reversal symmetry imply the relations $v_\sigma^+ = v_\uparrow^- = -v_\downarrow^- = v$ for the tunneling matrix elements [54].

We can simplify the problem further by introducing new fields, $\Phi_\sigma \sim v_\sigma^+ \psi_\sigma - v_{\bar\sigma}^- \psi_{\bar\sigma}^\dagger$, and rewriting the tunneling Hamiltonian as

$$H_v = v\sum_\sigma (|\sigma\rangle\langle 0|\Phi_\sigma + \text{H.c.}).$$

Clearly, in terms of this new field, our Hamiltonian (3) is just the mixed valence Anderson model [55]. Notice, however, that this mapping is only valid in equilibrium. Biasing the S-QD-N junction affects the hole and electron components of the field Φ_σ differently. Note also that, although the fields Φ_σ have no charge, the Hamiltonian conserves a corresponding "pseudocharge," $\tilde{Q}$ [56].

Fermi liquid theory.—The ground state of the Anderson model is a famous example of a local Fermi liquid. At temperatures and voltages below the so-called Fermi liquid scale, $eV, k_B T \ll T_{\text{FL}}$, one can follow Landau and Nozières [28], and describe its low temperature (low bias) behavior in terms of weakly and locally interacting quasiparticles $b_{\varepsilon\sigma}$ created by the field Φ_σ. Although Fermi liquid theory has been well-known since the seminal work of Nozières [28], it has only recently been extended to the Anderson model [29]. The structure of the Fermi liquid Hamiltonian follows from symmetries: it must conserve spin and the pseudo charge $\tilde{Q}$; however, unlike Nozières' original theory [28], it is not "electron-hole" symmetrical in terms of $\tilde{Q}$ [57]. Its relevant terms can be identified by power counting, yielding the simple structure

$$\begin{aligned}
H_{\text{FL}} &= \sum_\sigma \int_\varepsilon \varepsilon b_{\varepsilon\sigma}^\dagger b_{\varepsilon\sigma} + H_\alpha + H_\phi + \cdots \\
H_\alpha &= -\sum_\sigma \int_{\varepsilon_1,\varepsilon_2} \left[\frac{\alpha_1}{2\pi}(\varepsilon_1+\varepsilon_2) + \frac{\alpha_2}{4\pi}(\varepsilon_1+\varepsilon_2)^2\right] b_{\varepsilon_1\sigma}^\dagger b_{\varepsilon_2\sigma} \\
H_\phi &= \int_{\varepsilon_1,\ldots,\varepsilon_4} \left[\frac{\phi_1}{\pi} + \frac{\phi_2}{4\pi}\left(\sum_{i=1}^4 \varepsilon_i\right)\right] : b_{\varepsilon_1\uparrow}^\dagger b_{\varepsilon_2\uparrow} b_{\varepsilon_3\downarrow}^\dagger b_{\varepsilon_4\downarrow} :.
\end{aligned} \tag{4}$$

Here, $:\ldots:$ denotes normal ordering, and the annihilation operators, $b_{\varepsilon\sigma}$ destroy scattering states with the incoming and outgoing parts behaving as $\Phi_\sigma(r) \sim e^{-irk}e^{-irk_F}$ and $\sim S_\sigma e^{ikr}e^{irk_F}$, respectively, with k_F the Fermi momentum, $k = \varepsilon/v_F$, and $S_\sigma = e^{2i\delta_\sigma}$ the scattering amplitude. The terms in H_α generate energy dependent phase shifts, while the last two terms account for interactions, and are responsible for inelastic scattering. All Fermi liquid coefficients are functions of Γ and ΔE, with the hybridization providing the natural Fermi liquid scale throughout the parity-changing transition, $T_{\mathrm{FL}} \sim \Gamma \sim v^2$ [58]. Importantly, α_1, $\phi_1 \sim 1/\Gamma$, while α_2, $\phi_2 \sim 1/\Gamma^2$, but apart from these prefactors, they are all universal functions of the ratio $\Delta E/\Gamma$ (see [42] for more details).

