interesting discussions.

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[2]M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nucl. Sci. 7, 454 (1957).

[3]S. Barshay and R. E. Behrends, Phys. Rev. 114, 931 (1959); S. Iwao and J. Leitner, Nuovo Cimento 22, 904 (1961).

[4]In addition both these calculations omitted the factor $4\pi\alpha$ in the numbers given for the integrated branching ratio.

[5]For the meaning of symbols see reference 3.

REMARKS ON CABIBBO'S THEORY OF WEAK INTERACTIONS*

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We wish to make a few comments on Cabibbo's theory[1] of leptonic decays of strange particles based on the octet version of unitary symmetry.[2,3] Some of the points we wish to emphasize are:

(1) The quantitative results obtained by Cabibbo rest on the assumption that the $|\Delta S|=1$ currents are not renormalized at all (as would be the case for the vector part if unitary symmetry were exact).

(2) Under certain reasonable assumptions the vertex renormalization of the $|\Delta S|=1$ current responsible for K_{e3} decay can be estimated from a comparison of the decay width of the ρ meson and the M $[=K^*(888)]$ meson.

(3) If we take into account the above-mentioned renormalization effect, the mixing angle θ in Cabibbo's paper can be shown to be too large. With our new corrected angle the empirical beta-decay constant turns out to be in exact agreement with the theoretically expected value, i.e., there is no longer any discrepancy between the universality principle of Cabibbo and observation.

(4) It is very important to test separately the vector and the axial-vector part of the Cabibbo theory since his treatement of the axial-vector interaction appears to be on less secure grounds.

Recently Cabibbo proposed a theory of weak interactions in which the vector part of the leptonic decays of strongly interacting particles proceeds via the interaction

$$G[\bar{e}\gamma_\mu(1+\gamma_5)\nu_e+\bar{\mu}\gamma_\mu(1+\gamma_5)\nu_\mu][\cos\theta(j_\mu^{(1)}+ij_\mu^{(2)}) +\sin\theta(j_\mu^{(4)}+ij_\mu^{(5)})]+\text{H.c.}, \qquad (1)$$

where $j_\mu^{(i)}$ are the conserved or approximately conserved currents (of the F type) generated by the gauge transformations of the eightfold way, and G is the muon-decay coupling constant. From the decay rates of π_{e3} (known from the conserved vector-current theory for $\Delta S=0$ processes) and K_{e3}, the angle θ is determined to be 0.26. In contrast to the "old" universality principle the Cabibbo theory requires that the beta-decay constant now be given by $G\cos\theta$ which is numerically equal to $0.966G$. This is to be compared with the experimental beta-decay constant[4] (with radiative corrections taken into account)

$$(0.975\pm0.003)G \text{ from } \mathrm{O}^{14},$$
$$(0.985\pm0.003)G \text{ from } \mathrm{Al}^{26}, \qquad (2)$$

which means that there is about a 3% discrepancy in the decay rate.

We wish to point out that because of the approximate nature of unitary symmetry an exact agreement between the Cabibbo prediction and experiment is not expected. As is well known, the coupling constant associated with the $\Delta S=0$ current $j_\mu^{(1)}+ij_\mu^{(2)}$ is not renormalized by strong interactions because the current is divergenceless.[5] In contrast, the $\Delta S=1$ current $j_\mu^{(4)}+ij_\mu^{(5)}$ is not divergenceless in the real world in which the eightfold way is broken; so the coupling constant for a $|\Delta S|=1$ process is subject to renormalization. For instance, the effective coupling constant for $K^+\to\pi^0+e^++\nu$ should be given by

$$(1/\sqrt{2})G\sin\theta Z_1^{-1}(K\pi)Z_2^{1/2}(K)Z_2^{1/2}(\pi). \qquad (3)$$

This is to be compared with the coupling constant for $\pi^+\to\pi^0+e^++\nu$ which is just

$$\sqrt{2}\,G\cos\theta, \qquad (4)$$

where we have used the Ward identity

$$Z_1^{-1}(\pi\pi)Z_2(\pi)=1. \qquad (5)$$

The quantitative results given in Cabibbo's paper rest on the assumption that the product

$$Z_1^{-1}(K\pi)Z_2^{1/2}(K)Z_2^{1/2}(\pi) \tag{6}$$

is also equal to unity, which is impossible because of the finite $K-\pi$ mass difference.

