# COMPARISON OF A NEW $SU_3$ PREDICTION WITH EXPERIMENT

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The "eightfold way"[1] has been quite successful in describing the masses[2] of elementary particles and resonances, and in explaining the decay widths of various resonances.[3] Various other tests of this symmetry scheme have been proposed which concern themselves with reaction cross sections.[4] Because of the large number of $S$-matrix elements which usually occur, the simplest proposed tests have taken the form of equalities. Unfortunately, these predictions of equalities have been difficult to verify. This paper proposes a rule for reaction processes, which seems capable of experimental test.

Consider processes of the two types:

$$\text{meson} + \text{proton} \to \text{baryon resonance} + \text{meson}, \tag{1}$$

$$\text{meson} + \text{proton} \to \text{baryon resonance} + \text{vector meson}. \tag{2}$$

For the processes of type (1) the reactions treated are

$$K^+ + p \to N^{*++} + K^0, \tag{1a}$$

$$\pi^+ + p \to N^{*++} + \pi^0, \tag{1b}$$

$$\pi^+ + p \to N^{*++} + \eta, \tag{1c}$$

$$\pi^+ + p \to Y_1^{*+} + K^+. \tag{1d}$$

For the processes of type (2) the reactions treated are

$$K^+ + p \to N^{*++} + K^{*0}, \tag{2a}$$

$$\pi^+ + p \to N^{*++} + \rho^0, \tag{2b}$$

$$\pi^+ + p \to N^{*++} + \varphi^0, \tag{2c}$$

$$\pi^+ + p \to Y_1^{*+} + K^{*+}, \tag{2d}$$

where $\varphi^0$ is the $I=0$, $Y=0$ member of the vector-meson octet. Because of $\omega$-$\varphi$ mixing,[5] it is also necessary to consider the following reactions, in addition to the reactions (2a)-(2d):

$$\pi^+ + p \to N^{*++} + \omega^0, \tag{2e}$$

$$\pi^+ + p \to N^{*++} + \omega, \tag{2f}$$

$$\pi^+ + p \to N^{*++} + \varphi. \tag{2g}$$

$\omega^0$ is the vector-meson singlet, and $\omega$ and $\varphi$ denote the physically observed particles of masses 782 MeV and 1020 MeV, respectively.

In general, the meson (11) symmetry and the baryon (11) symmetry can couple together to make product symmetries (22), (11), (11), (30), (03), and (00). Each product symmetry defines a channel through which the reaction can proceed. In a similar fashion the right-hand sides of Eqs. (1) and (2) are described by coupling the baryon-resonance (30) symmetry and either the vector-meson or meson (11) symmetries to produce symsymmetries (41), (30), (22), and (11). One can introduce the energy- and angle-dependent amplitudes $A^{(\lambda\mu)}$ which are diagonal elements of the $S$ matrix and in terms of which one obtains expressions for the amplitudes for processes (1) and (2). Only those symmetries common to the left and right sides of (1) and (2) contribute to the amplitude, i.e., (22), (30), and two (11)'s. An important simplification occurs, however, if we restrict ourselves to incident $\pi^+$ and $K^+$ mesons. Then only the two $SU_3$ channels (22) and (30) contribute, and only the $N^*$ and $Y_1^*$ baryon resonances occur. The coefficients of the $A^{(\lambda\mu)}$ for the processes which we consider are given in Table I. The squares of the amplitudes, $|M_a|^2$, $|M_b|^2$, $|M_c|^2$, $|M_d|^2$, for the four reactions of types (1) and (2) depend on two independent complex amplitudes, $A^{(22)}$ and $A^{(30)}$. Therefore, in each case, one obtains the following relation:

$$|M_a|^2 = |M_b|^2 + 3|M_c|^2 - 3|M_d|^2. \tag{3}$$

Table I. Scattering amplitudes $A^{(\lambda\mu)}$ for production of baryon resonances in meson-baryon reactions. The coefficients result from taking scalar products of $SU_3$ functions for the case $((11)\otimes(11)|(30)\otimes(11))$.