Current and noise.—We now use the Fermi liquid theory outlined above to study nonequilibrium transport and noise. Our main purpose is to determine the expectation value and the noise of the current $\hat{I}(r) = ev_F\sum_\sigma[\psi_\sigma^\dagger(-r)\psi_\sigma(-r) - \psi_\sigma^\dagger(r)\psi_\sigma(r)]$ at some position $r > 0$, at low temperatures, and small bias voltages. For simplicity, we now focus on the case of electron-hole symmetry. Then, the connection between $\psi_{\uparrow/\downarrow}$ and $\Phi_{\uparrow/\downarrow}$ simplifies, and the current operator becomes

$$\hat{I}(r) = ev_F[b_\downarrow(-r)b_\uparrow(-r) - S_\downarrow b_\downarrow(r)S_\uparrow b_\uparrow(r) + \mathrm{H.c.}], \quad (5)$$

where we introduced the auxiliary quasiparticle fields $b_\sigma(x) \equiv \int d\varepsilon e^{ikx} b_{\varepsilon\sigma}$ and made use of the asymptotic structure of scattering states. Since $b_{\varepsilon\sigma}$ represents incoming scattering states, expectation values of their products are determined by the normal electrodes (see [42]). This enables us to compute $\langle\hat{I}\rangle$, or its correlation functions, perturbatively in the interaction terms of Eq. (4) using diagrammatic methods.

We now focus on the nonequilibrium regime, where the temperature is much smaller than the applied voltage bias, $eV \gg T \approx 0$. There the zeroth order expectation value of the current, e.g., simply yields $\langle\hat{I}\rangle = G_0V + \cdots$, where the linear conductance recovers the known expression for noninteracting electrons [11]

$$G_0 = \frac{2e^2}{h}[1 - \Re\mathrm{e}(S_\downarrow S_\uparrow)] = \frac{4e^2}{h}\sin^2(\delta_\uparrow + \delta_\downarrow), \quad (6)$$

with δ_σ the phase shifts of the (chargeless) quasiparticles at the Fermi energy, $S_\sigma = e^{2i\delta_\sigma}$. In the absence of magnetic field $\delta_\uparrow = \delta_\downarrow = \delta_0$, it is just a function of $\Delta E/\Gamma$, smoothly crossing over from 0 to $\pi/2$, and thereby producing a conductance resonance at $\Delta E \approx 0$ (see Fig. 4).

Let us first focus on the "unitary point," $\delta_0 = \pi/4$. There every incoming electron is perfectly reflected as a hole, and we find a noiseless current even in the presence of interactions (at least up to third order in the voltage V), similar to a perfectly transmitting, noninteracting point contact [59]. To determine the effective charge of the carriers, we therefore need to move slightly away from this unitary limit and, similar to Refs. [17,18,31], investigate the *backscattered* current, $\delta\hat{I}$, defined as the correction with respect to the maximal current, $I_u = 4e^2V/h$.

Close to the unitary (resonant) point of maximal conductance, $\delta_0 \to \pi/4 + \tilde{\delta}$, we can gain insight to the structure of the backscattered current perturbatively. At the level of the effective field theory, we can induce the change in the phase shift by adding a term, $\tilde{H}_\delta = -(\tilde{\delta}/\pi)\sum_\sigma \int_{\varepsilon,\varepsilon'} b_{\varepsilon\sigma}^\dagger b_{\varepsilon'\sigma}$, and we can treat its effect similarly to the other terms in Eq. (4). To see how $\tilde{\delta}$ generates the backscattered current, we rewrite $\tilde{H}_\delta$ in terms of "charged" unitary quasiparticle operators, $a_{\varepsilon\uparrow/\downarrow} = (b_{\varepsilon\uparrow/\downarrow} \mp b_{-\varepsilon\downarrow/\uparrow}^\dagger)/\sqrt{2}$ as

$$\tilde{H}_\delta = -(\tilde{\delta}/\pi)\sum_\sigma \int_{\varepsilon,\varepsilon'} (a_{\varepsilon\uparrow}^\dagger a_{\varepsilon'\downarrow}^\dagger + \mathrm{H.c.}).$$

