Collective Longitudinal Polarization in Relativistic Heavy-Ion Collisions at Very High Energy

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We study the polarization of particles in relativistic heavy-ion collisions at very high energy along the beam direction within a relativistic hydrodynamic framework. We show that this component of the polarization decreases much slower with center-of-mass energy compared to the transverse component, even in the ideal longitudinal boost-invariant scenario with nonfluctuating initial state, and that it can be measured by taking advantage of its quadrupole structure in the transverse momentum plane. In the ideal longitudinal boost-invariant scenario, the polarization is proportional to the gradient of temperature at the hadronization and its measurement can provide important information about the cooling rate of the quark-gluon plasma around the critical temperature.

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Global polarization of hadrons produced in relativistic heavy-ion collisions has been recently observed by the STAR experiment over a center-of-mass energy range between 7.7 and 200 GeV [1,2]. This finding confirms early proposals [3,4], later predictions based on local thermodynamic equilibrium of spin degrees of freedom [5,6], which provides a relation between polarization and relativistic vorticity, and it agrees quantitatively with the hydrodynamic model calculations [7–9] to a very good degree of accuracy. In the hydrodynamic framework, the distinctive feature of polarization is its proportionality to the gradients of the combined temperature and velocity fields [see Eq. (5)], so that its measurement is a stringent test of the hydrodynamic picture which is distinct and complementary to momentum spectra.

The experimental efforts, thus far, focused on the search of the average global polarization of Λ hyperons along the direction of the angular momentum of the plasma. This measurement requires the identification of the reaction plane in peripheral collision as well as its orientation, that is the direction of the total angular momentum vector $\mathbf{J}$. The average global polarization along $\mathbf{J}$ is also found to decrease rapidly as a function of center-of-mass energy [1], from few percent at $\sqrt{s_{NN}} = \mathcal{O}(10)$ GeV to few permille at $\sqrt{s_{NN}} = \mathcal{O}(100)$ GeV, in agreement with calculations based on the hydrodynamic model [8–10] as well with hybrid approaches [11,12]. In the TeV energy range, at the LHC, the global polarization along $\mathbf{J}$ is not seen [13], as it is most likely beyond experimental sensitivity.

It is of course desirable to check more—possibly distinctive—predictions of the hydrodynamic model besides the global polarization along $\mathbf{J}$. For instance, in Ref. [14], a connection between local vortical structures in event-by-event hydrodynamics and correlation of polarizations of two Λ hyperons in transverse and longitudinal (along the beam line) directions has been studied. In this Letter, we argue that in noncentral heavy-ion collisions, a nonzero longitudinal polarization of Λ with different transverse momenta p_T is a more generic effect present in a simple nonfluctuating hydrodynamic picture, and propose to measure it in experiment [15]. As it will be shown, this observable has several attractive features: (i) unlike the polarization along $\mathbf{J}$, it is sensitive only to the transverse expansion dynamics; (ii) it is found not to decrease rapidly as a function of center-of-mass energy (similar to longitudinal correlations in Ref. [14]) and it can be detected even at the LHC energy in the TeV range; (iii) it survives the "minimal vorticity" scenario of Bjorken longitudinal boost invariance; (iv) unlike the polarization component along the angular momentum, it does not require the identification of the orientation (hereafter, we use the term "orientation" in its mathematical sense, meaning discrete choice of orientation of a normal vector to the plane) of the reaction plane, thus greatly reducing the experimental labor. The effect is dominated by the geometry of collision; therefore, we do not include event-by-event fluctuations in this study.

Symmetries in relativistic nuclear collisions at very high energy.—In principle, (average) collisions of two identical nuclei at a finite impact parameter feature two initial discrete symmetries: rotation by an angle π around the total angular momentum axis and reflection in the transverse plane with respect to the reaction plane (see Fig. 1). Their combination implies an invariance by total reflection $\mathbf{x} \to -\mathbf{x}$. In the high energy limit, another (continuous) symmetry becomes plausible and it is commonly assumed in the hydrodynamic modeling of relativistic heavy-ion collisions, that is the invariance by Lorentz boost along the beam axis, also known as Bjorken longitudinal boost invariance. The straightforward consequence of the boost invariance is that any scalar function of space-time coordinates is independent of the space-time rapidity $\eta = (1/2)\log[(t+z)/(t-z)]$, where z