| Process \ $(\lambda\mu)$ | (22) | (30) |
|---|---|---|
| (a) $(K^+p\|N^{*++}K^0)(K^{*0})$ | $-\sqrt{3}/2$ | |
| (b) $(\pi^+p\|N^{*++}\pi^0)(\rho^0)$ | $-\sqrt{3}/4\sqrt{2}$ | $-\sqrt{3}/4$ |
| (c) $(\pi^+p\|N^{*++}\eta)(\varphi^0)$ | $-3/4\sqrt{2}$ | $1/4$ |
| (d) $(\pi^+p\|Y_1^{*+}K^+)(K^{*+})$ | $1/4$ | $-1/2\sqrt{2}$ |

The determination of $|M_{2c}|^2$ is complicated by $\omega$-$\varphi$ mixing.

Equation (3) is a general result, encompassing the predictions of specific models like the peripheral exchange model[6] which, indeed, do satisfy Eq. (3). The fact that Eq. (3) may or may not be satisfied by the experimental cross sections is a test only of $SU_3$, not of the peripheral exchange model.

We are now faced with the problem of comparing Eq. (3) with experiment. Since $SU_3$ symmetry is broken (the masses of particles in an $SU_3$ multiplet are different), there is at the moment no rigorous way of making this comparison. In the following analysis, we ignore the dynamical effects of the mass-splitting interaction on the reaction amplitudes which lead to Eq. (3). We assume that the best way to test Eq. (3) is to compare experimental cross sections for each channel at the same $Q$ value, where $Q = E^* - M_3 - M_4$, $E^*$ = total energy in the c.m. system, and $M_3, M_4$ are the masses of the two outgoing particles. This choice has the virtue that the thresholds for each reaction are superimposed at $Q = 0$ and that the opening up of new channels which, through unitarity, will affect the reactions of types (1) or (2) is also superimposed. The kinematic factors that relate cross sections to matrix elements squared are, of course, different for each channel. We assume that the amplitudes of Eq. (3) are relativistically invariant so that the relation between $|M|^2$ and $\sigma$ for two-body reactions is[7]

$$|M|^2 = (E^{*2} p_{\text{in}}/p_{\text{out}})\sigma \equiv F\sigma, \tag{4}$$

where $p_{\text{in}}$ and $p_{\text{out}}$ are the momenta of incident and outgoing particles in the c.m. system. In Fig. 1, the factor $F = (E^{*2} p_{\text{in}}/p_{\text{out}})$ is plotted vs $Q$ for each of the reactions of type (1) and type (2). The shapes of these curves are nearly the same, again suggesting the usefulness of comparing cross sections for different reactions at similar $Q$ values. With the above assumptions, Eq. (3) predicts the following relation between the experimental cross sections:

$$F_a\sigma_a = F_b\sigma_b + 3F_c\sigma_c - 3F_d\sigma_d. \tag{5}$$

Let us consider the reactions of type (1). Table II lists the pertinent, available experimental cross sections, together with associated values of $E^*$, $Q$, and $F$. In Fig. 2 the four terms of Eq. (5) are plotted individually as functions of $Q$. The lines are drawn roughly through the