The word "charged" must be used with caution: the operator $a_{\varepsilon\uparrow}^\dagger$ creates, namely, a scattering state, which at time $t \to -\infty$ represents an incoming electron of charge e on the normal side, while for $t \to \infty$, it is a reflected hole of charge $-e$. Scattering events generated by the term $\tilde{H}_\delta$ convert, e.g., an incoming electron state $a_{\varepsilon\uparrow}^\dagger|\mathrm{FS}\rangle$ into an incoming hole state $a_{-\varepsilon\downarrow}|\mathrm{FS}\rangle$ which, for time $t \to \infty$, represents a reflected *electron*. These reflected electrons *reduce* the transmitted current, and they contribute to the backscattered current, $\delta\hat{I}$ (see Fig. 2).

The rate of these backscattering events can be simply computed by applying Fermi's golden rule as

$$\Gamma_\delta = \sum_\sigma \left(\frac{2\pi}{\hbar}\right) \int_{\varepsilon_1\varepsilon_2} |\langle a_{\varepsilon_2,-\sigma}^\dagger H_\delta a_{\varepsilon_1\sigma}^\dagger\rangle|^2 \delta(\varepsilon_1 + \varepsilon_2), \quad (7)$$

yielding $\Gamma_\delta = 8\tilde{\delta}^2 eV/h$ and a backscattering current $\delta I_\delta = 2e\Gamma_\delta$. Similarly computing the rates of all scattering processes perturbatively, we arrive at

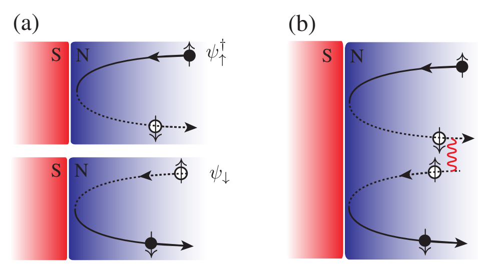

FIG. 2. (a) The "charged" quasiparticles $a_{\varepsilon\sigma}$ correspond to incoming electrons reflected as holes. Each unscattered quasiparticle therefore transfers a charge of $2e$ to the superconducting side. (b) A potential scattering event $a_{\varepsilon\uparrow}^\dagger|\mathrm{FS}\rangle \to a_{-\varepsilon\downarrow}|\mathrm{FS}\rangle$ creates an outgoing electron state, i.e., a reflected charge, and reduces the current by $2e$.

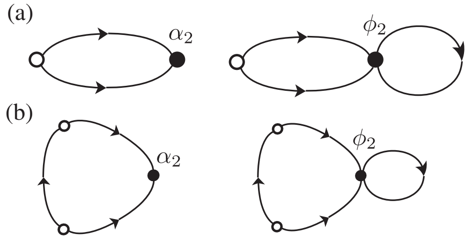

FIG. 3. First order diagrams describing the corrections to the current (a) and to the noise (b). Open circles represent current vertices, while filled dots correspond to interaction vertices. Only nonvanishing contributions are displayed.

$$\langle \delta\hat{I}\rangle = \frac{8e^2}{h}V\left\{2\tilde{\delta}^2 + \left(\frac{4}{3}\alpha_2 - \phi_2\right)\tilde{\delta}(eV)^2 + \cdots\right\}. \quad (8)$$

All processes that give contributions to $\langle \delta\hat{I}\rangle$ to this order turn out to be such that an incoming electron does not transfer its charge to the superconductor, but it is reflected back as an electron or as an electron and an electron-hole pair. Assuming that these are all independent Poissonian processes that reduce the current by $2e$, we immediately arrive at the conclusion that the shot noise is just $2e\langle\delta I\rangle$, yielding the effective charge (1).