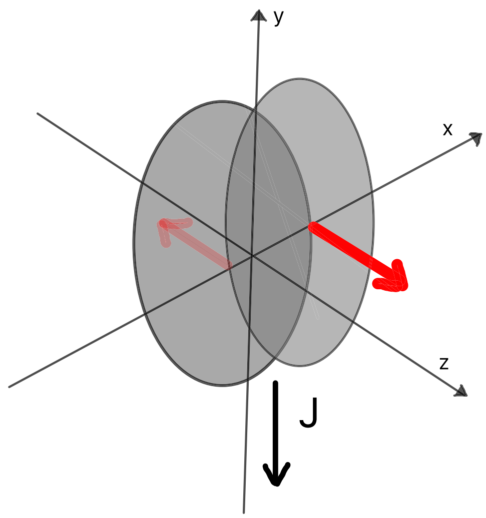

FIG. 1. A sketch of peripheral relativistic heavy-ion collisions. The system is symmetric for a rotation by 180° around the angular momentum and reflection with respect to the reaction plane xz. The axes x, y, z are the reference frame for this work.

is the Cartesian coordinate along the beam direction and t the time in the center-of-mass frame. Also, by combining reflection with boost invariance, one readily obtains that the space-time rapidity component of any vector field V vanishes; that is, $V^\eta = 0$.

An orthogonal (i.e., scalar product conserving) symmetry transformation for a general tensor field (including all special cases: scalar, vector, pseudovector, etc.) can be expressed as

$$T(\Lambda(x)) = D(\Lambda)T(x), \qquad (1)$$

where Λ is the linear symmetry transformation and $D(\Lambda)$ its representation matrix for the tensor field T. Suppose that we deal with a (tensor) function Θ of four-momentum which is expressed as an integral over a domain Ω, symmetric under the transformation Λ, of a tensor function of x and p:

$$\Theta(p) = \int_\Omega d\Omega T(x,p), \qquad (2)$$

with

$$T(\Lambda(x),\Lambda(p)) = D(\Lambda)T(x,p). \qquad (3)$$

Therefore,

$$\begin{aligned}\Theta(\Lambda(p)) &= \int_\Omega d\Omega T(x,\Lambda(p)) \\ &= \int_D d\Omega D(\Lambda)^{-1} T(\Lambda^{-1}(x),p) = D(\Lambda)^{-1} \\ &\quad \times \int_{\Lambda^{-1}(\Omega)} d\Omega' T(x',p) \\ &= D(\Lambda)^{-1}\int_\Omega d\Omega T(x',p) = D(\Lambda)^{-1}\Theta(\Lambda(p)),\end{aligned} \qquad (4)$$

where we have used Eq. (3) and the invariance of the domain Ω under the transformation Λ, implying $d\Omega' = d\Omega$ and $\Lambda(\Omega) = \Omega$. We are thus led to the conclusion that, when dealing with observables in momentum space like Eq. (2) with integrand fulfilling the condition Eq. (3), the spatial symmetries have counterparts in momentum space, as demonstrated by Eq. (4). For instance, this is the case for the particle momentum spectrum of fermions which can be written as an integral over the decoupling (or *particlization*) 3D hypersurface Σ:

$$\int_\Sigma d\Sigma_\lambda p^\lambda n_F = \int_\Sigma d\Sigma (n\cdot p) n_F,$$

where n is the unit vector perpendicular to the hypersurface Σ and n_F is the relativistic Fermi-Dirac distribution:

$$n_F = \frac{1}{e^{\beta\cdot p - \sum_j \mu_j q_j/T} + 1},$$

where $\beta = (1/T)u$ is the four-temperature vector. The reason is that any integrand function involving the scalar product of four-momentum and a symmetric vector field like $\beta(x)$ or $n(x)$ fulfills Eq. (3). For instance,

$$\beta(\Lambda(x))\cdot\Lambda(p) = \Lambda(\beta(x))\cdot\Lambda(p) = \beta(x)\cdot p.$$

For the specific case of relativistic heavy-ion collisions and longitudinal boost invariance, this means that the spectrum of final particles will be invariant under longitudinal boost in momentum space, which is independent of the rapidity $Y = (1/2)\log[(E+p_z)/(E-p_z)]$, and similarly for the reflection and discrete rotation symmetries.

