Σ-RADIATIVE DECAY AS A METHOD OF DETERMINING THE ANGULAR MOMENTUM OF THE Σ-PIONIC DECAY

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In this note we discuss a possible way of determining experimentally which of the Σ-hyperon pionic decays ($\Sigma^\pm \to \pi^\pm + n$) occurs via the S-wave and which via the P-wave channel. The present experimental values of the parameters $\alpha_{\Sigma^\pm}$ which describe the decay angular distributions[1] combined with the $\Delta I = 1/2$ rule for nonleptonic decays requires that one of the Σ's decays via P wave and the other via S wave.[2] Hence a strong test of the $\Delta I = 1/2$ rule is the confirmation that, in fact, the two Σ hyperons decay via different angular momentum channels. It is also of importance to know which decay mode is P-wave and which is S-wave. The only other experimental method known to us that will answer this question is the measurement of the neutron polarization in the decay of Σ hyperons of known polarization. This latter method is difficult and therefore has never been carried out. It is likely that the method we describe here is more feasible with presently available techniques.

We present here the calculation of the momentum distribution for the pion in the sigma center of mass in the decay

$$\Sigma^\pm \to n + \pi^\pm + \gamma,$$

assuming that the $\Sigma^\pm \to \pi^\pm + n$ decay mode occurs via S wave or via P wave. We present this distribution for Σ^+ decay in Fig. 1 where we neglect all magnetic-moment transition terms. The corrections to this approximation are discussed later on. We note, for example, that, with respect to the ordinary decay $\Sigma^+ \to \pi^+ + n$, the branching ratio for $q_\pi < 150$ MeV/c is 6.5×10^{-4} for S wave and 8.8×10^{-4} for P wave. This difference becomes larger for lower values of q_π. Although previous calculations of this branching ratio have appeared in the literature,[3] they have neglected to point out this difference.[4] This note is written to bring out the relevance of the radiative decay in determining the angular momentum of the $\Sigma^\pm \to \pi^\pm + n$ decay channel.

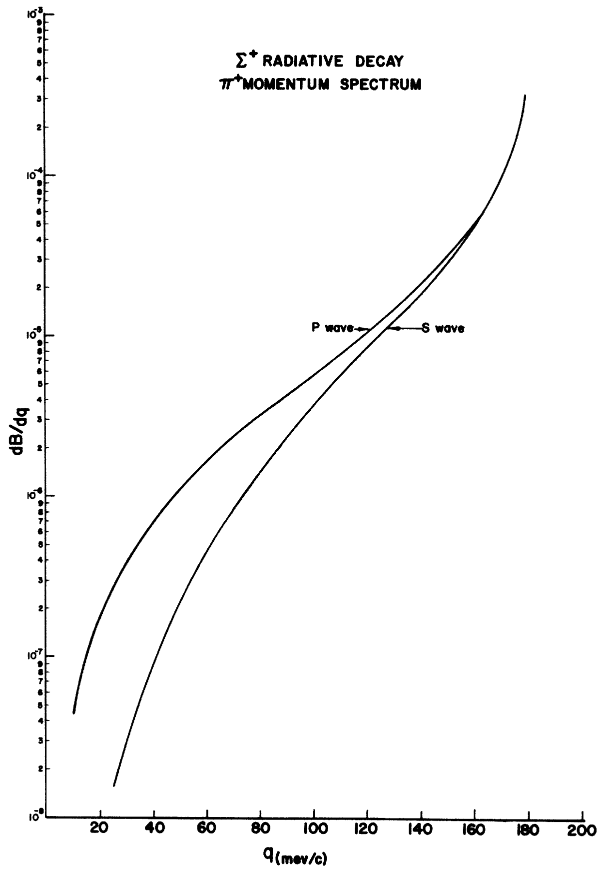

FIG. 1. π^+ momentum spectrum in Σ^+ radiative decay.

We calculate the matrix element from the Feynman diagrams shown in Fig. 2. We use the Lagrangian density[5]

$$L_{\mathrm{int}} = -G\bar{\psi}_n\gamma_\mu(a+ib\gamma_5)\psi_\Sigma(\partial_\mu - ieA_\mu)\varphi - e\bar{\psi}_\Sigma\gamma_\mu A_\mu\psi_\Sigma + \frac{1}{2}\sum_{i=\Sigma,n}\frac{e\mu_i}{2M_i}\bar{\psi}_i\sigma_{\mu\nu}\psi_i F_{\mu\nu} - \frac{1}{2}\frac{eG\mu_t}{(M_\Sigma+M_n)}\bar{\psi}_n\sigma_{\mu\nu}$$
$$\times(a'+ib'\gamma_5)\psi_\Sigma F_{\mu\nu}\varphi + ie\{(\partial_\mu\varphi^*)\varphi - \varphi^*\partial_\mu\varphi\}A_\mu + \mathrm{H.c.} \qquad (1)$$