We may naturally [illegible] Is there any way of estimating the product (6). In a strong-interaction theory in which the [illegible] of the vector-meson octet are coupled to the appropriate F-type currents, the sources of ρ^+ and M^+ [$=K^{*+}(888)$] are precisely the currents that appear in Cabibbo's theory of weak interactions. In fact, if ρ and M had zero masses, the coupling constants for $M^+ \to K^+ + \pi^0$ and $\rho^+ \to \pi^+ + \pi^0$ would be given by

$$\gamma_0 Z_1^{-1}(K\pi)Z_2^{1/2}(\pi)Z_2^{1/2}(K)Z_3^{1/2}(M) \tag{7}$$

and

$$2\gamma_0 Z_3^{1/2}(\rho), \tag{8}$$

where γ_0 is the unrenormalized coupling constant that characterizes the coupling of the vector-meson octet to the F-type vector currents. The actual situation, however, is a little more involved because of the finite rest mass of the vector meson; we must also take into account the momentum-transfer dependence of the vector-meson form factor since the renormalized coupling constant defined on the mass shell of the vector meson is not necessarily equal to the renormalized zero-momentum-transfer coupling constant given by (7) and (8).

Meanwhile, if we assume that the on-the-mass-shell coupling constants satisfy unitary symmetry, we can immediately calculate the decay width of the M meson once the decay width of the ρ meson is given:

$$\frac{\Gamma(M \to K+\pi)}{\Gamma(\rho \to \pi+\pi)} = \frac{3}{4}\frac{(p_{K\pi}^{3}/m_M^{2})}{(p_{\pi\pi}^{3}/m_\rho^{2})}. \tag{9}$$

This means that, if the ρ width is 110 MeV, then the M width must be 33 MeV which is to be compared with the actual decay width of the M meson estimated to be about 50 MeV. This discrepancy is attributed to violation of at least one of the following equalities:

$$Z_1(K\pi) = Z_2^{1/2}(K)Z_2^{1/2}(\pi), \tag{10a}$$

$$Z_3(\rho) = Z_3(M), \tag{10b}$$

$$V_{MK\pi}(0) = V_{\rho\pi\pi}(0), \tag{10c}$$

where $V_{MK\pi}(s)$ and $V_{\rho\pi\pi}(s)$ are the vector-meson form factors normalized so that $V_{MK\pi}(m_M^2) = V_{\rho\pi\pi}(m_\rho^2) = 1$.

Among the three possible reasons for the discrepancy, the most likely one appears to be violation of the Ward-like identity (10a) since the breakdown of unitary symmetry appears most serious for the pseudoscalar mesons because of the large $K-\pi$ mass difference. Relation (10b) is likely to hold because of the near degeneracy of the ρ and the M mass; in general, unitary symmetry appears to be good for the vector-meson octet (apart from complications arising from $\varphi\omega$ mixing). As for (10c) it is high-mass channels other than the resonant $K\pi$ or $\pi\pi$ channels that give structure to the $MK\pi$ or $\rho\pi\pi$ vertex[6]; so we may assume that the two form factors are slowly varying and similar.

Assuming that the discrepancy between the computed and the observed decay width of the M meson is due to (10a) alone, we have

$$Z_1^{-1}(K\pi)Z_2^{1/2}(K)Z_2^{1/2}(\pi) = (50/33)^{1/2} = (0.81)^{-1}. \tag{11}$$

Coming back to the Cabibbo theory, $\sin\theta$ is now given by [see Eq. (3)]

$$\sin\theta = 0.81 \sin\theta_{\text{Cabibbo}} = 0.206. \tag{12}$$

With our new angle θ the beta-decay constant becomes

$$G\cos\theta = 0.979G \tag{13}$$

in exact agreement with the observed beta-decay constant.