Polarization of emitted particles in high energy heavy-ion collisions.—These symmetries have remarkable consequences on the polarization of emitted particles, specifically on the single-particle mean spin vector $S^\mu(p)$. At the leading order, the mean spin vector is given by the formula [5]

$$S^\mu(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} p_\tau \frac{\int_\Sigma d\Sigma_\lambda p^\lambda \varpi_{\rho\sigma} n_F(1-n_F)}{\int_\Sigma d\Sigma_\lambda p^\lambda n_F}, \qquad (5)$$

where ϖ is the thermal vorticity, that is,

$$\varpi_{\mu\nu} \equiv \frac{1}{2}(\partial_\nu\beta_\mu - \partial_\mu\beta_\nu) \qquad (6)$$

and Σ is the decoupling hypersurface (see also Refs. [16,17]). It is important to stress that in Eq. (5), the Cartesian coordinates in the integrand are understood as they are the only ones making sense of an integral of a vector field. The $S(p)$ is a pseudovector in momentum space, so, unlike polar vectors, the component along the beam line S^z (henceforth defined as *longitudinal*) at $Y = 0$ can be nonvanishing and must feature a quadrupole pattern in the transverse momentum plane like that shown in Fig. 2. Particularly, the rotation-reflection symmetries imply that S^z has a Fourier decomposition involving only the sine of even multiples of the azimuthal angle φ:

$$S^z(\mathbf{p}_T, Y=0) = \frac{1}{2}\sum_{k=1}^{\infty} f_{2k}(p_T)\sin 2k\varphi. \tag{7}$$

A nice feature of Eq. (7) is that the sign of S^z in the transverse momentum plane does not depend on the reaction plane orientation, for a fixed handedness of the reference frame. The basically quadrupole pattern of the longitudinal component of the spin vector had been observed in numerical hydrodynamic calculations [8,18], with a remarkable feature that S^z has an absolute magnitude larger than those of the transverse components and we will delve into this feature later on.

Longitudinal boost invariance has further consequences for the spin vector in momentum space. Because of the boost and reflection invariance, a vector field at $z=0$ must have a vanishing Cartesian longitudinal component $V^z=0$. Likewise, the Cartesian longitudinal component of any vector field in momentum space must be vanishing at midrapidity $Y=0$. This implies that the transverse components of a pseudovector field in momentum space must be vanishing; that is,

$$S^x(\mathbf{p}_T, Y=0) = S^y(\mathbf{p}_T, Y=0) = 0.$$

The current experimental evidence seems to bear out the asymptotic longitudinal boost-invariance scenario insofar as S^y, which is the component perpendicular to the reaction plane, is found to steadily decrease as center-of-mass energy increases. This has been observed by the experiment STAR [1] and confirmed by a null result from ALICE [13] at $\sqrt{s_{NN}} = 2.76$ TeV. These observations are in agreement with numerical calculation of the polarization of Λ hyperons carried out in Refs. [8,18] with hydrodynamic models asymptotically fulfilling longitudinal boost invariance. In fact, according to these calculations the longitudinal component of the mean spin vector S^z turns out to be sizably larger in magnitude than the transverse ones from $\sqrt{s_{NN}} \approx 60$ GeV onwards and it is thus reasonable to surmise that it will survive at the highest center-of-mass energy in the TeV range.

Indeed, it can be shown that this component does not vanish even in the exact boost-invariant scenario with no initial state fluctuations and that it decreases slowly with increasing center-of-mass energy. For the sake of simplicity, let us demonstrate that with an explicit calculation by assuming that the fluid is ideal, uncharged, and that the *initial* transverse velocities u^x, u^y vanish. Accumulated evidence in relativistic heavy-ion collisions indicates that these are reasonable approximations at very high energy. Under such assumptions, it is known that a particular antisymmetric tensor, the T vorticity,

$$\Omega_{\mu\nu} = \partial_\mu(Tu_\nu) - \partial_\nu(Tu_\mu), \tag{8}$$

vanishes at all times [18,19], as a consequence of the equations of motion. In this case, the thermal vorticity reduces to [18]

$$\varpi_{\mu\nu} = \frac{1}{T}(A_\mu u_\nu - A_\nu u_\mu), \tag{9}$$

A being the four-acceleration field. This form of the thermal vorticity shows its entirely relativistic nature, its spatial part being proportional to $(\mathbf{a}\times\mathbf{v})/c^2$ in the classical units. If we now substitute Eq. (9) in Eq. (5), we get