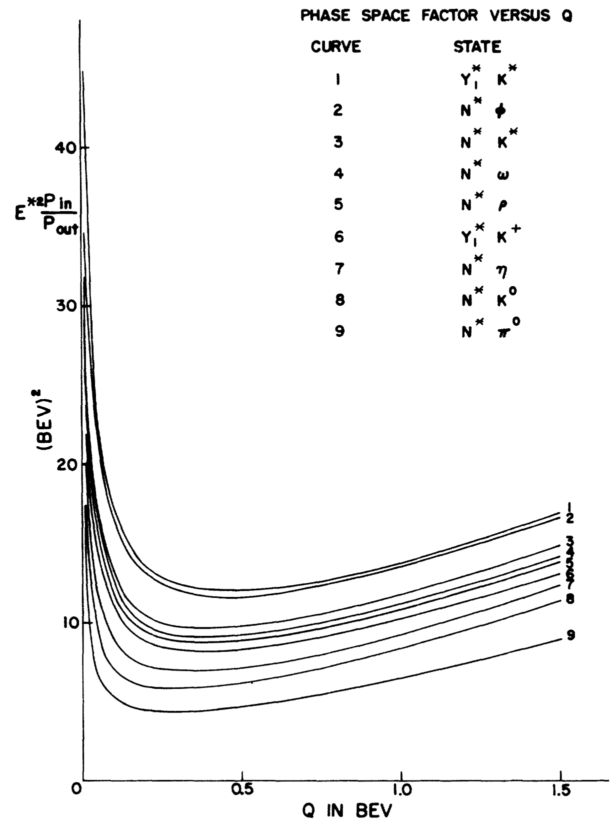


FIG. 1. The factor $F = E^{*2} p_{\text{in}}/p_{\text{out}}$ vs $Q$ for reactions of type (1a)-(1d) and (2a)-(2f). $E^*$, $p_{\text{in}}$, $p_{\text{out}}$, and $Q$ are the total energy, the incident momentum, the outgoing momentum, and the total outgoing kinetic energy in the c.m. system.

data points to serve as a guide. At present, a complete comparison of the left-hand side (LHS) and right-hand side (RHS) of Eq. (5) can be made only at $Q \approx 300$ MeV and 500 MeV. As can be seen from Fig. 2, at $Q \approx 300$ MeV (expressing $\sigma F$ in units of $\text{BeV}^2$-mb), LHS = $25 \pm 6$, and RHS = $28 \pm 6$. At $Q \approx 500$ MeV, LHS = $21 \pm 5$ and RHS = $19 \pm 5$. At $Q$ values below ~120 MeV, the data obviously do not satisfy Eq. (5). In this threshold region, the broad $N^{*++}$ is only partially excited, so that Eq. (5) may not be reliable. Above 500 MeV a detailed test of Eq. (5) requires more and better data than are available, but there is no indication of a large violation of Eq. (5). In general, for reactions of type (1), the $SU_3$ prediction of Eq. (5) is quite well satisfied.

The analysis of reactions of type (2), involving vector mesons, is complicated by $\omega$-$\varphi$ mixing.[5] Since the cross section for $\varphi^0$ production, $\sigma_{2c}$, is not directly measurable, we try to calculate it from $\sigma_{2f}(\omega)$ and $\sigma_{2g}(\varphi)$ (defined previously) which can be measured. Recall that we may write

$$|\omega_0\rangle = \sin\lambda|\varphi\rangle + \cos\lambda|\omega\rangle, \qquad (6)$$

$$|\varphi_0\rangle = \cos\lambda|\varphi\rangle - \sin\lambda|\omega\rangle, \qquad (7)$$

where the mixing angle $\lambda$ equals $38°$; $\cos^2\lambda = 0.6$, $\sin^2\lambda = 0.4$, and $2\sin\lambda\cos\lambda \cong 1$. The squares of the amplitudes $|M_{2c}|^2$ and $|M_{2e}|^2$ may be written in terms of the physical amplitudes $|M_{2f}|^2$ and $|M_{2g}|^2$ and a cross term $\mathrm{Re}(M_{2f}M_{2g}^*)$, as follows:

$$|M_{2c}|^2 = 0.6|M_{2g}|^2 + 0.4|M_{2f}|^2 - \mathrm{Re}(M_{2g}M_{2f}^*), \qquad (6)$$

$$|M_{2e}|^2 = 0.4|M_{2g}|^2 + 0.6|M_{2f}|^2 + \mathrm{Re}(M_{2g}M_{2f}^*). \qquad (7)$$

Inasmuch as we have two equations with three unknown quantities, $|M_{2c}|^2$, $|M_{2e}|^2$, and $\mathrm{Re}(M_{2g}M_{2f}^*)$, only lower and upper bounds can be obtained for $|M_{2c}|^2$. However, since $\sigma_{2g} \ll \sigma_{2f}$, the uncertainty introduced by this situa-

Table II. Data for processes of type (1). Meson + proton → baryon resonance + pseudoscalar meson.

