WIDE-ANGLE PROTOPRODUCTION OF MU PAIRS*

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A study has been made of the photoproduction of μ pairs from carbon using a 5-BeV bremsstrahlung beam at the Cambridge Electron Accelerator. We report here the initial experimental results which when compared with theory[1,2] provide a test of the validity of the quantum electrodynamic (QED) description of the muon propagator at squared four-momentum transfers q_m^2 up to about 8 F^{-2}.

The μ-pair detector was a 154-counter hodoscope arranged in two similar arrays placed symmetrically on either side of the γ beam, as shown in Fig. 1. The counters nearest the target defined the polar and azimuthal angles of each member of the muon pair. The range of polar angles θ covered was from about 4.5° to about 11.5° and was divided into nine equal intervals. The range of azimuthal angles, φ_1 and φ_2, on each side was ±15° centered about φ_1-φ_2 = 180°. Each 30° interval consisted of five 6° counters. The polar- and azimuthal-angle defining counters were placed behind three feet of iron in order to reduce the singles rate to an acceptable level. During early runs these rates were determined experimentally in attentuation measurements.[3] A layer of eight polar-angle counters was placed behind the layer of nine counters in order to reject neutrals.

In each array 12 inches of iron followed the angular counters and was in turn followed by a trigger array. Thus 48 inches of iron preceeded the trigger arrays in order to attenuate the pion flux. Each trigger array consisted of two layers of five component counters each, separated by three inches of iron. A quadruple fast coincidence of pulses from the four layers was used as a μ-pair signature. This coincidence was used to

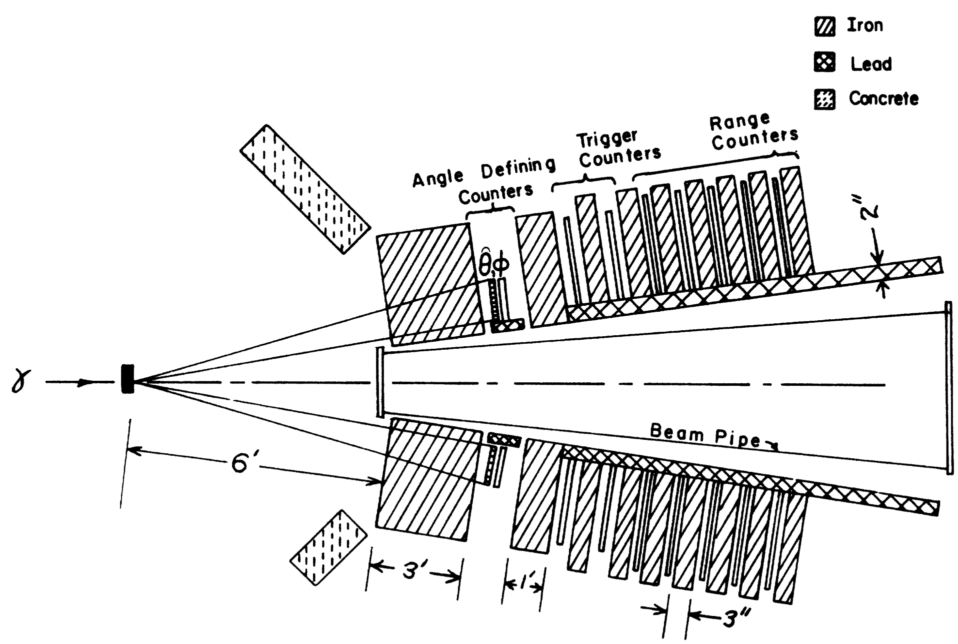

FIG. 1. Schematic of setup for wide-angle μ-pair experiment.

gate flipflops which could store a count from each hodoscope counter. All flipflop states and additional information describing running conditions were then stored on magnetic tape after each gate pulse. Delayed coincidences used to determine the chance rate were also recorded and tagged with a distinguishing label.

In each array range counters, placed behind the trigger layers, measured muon energies between 1.8 and 2.4 BeV, in five intervals. Consecutive layers of range counters were separated by 3 inches of iron. Each range counter was composed of nine component counters.

Beyond 10 radiation lengths of iron, the charged particles from the target giving rise to quadruple coincidences consisted of one component which exhibited an attenuation length in iron which was in excellent agreement with measurements of π attenuation.[4] A second charged component had a distinctly different dependence on iron thickness. With the back layer of trigger counters shielded by $4\frac{1}{4}$ feet of iron, this latter charged component accounted for about 95% of the detected charged particle pairs. These we identify as muon pairs. The yield of detectable K-meson pairs (kinetic energy of each member greater than 1.8 BeV) was negligible because of the very small portion of the bremsstrahlung spectrum available for such production, and because of the small production cross section.