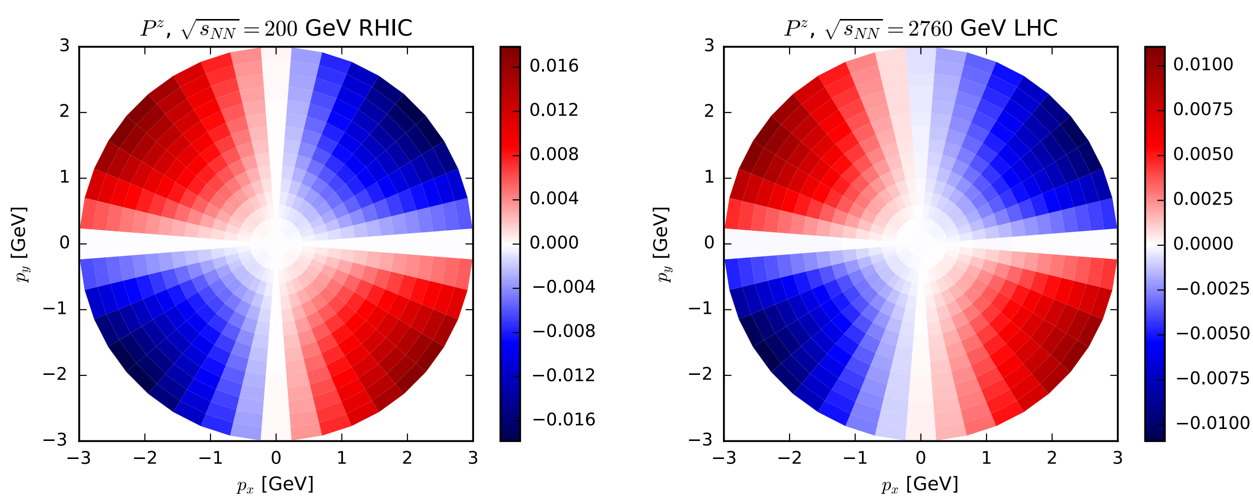

FIG. 2. Map of longitudinal component of polarization of midrapidity Λ from a hydrodynamic calculation corresponding to 20%–50% central Au-Au collisions at $\sqrt{s_{NN}} = 200$ GeV (left) and 20%–50% central Pb-Pb collisions at $\sqrt{s_{NN}} = 2760$ GeV (right).

$$S^{\mu}(p) = -\frac{1}{4m}\epsilon^{\mu\rho\sigma\tau}p_{\tau}\frac{\int_{\Sigma} d\Sigma_{\lambda}p^{\lambda}A_{\rho}\beta_{\sigma}n_F(1-n_F)}{\int_{\Sigma} d\Sigma_{\lambda}p^{\lambda}n_F}, \quad (10)$$

which shows that $S^z(p)$ can get contributions from the vector product of fields and momenta in the transverse plane, where they are expected to significantly develop even in the case of longitudinal boost invariance. The uncharged perfect fluid equations of motion can be written as

$$A_{\rho} = \frac{1}{T}\nabla_{\rho}T = \frac{1}{T}(\partial_{\rho}T - u^{\rho}u\cdot\partial T).$$

If we plug the above acceleration expression into Eq. (11), only the first term with $\partial_{\rho}T$ gives a finite contribution as the second term vanishes owing to the presence of the $\beta_{\sigma}u_{\rho}$ factor and the Levi-Civita tensor. Furthermore, since

$$\frac{\partial}{\partial p^{\sigma}}n_F = -\beta_{\sigma}n_F(1-n_F),$$

we can rewrite Eq. (10) as

$$S^{\mu}(p) = \frac{1}{4mT}\epsilon^{\mu\rho\sigma\tau}p_{\tau}\frac{\int_{\Sigma} d\Sigma_{\lambda}p^{\lambda}\frac{\partial n_F}{\partial p^{\sigma}}\partial_{\rho}T}{\int_{\Sigma} d\Sigma_{\lambda}p^{\lambda}n_F}. \quad (11)$$