The previous perturbative result can be generalized to arbitrary values of δ by incorporating the phase shift δ in the definition of the quasiparticle operators $a_{\varepsilon\sigma}$ and the current operator, and then using the Keldysh approach to perform perturbation theory in the Fermi liquid coefficients [42]. The leading corrections to the current are shown as an example in Fig. 3, and yield

$$\langle\hat{I}\rangle = \frac{2e^2}{h}V\left\{2\sin^2 2\delta - \frac{\pi\chi_c'}{3}\sin 4\delta(eV)^2 + \cdots\right\}, \quad (9)$$

where we made use of a Fermi liquid relation, relating the derivative of the charge susceptibility of the effective Anderson model [29] to the Fermi liquid coefficients as, $\frac{4}{3}\alpha_2 - \phi_2 = -\pi\chi_c'/3$. By expanding this formula around $\delta_0 = \pi/4$, we recover Eq. (8).

The shot noise can be computed along similar lines [42]. It can be expressed as a power series

$$S(V,\Delta E) = \frac{2e^3}{h}V\sum_{n=0}^{\infty} s_n(\Delta E)(eV/\Gamma)^n, \quad (10)$$

with the first few dimensionless coefficients given by $s_0 = \sin^2 4\delta$, $s_1 = 0$ and $s_2 = -\Gamma^2(\pi\chi_c'/3)\sin 8\delta$. Remarkably, at the sweet point of maximal conductance, $\delta = \delta_0 = \pi/4$, both s_0 and s_2 vanish, and the current remains noiseless up to $\sim(eV/\Gamma)^4$ even in the presence of interactions (see Fig. 4). Similar to the dimensionless linear conductance, $g \equiv G/G_0 = \sin^2(2\delta)$, the dimensionless coefficients s_n are universal functions of the ratio $\Delta E/\Gamma$. These functions can all be determined using Bethe ansatz or numerical renormalization group calculations [29], and we have displayed them in Fig. 4.

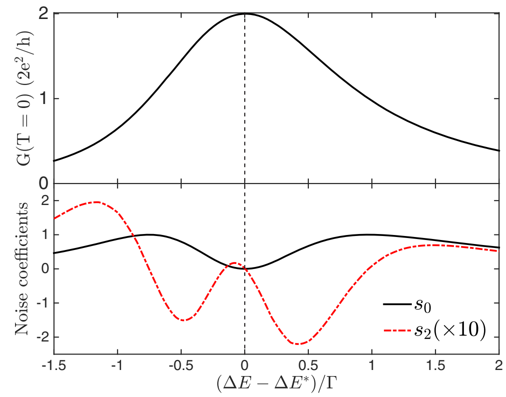

FIG. 4. Dimensionless conductance (top panel) and the first nonvanishing dimensionless noise coefficients, s_0 and s_2 (bottom panel). The coefficient s_2 is magnified 10 times. The noise coefficients vanish at the point of maximal conductance, $\Delta E = \Delta E^*$, where $\delta = \delta_0 = \pi/4$.

We are now in the position of expressing the effective charge e^* as a function of ΔE and V,

$$\frac{e^*}{e} \equiv \frac{S}{e\delta I} = \frac{\sin^2 4\delta - \frac{\pi\chi_c'}{3}\sin 8\delta(eV)^2 + \cdots}{2\cos^2 2\delta + \frac{\pi\chi_c'}{3}\sin 4\delta(eV)^2 + \cdots}. \quad (11)$$