Turning our attention to the axial-vector interaction we see that the predictions of the axial-vector part of the Cabibbo theory are on somewhat less secure grounds. First of all, even if unitary symmetry were exact, the conserved current principle in the usual sense would not be applicable to axial-vector currents; hence renormalization corrections are expected to be large. What is worse, we know of no examples in which an axial-vector matrix element can be related to some measurable strong (or electromagnetic) matrix element. So when the predictions of the Cabibbo theory turn out to disagree with observation, we cannot decide whether the theory itself is wrong or the renormalization corrections mask the symmetry of the bare Lagrangian. For this reason it is important to test separately the V and the A part of the Cabibbo theory. As an example, if the V part of the Λ beta decay is given by the Cabibbo theory, we must have

$$\Gamma(\Lambda \to p + e^- + \nu) = 2.15\times 10^7 \sec^{-1} \sin^2\theta[1+3|x|^2], \quad (14)$$

where x is the ratio of the axial to the vector coupling constant. Even if the Cabibbo theory fails for the A part, $|x|$ can still be predicted theoretically once the branching ratio for $\Lambda \to p + e^- + \nu$ is given experimentally. Using the observed branching ratio[7] $(0.82 \pm 0.13)\times 10^{-3}$, we obtain

$$|x| = 0.94 \pm 0.12 \text{ for } \sin\theta = 0.206,$$

which is consistent with observation.[8]

In spite of our critical remarks on the A part of the Cabibbo theory, it is remarkable that the mixing angle for the axial-vector interaction deduced from the decay rates of $K_{\mu 2}$ and $\pi_{\mu 2}$ using the simple effective interaction

$$(f/m)(\cos\theta\partial_\mu \pi^+ + \sin\theta\partial_\mu K^+)\bar{\nu}\gamma_\mu(1+\gamma_5)\mu \quad (15)$$

agrees almost exactly with the mixing angle for the vector interaction deduced from the decay rates of K_{e3} and π_{e3}. Another remarkable feature is that the D-vs-F mixing ratio obtained by Cabibbo for the axial-vector interaction of hyperon beta decay agrees very well with the D-vs-F ratio for the strong interactions of the pseudoscalar mesons with the baryons estimated by several authors[9] in an attempt to explain the low-lying baryon isobars. This might not be too surprising if a generalized Goldberger-Treiman relation of the type

$$(m_\Lambda + m_N)G_{N\Lambda}^{(A)} = (f/m)\sin\theta g_{K\Lambda N}, \text{ etc.} \quad (16)$$

is valid where $G_{N\Lambda}^{(A)}$ and $g_{K\Lambda N}$ are, respectively, the axial-vector constant for Λ beta decay and the strong $K\Lambda N$ constant (of the ps-ps type). The validity of such relations may also be necessary if we are to justify Cabibbo's procedure of using the same angle θ for both $K_{\mu 2}$ decay and hyperon beta decay. (Note, however, that this type of reasoning requires that the pseudoscalar couplings of the K mesons to the baryons must be as strong as are required by unitary symmetry.) In any case, if these remarkable features are not accidental, we must conclude the following: (a) The renormalization corrections for the axial-vector currents are small, or else they satisfy unitary symmetry. (b) There must be a deep connection between the axial-vector currents that appear in the weak interactions and the pseudoscalar densities that appear in the strong interactions.

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[6]For detailed discussions of vector-meson form factors, see M. Gell-Mann and F. Zachariasen, Phys. Rev. 124, 953 (1961).

[7]Taken from the summary talk by A. Rousset, in the Proceedings of the International Conference on Fundamental Aspects of Weak Interactions, Brookhaven National Laboratory, Upton, New York, September 1963 (unpublished).

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