We can now integrate by parts the numerator in the above equation:

$$\int_{\Sigma} d\Sigma_{\lambda}p^{\lambda}\frac{\partial n_F}{\partial p^{\sigma}}\partial_{\rho}T = \frac{\partial}{\partial p^{\sigma}}\int_{\Sigma} d\Sigma_{\lambda}p^{\lambda}n_F\partial_{\rho}T - \int_{\Sigma} d\Sigma_{\sigma}n_F\partial_{\rho}T.$$

Another very reasonable assumption is that the decoupling hypersurface at high energy is described by the equation $T = T_c$, where T_c is the QCD pseudocritical temperature. This entails that the normal vector to the hypersurface is the gradient of temperature. Then the final expression of the mean spin vector is

$$S^{\mu}(p) = \frac{1}{4mT}\epsilon^{\mu\rho\sigma\tau}p_{\tau}\frac{\frac{\partial}{\partial p^{\sigma}}\int_{\Sigma} d\Sigma_{\lambda}p^{\lambda}n_F\partial_{\rho}T}{\int_{\Sigma} d\Sigma_{\lambda}p^{\lambda}n_F}. \quad (12)$$

The longitudinal component of the mean spin vector S^z thus depends on the value of the temperature gradient on the decoupling hypersurface and its measurement can provide information thereupon. A simple solution of the above integral appears under the assumption of isochronous decoupling hypersurface, with the temperature field only depending on the Bjorken time $\tau = \sqrt{t^2 - z^2}$. In this case the parameters describing the hypersurface are x, y, η with $\tau = \text{const}$, and the only contribution to the numerator of Eq. (12) arises from $\rho = 0$:

$$\int d\Sigma_{\lambda}p^{\lambda}n_F\frac{dT}{d\tau}\cosh\eta.$$

At $Y = 0$, the factor $\cosh\eta$ can be approximated with 1 because of the exponential falloff $\exp[-(m_T/T)\cosh\eta]$ involved in n_F; therefore,

$$S^z(\mathbf{p}_T, Y=0)\hat{\mathbf{k}} \simeq -\frac{dT/d\tau}{4mT}\hat{\mathbf{k}}\frac{\partial}{\partial\varphi}\log\int_{\Sigma} d\Sigma_{\lambda}p^{\lambda}n_F,$$

where φ is the transverse momentum azimuthal angle, counting from the reaction plane. In the above equation the longitudinal spin component is a function of the spectrum alone at $Y = 0$. By expanding it in Fourier series in φ and retaining only the elliptic flow term, one obtains

$$\begin{aligned} S^z(\mathbf{p}_T, Y=0) &\simeq -\frac{dT/d\tau}{4mT}\frac{\partial}{\partial\varphi}2v_2(p_T)\cos 2\varphi \\ &= \frac{dT}{d\tau}\frac{1}{mT}v_2(p_T)\sin 2\varphi, \end{aligned} \quad (13)$$

meaning, comparing this result to Eq. (7), that in this case

$$f_2(p_T) = 2\frac{dT}{d\tau}\frac{1}{mT}v_2(p_T).$$

This simple formula only applies under special assumptions with regard to the hydrodynamic temperature evolution, but it clearly shows the salient features of the longitudinal polarization at midrapidity as a function of transverse momentum and how it can provide direct information on the temperature gradient at hadronization. It also shows, as has been mentioned, that it is driven by physical quantities related to transverse expansion and that it is independent of longitudinal expansion.