The first term in this density represents a gauge-invariant phenomenological description for the pionic decay of the sigma, assuming a vector-type interaction. We describe the effects of the static moments of the sigma and the nucleon by the phenomenological third term. The term involving the "transition" magnetic moment, μ_t, is a phenomenological gauge-invariant representation of graphs in which the three particles (π, n, γ) emerge from a "black box." In general, the quantities a, b, a', b' are functions of the pion momentum and angle. We will suppose that they are weakly dependent on these variables for the range of pion momenta involved in these decays. Hence we treat them as constants.

The matrix element for the decay becomes, after some algebra,

$$M = i(4\pi\alpha)^{1/2}G\bar{U}_n\Big\{[(M_\Sigma - M_n)a - i(M_\Sigma + M_n)b\gamma_5]\Big[-\frac{\epsilon\cdot q}{q\cdot k} + \frac{\not\epsilon\not k}{2Q\cdot k} - \frac{\mu_\Sigma}{2M_\Sigma}\Big(\not\epsilon - \frac{\not\epsilon\not k}{k} + \frac{\not\epsilon\not k}{M_\Sigma \mp M_n}\Big) \mp \frac{\mu_\Sigma}{2M_\Sigma}$$
$$\times\Big(\not\epsilon - \frac{\not\epsilon\not k}{k} + \frac{\not\epsilon\not k}{M_\Sigma \mp M_n}\Big) \mp \frac{\mu_n}{2M_n}\Big(\not\epsilon + \frac{q\cdot\epsilon}{P_n\cdot k}\not k \pm \frac{M_n}{P_n\cdot k}\not\epsilon\not k - \frac{\not\epsilon\not k}{M_\Sigma \mp M_n}\Big)\Big] + \frac{\mu_t}{M_n + M_\Sigma}(a'+ib'\gamma_5)\not\epsilon\not k\Big\}\mu_\Sigma, \qquad (2)$$

where $b=0$ and the top sign refer to the S-wave decay, $a=0$ and the bottom sign refer to the P-wave decay. Also ϵ represents the photon polarization and $\alpha = 1/137$.

As a first approximation we set $\mu_\Sigma = \mu_n = \mu_t = 0$, and we get for the branching ratio

$$B = R(\Sigma^+ \to n + \pi^+ + \gamma)/R(\Sigma^+ \to \pi^+ + n) \qquad (3)$$

as a function of the pion momentum in the Σ^+

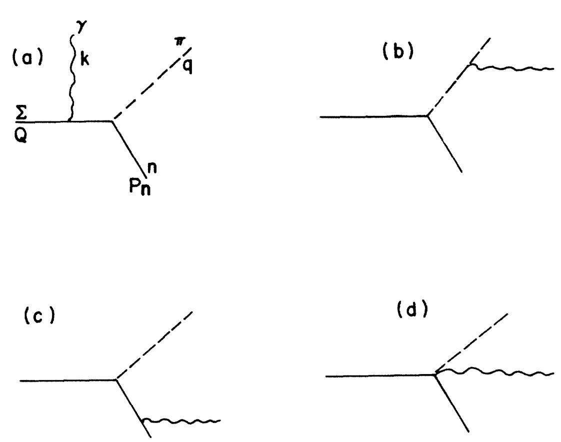

FIG. 2. Feyman diagrams in Σ^+ radiative decay.

center of mass

$$\frac{dB}{dq} = \frac{\alpha}{\pi P}\left\{\frac{\beta(E-\omega)}{2(C-E)}\ln\left(\frac{M_\Sigma - \omega + q}{M_\Sigma - \omega - q}\right) + \frac{q}{E-\omega}\left[\ln\left(\frac{1+\beta}{1-\beta}\right) - 2\beta\right]\right\}, \qquad (4)$$

where $P(E)$ = momentum (energy) of the π in the decay $\Sigma^+ \to n + \pi^+$, $q(\omega)$ = momentum (energy) of the π in the decay $\Sigma^+ \to n + \pi^+ + \gamma$, $\beta = q/\omega$,

$$C = M_\Sigma + M_n \text{ for } S \text{ wave,}$$
$$= M_\Sigma - M_n \text{ for } P \text{ wave.}$$

This is the distribution shown in Fig. 1. We note that the calculation is identical for Σ^- decay where in Eq. (4) instead of the Σ^+ we use the Σ^- decay parameters. Also we note that the spectrum arising from the charge terms is the same for a scalar-pseudoscalar interaction as for the vector-pseudovector interaction of Eq. (1).