For small voltages, this formula yields an effective charge $e^* = 2e(1-\delta g)$, while for larger voltages, this value crosses over to $e^* = 2e(1-2\delta g)$, with $\delta g = 1 - \sin^2(2\delta) = \cos^2(2\delta)$ representing the reduction of the $T = 0$ temperature dimensionless linear conductance with respect to its maximal, unitary value. Unlike a usual, strongly interacting Fermi liquid, the effective charge remains close to $2e$ in the vicinity of the unitary scattering regime, even beyond the linear voltage regime. The effective charge of the chargeless Fermi liquid is thus that of Cooper pairs $e^* \approx 2e$, and it only slightly deviates from this value in spite of the strong electron-electron interactions. This result is not unexpected for the lowest order, elastic terms in Eq. (11), which are expected, and it indeed agrees with results on noninteracting S-N junctions [12] and mean field calculations [22]. They are completely unexpected in the case of the interaction-generated terms, $\sim(eV)^2$, and are related to the lack of $4e$ charge transfer processes.

Although we focused on the local moment regime and restricted ourselves to $\Gamma < \Delta_S$, most of our results are quite general. In fact, our Fermi liquid approach does not rely on these assumptions, and it gives a valid description for temperatures and voltages below the Fermi liquid scale, $\{eV, k_BT\} \ll T_{\mathrm{FL}}$. Our results do not depend on the asymmetry of the junction either, and they are barely influenced by other external perturbations too. An external magnetic field, e.g., splits the phase shifts $\delta_{\uparrow,\downarrow}$, but the expressions above depend only on the sum, $\delta_\uparrow + \delta_\downarrow$, and are therefore independent of the applied field to the order discussed here. We only assumed electron-hole symmetry, a valid assumption for a half-filled quantum dot levels. We expect, however, that even electron-hole symmetry breaking just reduces the value of the maximum conductance, but it should not influence the effective charge $e^{\star} \approx 2e$ at the transition [60].

Measuring the effective charge should be possible with current-day technology. Indeed, Shiba transitions in half-filled quantum dots have been demonstrated by several groups [23,25,61] for typical quantum dot parameters [62]. Since tuning the tunneling rates is a part of the usual modern architecture [23,25], it should be possible to reach the Fermi liquid regime, $k_B T < \Gamma$, and verify our predictions without much difficulty.

We would like to thank Pascal Simon for fruitful discussions. This work has been supported by the Hungarian research Grant No. OTKA K105149, Romanian UEFISCDI Grants No. PN-II-RU-TE-2014-4-0432 and No. PN-III-P4-ID-PCE-2016-0032, and Polish NCN Grant No. DEC-2013/10/E/ST3/00213.