| Reaction | $E^*$ (BeV) | $Q$ (BeV) | $F$ (BeV$^2$) | $\sigma$ (mb) | $\sigma F$ | Reference |
|---|---|---|---|---|---|---|
| $N^{*++}K^0$ | 1.745 | 0.014 | 15.5 | $1.90 \pm 0.20$ | $29.50 \pm 5.0$ | a |
| | 1.859 | 0.122 | 6.5 | $3.60 \pm 0.50$ | $23.50 \pm 4.0$ | b |
| | 2.000 | 0.280 | 5.9 | $2.80 \pm 1.00$ | $16.52 \pm 6.0$ | c |
| | 2.010 | 0.290 | 5.9 | $4.90 \pm 1.00$ | $28.90 \pm 6.0$ | d |
| | 2.225 | 0.489 | 6.3 | $3.10 \pm 0.80$ | $21.10 \pm 5.0$ | e |
| | 2.615 | 0.878 | 7.9 | $0.80 \pm 0.20$ | $6.32 \pm 2.0$ | f |
| $N^{*++}\pi^0$ | 1.387 | 0.014 | 9.7 | $0.40^{+0.40}_{-0.20}$ | $3.90^{+3.9}_{-2.0}$ | g |
| | 1.495 | 0.122 | 4.9 | $2.40 \pm 0.80$ | $11.80 \pm 3.9$ | h |
| | 1.686 | 0.313 | 4.4 | $5.30 \pm 0.40$ | $23.30 \pm 1.8$ | i |
| | 1.692 | 0.319 | 4.4 | $7.40 \pm 0.40$ | $32.60 \pm 1.8$ | j |
| | 1.875 | 0.502 | 4.7 | $4.00 \pm 1.00$ | $18.80 \pm 4.7$ | j |
| | 2.290 | 0.917 | 6.2 | $0.47 \pm 0.06$ | $2.90 \pm 0.1$ | k |
| | 2.500 | 1.127 | 7.1 | $0.36 \pm 0.06$ | $2.60 \pm 0.1$ | k |
| $N^{*++}\eta$ | 1.790 | 0.004 | 37.2 | $0.04 \pm 0.02$ | $1.50 \pm 0.8$ | l |
| | 1.875 | 0.099 | 9.2 | $0.06 \pm 0.02$ | $0.55 \pm 0.2$ | l |
| | 2.410 | 0.624 | 7.6 | $0.10 \pm 0.03$ | $0.76 \pm 0.3$ | k |
| | 2.715 | 0.929 | 8.9 | $<0.13 \pm 0.03$ | $<1.20 \pm 0.3$ | m |
| $Y_1^{*+}K^+$ | 2.185 | 0.306 | 8.3 | $0.08 \pm 0.03$ | $0.66 \pm 0.2$ | n |
| | 2.410 | 0.531 | 8.4 | $0.11 \pm 0.04$ | $0.92 \pm 0.3$ | k |
| | 2.715 | 0.836 | 9.5 | $<0.02 \pm 0.01$ | $<0.29 \pm 0.1$ | m |

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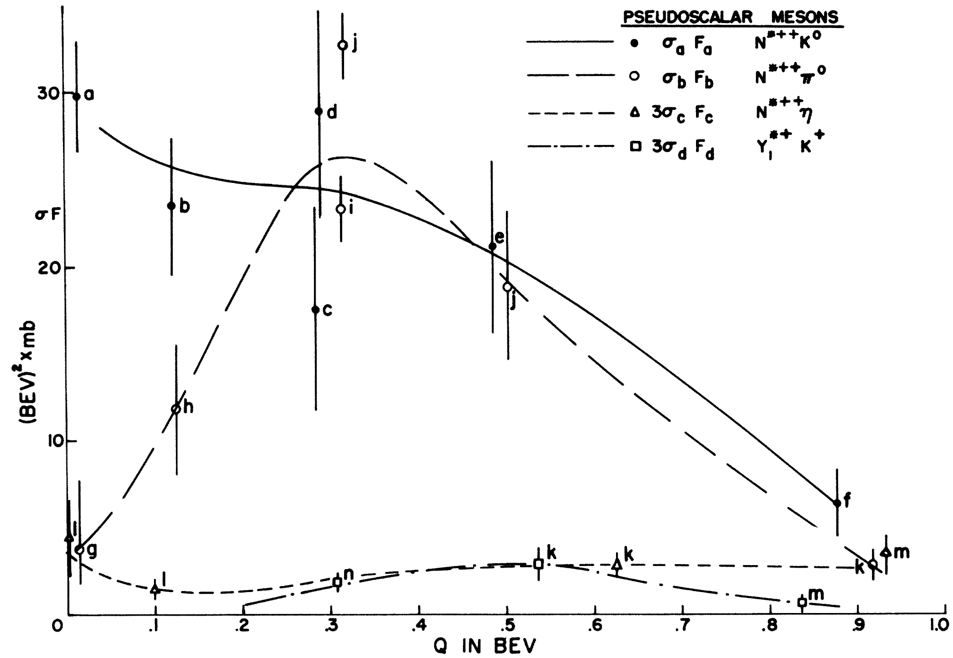