In addition we have calculated the correction to this graph due to the anomalous magnetic moment of the sigma and nucleon. The magnetic-

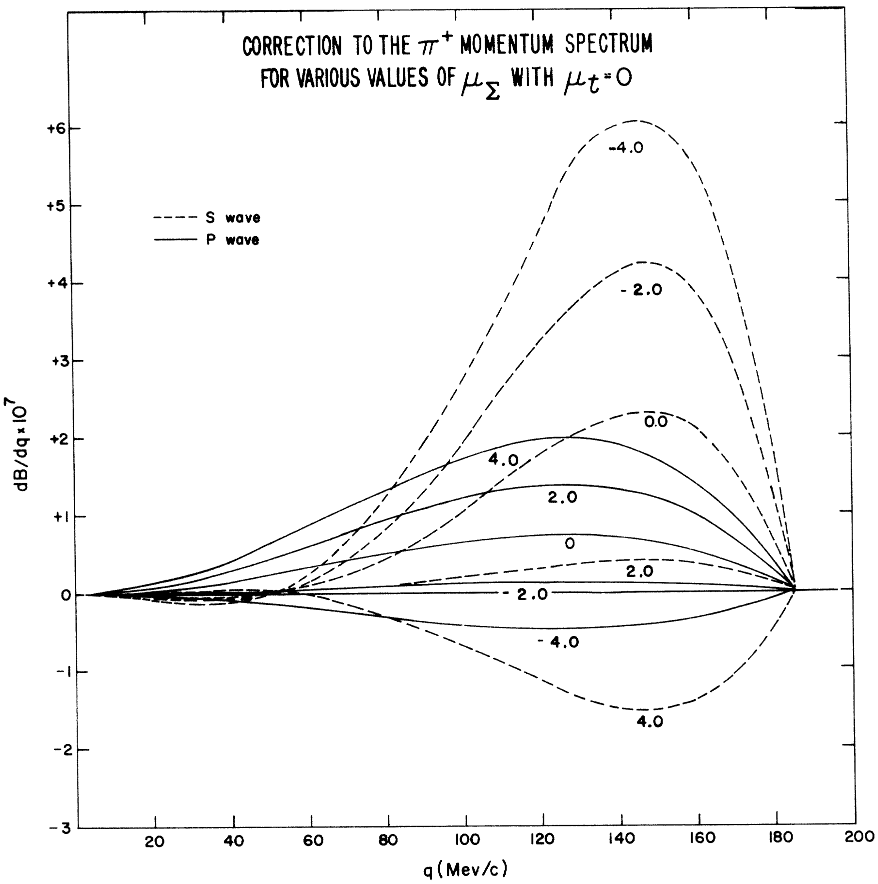

FIG. 3. Correction to the π^+ momentum spectrum for various values of μ_Σ.

moment contributions are somewhat dependent on the form of the interaction used for the decay. For the Lagrangian in Eq. (1) these contributions to Σ^+ decay are shown in Fig. 3. The corrections are proportional to $\mu_\Sigma/M_\Sigma \mp \mu_n/M_n$ for S wave and P wave, respectively, and are opposite in sign for Σ^+ and Σ^- decays. We note that for μ_Σ as large as ± 3.0 baryon magnetons, the change in the difference of the rates is only 10% for $q \approx 80$ MeV/c and is even less for higher values of q. These corrections are not very different in the case of a scalar-pseudoscalar interaction.

These are other types of corrections whose magnitude we do not know reliably. One is the effect of the transition magnetic moment μ_t. It is clear from Eq. (2) that for μ_t of the order of the static magnetic moment of the nucleon, this effect cannot alter the substantial difference between the P- and S-wave rates for $q \leqslant 120$ MeV/c. In addition there are other diagrams which are not included in this Lagrangian. In particular are graphs involving the transition between the hyperon and the nucleon with emission of a photon followed (or preceded) by the strong emission of a pion by a baryon. One such graph would involve the decay $\Sigma^+ \rightarrow p + \gamma$. These diagrams might contribute appreciably to the rate only for large photon momenta.