Experimental data were corrected for rates with the target removed, chance rates, counter efficiencies, Coulomb scattering losses, and backgrounds resulting from π-pair production. The latter gives rise to (π,π), (π,μ), and (μ,μ) backgrounds, where the muons originate from π decays in flight. Data on these backgrounds were obtained by measuring the charged particle rates behind two feet of iron where π pairs predominate. These measurements were made as a function of the γ end-point energy, and only the yields arising from photon energies which would permit detection of the π's or their decay μ's with our normal shielding arrangement were considered. These data gave the π-pair angular distribution, which within the statistics was uniform, but did not give the energy distribution. Pessimistic but reasonable π-pair energy spectra were assumed[5] in order to include range effects in the attenuation of the π's, compute the fraction of π's decaying into μ's, and make an estimate of the number of μ's with sufficient energy for detection. Except at the largest angles the backgrounds arising from π pairs were negligible. For example, for events symmetric in θ that were detected by our seventh θ counters $(\theta_1 \approx \theta_2 \approx 9.5°)$, the percentage backgrounds compared to the μ-pair rate were as follows:

$$(\pi,\pi)=4.5\%,\quad (\pi,\mu)=3.7\%,\quad (\mu,\mu)=6.2\%.$$

The background yields have an uncertainty of about 33% and the subsequent errors, which are included in the systematic error, are small compared to the statistical uncertainty in the final results.

The theoretical comparison with the data[6] employed a covariant calculation of pair production in first Born approximation.[7] The elastic form factor of carbon was taken into account by using an analytic expression for the carbon form factor that was derived from electron-scattering experiments.[8] The assumption was made, as is generally done, that the form factor for scattering a virtual e or μ is the same as that for a real e or μ.[9] The uncertainty in the form factor is included in the systematic errors. The form factor error is a monotonically increasing function of q_N^2 (q_N is the four-momentum transfer to the nucleus) starting at negligible values and rising to about 6% for the highest average values of q_N^2 that were used in obtaining the present results. This error is small compared to the statistical error in the corresponding points.

Cross sections were evaluated for all combinations of θ_1, θ_2, E_1, E_2, φ_1, and φ_2. Each cross section was calculated from a six-dimensional integral over the acceptance intervals of the variables. These results were then folded with the Molière distribution function for multiple Coulomb scattering due to three feet of iron, with the target thickness, and with the finite beam size. The results were also corrected for inelastic μ-pair production with and without π production. This was done using the calculations of Drell and Walecka[10] for inelastic pair production, sum rules[11] for inelastic electron scattering, inelastic electron-scattering spectra from carbon,[12] and results from the electroproduction of pions.[13] Inelastic pair production accounted for a negligible fraction of events at low q_m^2 and for about 8% at the highest value of q_m^2 used in the present results. The uncertainty of this yield is also included in the systematic error. Compton terms and radiative corrections are negligible and charge conjugation arguments show that interference terms between Compton and Bethe-Heitler diagrams vanish.[2]

Table I. R vs q_m^2 for five selections of data in which $\theta_1 \approx \theta_2$.

$\lvert q_m^2 \rvert$	$\lvert\theta_2-\theta_1\rvert = 0.16°$		$\lvert\theta_2-\theta_1\rvert = 0.61°$		$\lvert\theta_2-\theta_1\rvert = 0.93°$		$\lvert\theta_2-\theta_1\rvert = 0.42°$		$\lvert\theta_2-\theta_1\rvert = 0.35°$		Systematic[b]
(F^{-2})	R[a]	ΔR[a]	R	ΔR	R	ΔR	R	ΔR	R	ΔR	error
1.34	0.970	0.166	1.252	0.308	0.969	0.389	1.119	0.133	1.201	0.284	0.012
1.87	1.360	0.108	1.274	0.127	1.277	0.125	1.179	0.079	1.175	0.093	0.014
2.48	1.253	0.118	1.037	0.110	1.219	0.123	1.082	0.094	1.053	0.104	0.017
3.16	1.102	0.139	1.147	0.138	0.971	0.159	1.123	0.120	1.287	0.121	0.023
3.91	0.715	0.172	1.062	0.182	0.865	0.208	1.089	0.147	0.894	0.155	0.031
4.74	1.110	0.242	1.102	0.251	0.964	0.270	1.389	0.208	1.664	0.366	0.045
5.65	0.854	0.372	1.320	0.340	0.885	0.372	1.185	0.328	0.958	0.281	0.067
6.63							1.090	0.617	2.889	0.673	0.110
6.90	0.844	0.526	1.625	0.477	2.216	0.565					0.110
7.86							1.799	0.645	1.324	0.717	0.104

[a] $R = \sigma_{\text{exp}}/\sigma_{\text{theor}}$. ΔR is the error in R corresponding to one standard deviation.
[b] Systematic error includes error in the elastic and inelastic form factor and the uncertainty in the π background.