FIG. 2. Experimental values of $F_{1a}\sigma_{1a}$, $F_{1b}\sigma_{1b}$, $3F_{1c}\sigma_{1d}$, and $3F_{1d}\sigma_{1d}$ vs $Q$. The lines are drawn roughly through the data points simply as a guide. The number next to each data point is the reference number of the data source (Table II).

tion is small. In this discussion of $\omega$-$\varphi$ mixing, we have ignored the dynamical effects of this mixing on the production amplitudes, analogous to our neglect of the $SU_3$ mass splitting interaction.

Table III lists the available data on reactions of type (2), together with the associated values of $E^*$, $Q$, and $F$. Note that $\sigma_{2g}$ is very small,

Table III. Data for processes of type (2). Meson + proton → baryon resonance + vector meson.

| Reaction | $E^*$ (BeV) | $Q$ (BeV) | $F$ (BeV²) | $\sigma$ (mb) | $\sigma F$ | Reference |
|---|---|---|---|---|---|---|
| $N^{*++}K^{*0}$ | 2.225 | 0.099 | 13.4 | 1.65 ± 0.20 | 22.10 ± 2.7 | a |
| | 2.340 | 0.214 | 10.2 | 1.55 ± 0.20 | 15.80 ± 2.0 | b |
| | 2.615 | 0.489 | 9.8 | 0.65 ± 0.30 | 6.37 ± 2.9 | c |
| $N^{*++}\rho^0$ | 2.185 | 0.214 | 9.4 | 1.20 ± 0.20 | 11.28 ± 1.9 | d |
| | 2.290 | 0.302 | 8.9 | 0.61 ± 0.18 | 5.40 ± 1.6 | e |
| | 2.410 | 0.422 | 8.8 | 0.56 ± 0.16 | 5.00 ± 1.5 | e |
| | 2.500 | 0.512 | 9.0 | 0.39 ± 0.12 | 3.50 ± 1.1 | e |
| | 2.715 | 0.727 | 9.6 | 0.85 ± 0.25 | 8.20 ± 2.1 | f |
| $N^{*++}\omega$ | 2.185 | 0.165 | 10.4 | 1.10 ± 0.20 | 11.40 ± 2.1 | d |
| | 2.290 | 0.270 | 9.4 | 0.83 ± 0.16 | 7.80 ± 1.6 | e |
| | 2.410 | 0.390 | 9.2 | 0.85 ± 0.17 | 7.80 ± 1.6 | e |
| | 2.500 | 0.480 | 9.2 | 0.50 ± 0.15 | 4.60 ± 1.0 | e |
| | 2.715 | 0.695 | 9.9 | 0.53 ± 0.06 | 5.20 ± 0.6 | f |
| $N^{*++}\varphi$ | 2.715 | 0.457 | 11.6 | <0.02 | <0.20 | f |
| $Y_1^{*+}K^{*+}$ | 2.410 | 0.160 | 14.5 | 0.09 ± 0.03 | 1.31 ± 0.4 | e |
| | 2.715 | 0.422 | 12.1 | 0.02 ± 0.01 | 0.24 ± 0.1 | f |

[a] See reference e, Table II.
[b] R. Kraemer et al. (to be published).
[c] See reference f, Table II.
[d] See reference n, Table II.
[e] See reference k, Table II.
[f] See reference m, Table II.