In summary, we emphasize that a 100% difference exists between the radiative decay of Σ^+ and Σ^- for $q < 120$ MeV/c if the main contribution to the radiative rate is due to the diagrams in Fig. 2. Other diagrams if large in magnitude may change the pion spectrum so as to change the difference in the spectra for S and P wave for low pion momenta. Such a result by itself is also of interest. An experimental measurement of the pion spectra in Σ^+ and Σ^- radiative decay where one agrees with S wave and the other with P wave as shown in Fig. 1 should be a confirmation of the $\Delta I = 1/2$ rule.

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[1]Robert D. Tripp, Mason B. Watson, and Massimiliano Ferro-Luzzi, Phys. Rev. Letters 9, 66 (1962).

[2]M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nucl. Sci. 7, 454 (1957).

[3]S. Barshay and R. E. Behrends, Phys. Rev. 114, 931 (1959); S. Iwao and J. Leitner, Nuovo Cimento 22, 904 (1961).

[4]In addition both these calculations omitted the factor $4\pi\alpha$ in the numbers given for the integrated branching ratio.

[5]For the meaning of symbols see reference 3.

REMARKS ON CABIBBO'S THEORY OF WEAK INTERACTIONS*

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We wish to make a few comments on Cabibbo's theory[1] of leptonic decays of strange particles based on the octet version of unitary symmetry.[2,3] Some of the points we wish to emphasize are:

(1) The quantitative results obtained by Cabibbo rest on the assumption that the $|\Delta S|=1$ currents are not renormalized at all (as would be the case for the vector part if unitary symmetry were exact).

(2) Under certain reasonable assumptions the vertex renormalization of the $|\Delta S|=1$ current responsible for K_{e3} decay can be estimated from a comparison of the decay width of the ρ meson and the M $[=K^*(888)]$ meson.

(3) If we take into account the above-mentioned renormalization effect, the mixing angle θ in Cabibbo's paper can be shown to be too large. With our new corrected angle the empirical beta-decay constant turns out to be in exact agreement with the theoretically expected value, i.e., there is no longer any discrepancy between the universality principle of Cabibbo and observation.

(4) It is very important to test separately the vector and the axial-vector part of the Cabibbo theory since his treatement of the axial-vector interaction appears to be on less secure grounds.

Recently Cabibbo proposed a theory of weak interactions in which the vector part of the leptonic decays of strongly interacting particles proceeds via the interaction

$$G[\bar{e}\gamma_\mu(1+\gamma_5)\nu_e+\bar{\mu}\gamma_\mu(1+\gamma_5)\nu_\mu][\cos\theta(j_\mu^{(1)}+ij_\mu^{(2)})+\sin\theta(j_\mu^{(4)}+ij_\mu^{(5)})]+\text{H.c.}, \qquad (1)$$

where $j_\mu^{(i)}$ are the conserved or approximately conserved currents (of the F type) generated by the gauge transformations of the eightfold way, and G is the muon-decay coupling constant. From the decay rates of π_{e3} (known from the conserved vector-current theory for $\Delta S=0$ processes) and K_{e3}, the angle θ is determined to be 0.26. In contrast to the "old" universality principle the Cabibbo theory requires that the beta-decay constant now be given by $G\cos\theta$ which is numerically equal to $0.966G$. This is to be compared with the experimental beta-decay constant[4] (with radiative corrections taken into account)

$$(0.975\pm0.003)G \text{ from } O^{14},$$
$$(0.985\pm0.003)G \text{ from } Al^{26}, \qquad (2)$$

which means that there is about a 3% discrepancy in the decay rate.

We wish to point out that because of the approximate nature of unitary symmetry an exact agreement between the Cabibbo prediction and experiment is not expected. As is well known, the coupling constant associated with the $\Delta S=0$ current $j_\mu^{(1)}+ij_\mu^{(2)}$ is not renormalized by strong interactions because the current is divergenceless.[5] In contrast, the $\Delta S=1$ current $j_\mu^{(4)}+ij_\mu^{(5)}$ is not divergenceless in the real world in which the eightfold way is broken; so the coupling constant for a $|\Delta S|=1$ process is subject to renormalization. For instance, the effective coupling constant for $K^+\to\pi^0+e^++\nu$ should be given by

$$(1/\sqrt{2})G\sin\theta Z_1^{-1}(K\pi)Z_2^{1/2}(K)Z_2^{1/2}(\pi). \qquad (3)$$

This is to be compared with the coupling constant for $\pi^+\to\pi^0+e^++\nu$ which is just

$$\sqrt{2}\,G\cos\theta, \qquad (4)$$

where we have used the Ward identity

$$Z_1^{-1}(\pi\pi)Z_2(\pi)=1. \qquad (5)$$