[1] J. Basset, A. Y. Kasumov, C. P. Moca, G. Zaránd, P. Simon, H. Bouchiat, and R. Deblock, Phys. Rev. Lett. **108**, 046802 (2012).
[2] Y. Meir and A. Golub, Phys. Rev. Lett. **88**, 116802 (2002).
[3] Y. C. Chung, M. Heiblum, and V. Umansky, Phys. Rev. Lett. **91**, 216804 (2003).
[4] Y. Yamauchi, K. Sekiguchi, K. Chida, T. Arakawa, S. Nakamura, K. Kobayashi, T. Ono, T. Fujii, and R. Sakano, Phys. Rev. Lett. **106**, 176601 (2011).
[5] J. C. Cuevas, A. Martín-Rodero, and A. LevyYeyati, Phys. Rev. Lett. **82**, 4086 (1999).
[6] M. Dolev, Y. Gross, Y. C. Chung, M. Heiblum, V. Umansky, and D. Mahalu, Phys. Rev. B **81**, 161303 (2010).
[7] J. Gabelli and B. Reulet, Phys. Rev. Lett. **100**, 026601 (2008).
[8] A. A. Clerk, M. H. Devoret, S. M. Girvin, F. Marquardt, and R. J. Schoelkopf, Rev. Mod. Phys. **82**, 1155 (2010).
[9] R. Walser and P. Zoller, Phys. Rev. A **49**, 5067 (1994).
[10] H. Paul, Rev. Mod. Phys. **54**, 1061 (1982).
[11] Y. Blanter and M. Buttiker, Phys. Rep. **336**, 1 (2000).
[12] M. J. M. de Jong and C. W. J. Beenakker, Phys. Rev. B **49**, 16070 (1994).
[13] H. E. van den Brom and J. M. van Ruitenbeek, Phys. Rev. Lett. **82**, 1526 (1999).
[14] J. Gabelli and B. Reulet, Phys. Rev. B **87**, 075403 (2013).
[15] R. de Picciotto, M. Reznikov, M. Heiblum, V. Umansky, G. Bunin, and D. Mahalu, Nature (London) **389**, 162 (1997).
[16] L. Saminadayar, D. C. Glattli, Y. Jin, and B. Etienne, Phys. Rev. Lett. **79**, 2526 (1997).
[17] A. Bid, N. Ofek, M. Heiblum, V. Umansky, and D. Mahalu, Phys. Rev. Lett. **103**, 236802 (2009).
[18] E. Sela, Y. Oreg, F. von Oppen, and J. Koch, Phys. Rev. Lett. **97**, 086601 (2006).
[19] M. Ferrier, T. Arakawa, T. Hata, R. Fujiwara, R. Delagrange, R. Weil, R. Deblock, R. Sakano, A. Oguri, and K. Kobayashi, Nat. Phys. **12**, 230 (2016).
[20] M. Ferrier, T. Arakawa, T. Hata, R. Fujiwara, R. Delagrange, R. Deblock, Y. Teratani, R. Sakano, A. Oguri, and K. Kobayashi, Phys. Rev. Lett. **118**, 196803 (2017).
[21] D. W. Abraham, C. J. Lobb, M. Tinkham, and T. M. Klapwijk, Phys. Rev. B **26**, 5268 (1982).
[22] P. Schwab and R. Raimondi, Phys. Rev. B **59**, 1637 (1999).
[23] R. S. Deacon, Y. Tanaka, A. Oiwa, R. Sakano, K. Yoshida, K. Shibata, K. Hirakawa, and S. Tarucha, Phys. Rev. Lett. **104**, 076805 (2010).
[24] W. Chang, V. E. Manucharyan, T. S. Jespersen, J. Nygård, and C. M. Marcus, Phys. Rev. Lett. **110**, 217005 (2013).
[25] E. J. H. Lee, X. Jiang, M. Houzet, R. Aguado, C. M. Lieber, and S. De Franceschi, Nat. Nanotechnol. **9**, 79 (2014).
[26] J. Schindele, A. Baumgartner, R. Maurand, M. Weiss, and C. Schönenberger, Phys. Rev. B **89**, 045422 (2014).
[27] R. Delagrange, R. Weil, A. Kasumov, M. Ferrier, H. Bouchiat, and R. Deblock, Phys. Rev. B **93**, 195437 (2016).
[28] P. Nozières, J. Low Temp. Phys. **17**, 31 (1974).
[29] C. Mora, C. P. Moca, J. von Delft, and G. Zaránd, Phys. Rev. B **92**, 075120 (2015).
[30] A. Golub, Phys. Rev. B **73**, 233310 (2006).
[31] A. Kaminski, Y. V. Nazarov, and L. I. Glazman, Phys. Rev. B **62**, 8154 (2000).
[32] C. Mora, X. Leyronas, and N. Regnault, Phys. Rev. Lett. **100**, 036604 (2008).
[33] B.-R. Choi, A. E. Hansen, T. Kontos, C. Hoffmann, S. Oberholzer, W. Belzig, C. Schönenberger, T. Akazaki, and H. Takayanagi, Phys. Rev. B **72**, 024501 (2005).
[34] Y. Ronen, Y. Cohen, J.-H. Kang, A. Haim, M.-T. Rieder, M. Heiblum, D. Mahalu, and H. Shtrikman, Proc. Natl. Acad. Sci. U.S.A. **113**, 1743 (2016).
[35] H. Shiba, Prog. Theor. Phys. **40**, 435 (1968).
[36] Here $\mathcal{J} \sim \Gamma_S/\rho U$, with Γ_S representing the tunneling rate from the dot to the superconductor, ρ the density of states at the Fermi level in the superconductor's normal state, and U as the electron-electron interaction on the dot.
[37] The location of the phase transition depends slightly on the strength of the tunneling rate to the superconductor, Γ_S with respect to the superconducting gap Δ_S, but typically occurs when $\rho\mathcal{J} \sim \Gamma_S/U \approx 0.1 - 0.5$ (see Ref. [39]).
[38] J. Bauer, A. Oguri, and A. C. Hewson, J. Phys. Condens. Matter **19**, 486211 (2007).
[39] T. Meng, S. Florens, and P. Simon, Phys. Rev. B **79**, 224521 (2009).
[40] R.Žitko, J. S. Lim, R. López, and R. Aguado, Phys. Rev. B **91**, 045441 (2015).
[41] S. Droste, J. Splettstoesser, and M. Governale, Phys. Rev. B **91**, 125401 (2015).
[42] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.120.016803 for details on Fermi liquid theory and numerical renormalization group calculations, which includes Refs. [11,28,29,35,43–50].
[43] P. W. Anderson, Phys. Rev. **124**, 41 (1961).
[44] A. Kamenev and A. Levchenko, Adv. Phys. **58**, 197 (2009).