Polarization of Λ *hyperons along the beam line*.—The above conclusion is confirmed by more realistic 3D viscous hydrodynamic simulations of heavy-ion collisions using the averaged initial state from the Monte Carlo Glauber model with its parameters set as in Ref. [20]. We have calculated the polarization vector $\mathbf{P}^* = 2\mathbf{S}^*$ of primary Λ hyperons with $Y = 0$ in their rest frame (note that at midrapidity $S^{*z} = S^z$). The resulting transverse momentum dependence of P^{*z} is shown in Fig. 2 for 20%–50% central Au-Au collisions at $\sqrt{s_{NN}} = 200$ (RHIC) and 20%–50% Pb-Pb collisions at $\sqrt{s_{NN}} = 2760$ GeV (LHC). The corresponding second harmonic coefficients f_2 are displayed in Fig. 3 for four different collision energies: 7.7, 19.6 GeV (calculated with initial state from the UrQMD cascade [21]), 200, and 2760 GeV (with the initial state from Monte Carlo Glauber model [20]). It is worth noting that, while the P^y component, along the angular momentum, decreases by about a factor of 10 between $\sqrt{s_{NN}} = 7.7$ and 200 GeV, f_2 decreases by only 35%. We also find that the mean, p_T integrated value of f_2 stays around 0.2% at all collision energies, owing to two compensating effects: decreasing p_T differential $f_2(p_T)$ and increasing mean p_T with increasing collision energy. The P^y component in our calculations is produced in noncentral collisions only due to anisotropic transverse expansion (elliptic flow), whereas in central collisions the initial state fluctuations dominate, as shown in Ref. [14]. The magnitude of the resulting correlation function [which has a $\cos(2\Delta\phi)$ shape] is similar to the one obtained in Ref. [14].

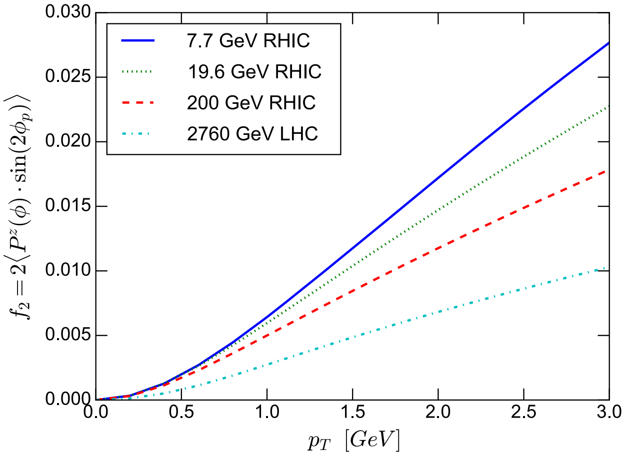

FIG. 3. Second harmonic of the longitudinal component of Λ polarization f_2 from hydrodynamic simulations as a function of p_T for different energies.

In principle, the longitudinal polarization of Λ hyperons can be measured in a similar fashion as for the component perpendicular to the reaction plane, i.e., by studying the distribution of p^{*z}, which is the longitudinal component of the momentum of the decay proton in the Λ rest frame, according to the formula

$$\frac{dN}{d\Omega} = \frac{1}{4\pi}(1 + \alpha \mathbf{P}^* \cdot \hat{\mathbf{p}}^*), \tag{14}$$

where $\alpha = 0.642$ is the known Λ weak decay constant. For Λ at midrapidity, both the longitudinal polarization component and the proton p^{*z} are the same as in the quark-gluon plasma (QGP) frame, so the longitudinal momentum distribution of the decay proton is a direct probe of the mean spin vector in the QGP frame; for the general case, a boost must be performed, but the method is basically the same. Hence, at $Y = 0$, the average sign of the p^z will follow the pattern shown in Fig. 2 for S^z, as a function of the azimuthal angle with respect to the reaction plane, with a leading behavior $\sin 2\varphi$. The probability P_s that the decay proton has a sign s reads:

$$P_s = \frac{1}{2} + \frac{s\alpha}{4} P^{*z},$$

so that the mean sign is just $(\alpha/4)P^{*z}$.

In summary, we have shown that local thermodynamic equilibrium of the spin degrees of freedom and the hydrodynamic model predict a global pattern of polarization along the beam line in relativistic heavy-ion collisions at very high energy even in a minimal scenario of longitudinal boost invariance, ideal fluid, and no initial state fluctuations. We have shown that the polarization component along the beam line has a typical quadrupole structure of $\mathbf{p}_T$ dependence similar to elliptic flow, by virtue of which the identification of the orientation of the reaction plane is not necessary. Its measurement is a crucial test of the hydrodynamic model and of its initial conditions and can provide important and unique information about the temperature gradient at the decoupling stage, when the QGP hadronizes around the critical temperature. Calculations in a realistic implementation of the hydrodynamic model indicate that its value is within the current reach of the experiments at RHIC and LHC energies.

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