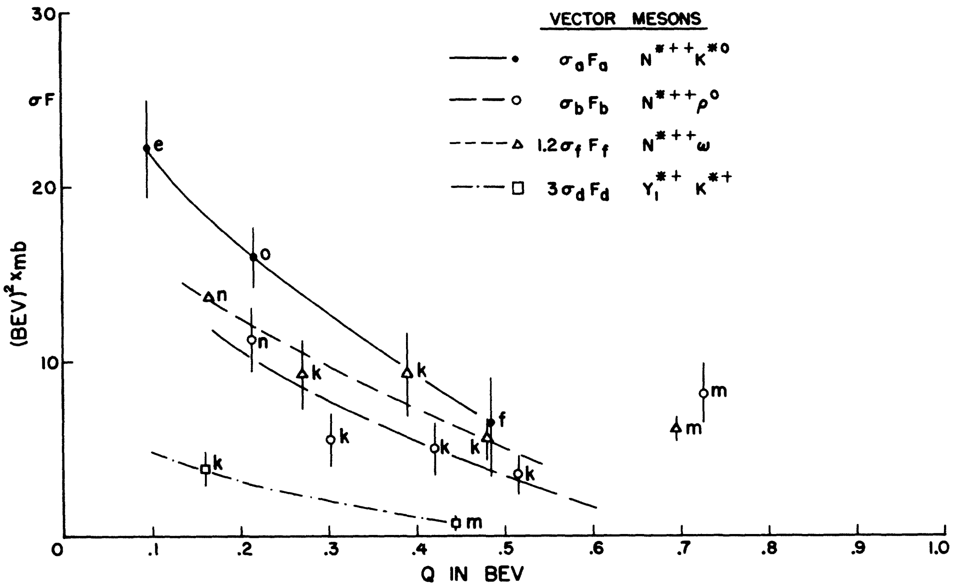


FIG. 3. Experimental values of $F_{2a}\sigma_{2a}$, $F_{2b}\sigma_{2b}$, $3F_{2c}\sigma_{2c}$, and $3F_{2d}\sigma_{2d}$ vs $Q$. The unmeasured quantity $3F_{2c}\times\sigma_{2c}$ is approximated by $3(0.4)F_{2f}\sigma_{2f}$. The lines are drawn roughly through the data points simply as a guide. The number next to each data point is the reference number of the data source (Table II).

so it seems that it is a good approximation to set

$$F_{2c}\sigma_{2c} = |M_{2c}|^2 \approx 0.4|M_{2f}|^2 = 0.4F_{2f}\sigma_{2f}. \quad (8)$$

Figure 3 displays each of the four terms of Eq. (5) versus $Q$. Again the lines are drawn through the data points simply as a guide. Figure 3 shows some similarity of the $Q$ dependences of $F_{2a}\sigma_{2a}$, $F_{2b}\sigma_{2b}$, and $F_{2c}\sigma_{2c}$. The $N^*\rho$ and $N^*\omega$ cross sections at $Q \sim 700$ MeV do not follow the downward trend of the other data. At $Q=210$ MeV the LHS of Eq. (5) is $16\pm2$ and the RHS is $20\pm3$. At $Q=450$ MeV the LHS is $7.5\pm2.9$, while the RHS is $9.8\pm3.5$. The agreement between the LHS and RHS of Eq. (5) for vector-meson production is quite good, over the whole region for which complete sets of data exist. It should be noted that the extraction of the partial cross sections for quasi two-body reactions from the data is difficult because of the possible presence of complicated interference and background effects.

In conclusion, we find that the reaction predictions of $SU_3$ embodied in Eq. (5) are, on the whole, in reasonable agreement with experiment, when comparisons are made at equal $Q$ values. Although individual cross sections at a given $Q$ may differ by factors of 10, Eq. (5) is satisfied to better than 50%. The scatter of experimental cross sections points up the difficulty in evaluating the reactions of types (1) and (2) and indicates the need for the accumulation of more precise data over a wide range of $Q$ values. This will allow a more rigorous test of our $SU_3$ predictions. It would also appear fruitful to test the equalities predicted by $SU_3$ symmetry,[4] using the same method of comparison described above.

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