[45] C. Mora, P. Vitushinsky, X. Leyronas, A. A. Clerk, and K. Le Hur, Phys. Rev. B **80**, 155322 (2009).

[46] M. J. M. de Jong and C. W. J. Beenakker, Phys. Rev. B **49**, 16070 (1994).

[47] A. C. Hewson, *The Kondo Problem to Heavy Fermions* (Cambridge University Press, Cambridge, 1993).

[48] K. G. Wilson, Rev. Mod. Phys. **47**, 773 (1975).

[49] O. Sakai, Y. Shimizu, H. Shiba, and K. Satori, J. Phys. Soc. Jpn. **62**, 3181 (1993).

[50] Y. Tanaka, N. Kawakami, and A. Oguri, J. Phys. Soc. Jpn. **76**, 074701 (2007).

[51] A. Oguri, Y. Tanaka, and J. Bauer, Phys. Rev. B **87**, 075432 (2013).

[52] J. C. Cuevas, A. Levy Yeyati, and A. Martín-Rodero, Phys. Rev. B **63**, 094515 (2001).

[53] The effective Hamiltonian (3) is very general and holds beyond the local moment regime, too.

[54] We also checked these relations via the numerical renormalization group calculations presented in [42].

[55] A mapping to the mixed valence Anderson model was also proposed in Refs. [50,51] in the particular case of an infinite superconducting gap.

[56] $\tilde{Q}$ is associated with a U(1) symmetry, generated by gauge transformations, $\Phi_\sigma \to e^{i\alpha}\Phi_\sigma$, $|\pm\rangle \to e^{i\alpha}|\pm\rangle$.

[57] C. Mora, Phys. Rev. B **80**, 125304 (2009).

[58] While for $\Delta E \approx 0$ one finds $T_{\mathrm{FL}} \sim \Gamma$, but Γ and T_{FL} become very different far from the transition.

[59] B. J. van Wees, H. van Houten, C. W. J. Beenakker, J. G. Williamson, L. P. Kouwenhoven, D. van der Marel, and C. T. Foxon, Phys. Rev. Lett. **60**, 848 (1988).

[60] A generalization of the Fermi liquid theory to the case of electron-hole symmetry breaking is possible but not straightforward.

[61] A. Jellinggaard, K. Grove-Rasmussen, M. H. Madsen, and J. Nygård, Phys. Rev. B **94**, 064520 (2016).

[62] For typical experiments U, Γ_s, and Δ_s are all in the range of a few Kelvins, and Γ, ΔE, V, and T must therefore all be below a few hundred mK.