The dependence of the cross section on q_m^2 has been investigated from about 1.3 F^{-2} to about 8 F^{-2}. Table I shows five separate selections of data representing pair production for $\lvert\theta_2-\theta_1\rvert \leq 0.9°$. This represents about half the data taken, and was selected because of the comparatively small contamination from inelastic effects, small uncertainty in the form factor, and because q_m^2 is fairly well defined. For these data q_N^2 lies between 0.01 F^{-2} and 1.0 F^{-2}; 95% of these data correspond to $q_N^2 < 0.4$ F^{-2}. Figure 2 shows an example of the dependence of the cross section on θ_1 for one set of data from Table I.

A least-squares fit to all of the data of Table I gives

$$R = (1.18 \pm 0.15)[1-(0.011 \pm 0.021)\lvert q_m^2 \rvert],$$

where $R = \sigma_{\text{exp}}/\sigma_{\text{theor}}$. The χ^2 probability for this fit is 15%. The errors quoted correspond to one standard deviation and are combined from statistical and systematic uncertainties. The error in the slope from statistics alone is ± 0.0184, whereas the major part of the uncertainty in the normalization is from systematic error which is 12%. If a breakdown model such as that proposed by Drell[1] is used, we may compare these data with the results of other experiments. However, such models are arbitrary. Following Drell we replace the rationalized muon propagator

$$1/(q_m^2-m_\mu^2) \to 1/(q_m^2-m_\mu^2)-1/(q_m^2-m_\mu^2-\hbar^2\Lambda_\mu^2),$$

and find that with 95% confidence $(1/\Lambda_\mu)^2 < (0.16\ \text{F})^2$. For the same confidence level the Frascati measurement of muon pair production[14] yields $(1/\Lambda_\mu)^2 < (0.23\ \text{F})^2$. The g-2 experiment[15] yields $(1/\Lambda_\mu)^2 < (0.1\ \text{F})^2$ if all deviation from theory is entirely attributed to the muon propagator. In a model-independent sense the present experi-

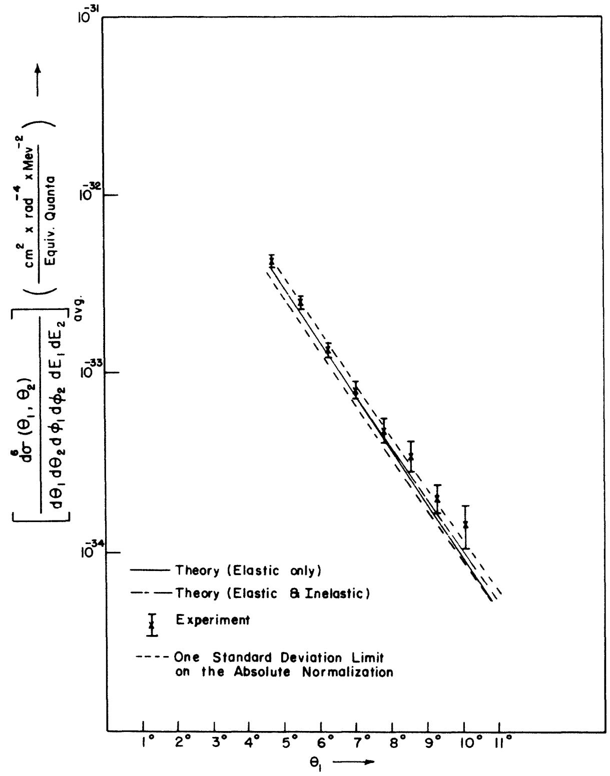

FIG. 2. Angular distribution of the μ-pair cross section for $\lvert\theta_2-\theta_1\rvert = 0.42°$. The acceptance intervals for the yields used in obtaining the cross sections shown are as follows: $\Delta\varphi_1 = \Delta\varphi_2 = 30°$, $\Delta E_1 = \Delta E_2 = 587$ MeV, $\Delta\theta_1 = \Delta\theta_2 = 0.764°$, $\lvert\varphi_1-\varphi_2-180°\rvert \leq 30°$, $1820 \leq (E_1, E_2) \leq 2407$ MeV. The theoretical cross sections include the effects of folding as discussed in the text and the experimental points are absolute cross-section measurements.

ment has found agreement with the predictions of QED, in both slope and normalization, at values of q_m^2 about an order of magnitude larger than that previously attained in the pair-production process.

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[3]We wish to acknowledge the cooperation of the Computation Center at Northeastern University where Monte Carlo calculations on production of neutrons were done in order to facilitate initial experimental design.

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[5]We are grateful to Francis E. Low for helpful discussions on this problem.

[6]We wish to acknowledge the cooperation of the Massachusetts Institute of Technology Computation Center where all of the off-line data processing and many of the numerical calculations were done.

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[9]This experiment produced sufficient data to permit investigation of the yields vs q_N^2 for various limited ranges of q_m^2. This will enable us empirically to investigate the carbon form factor for muons off the mass shell.

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THEORY OF RADIATIVE CORRECTIONS FOR NON-ABELIAN GAUGE FIELDS

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In a series of unpublished researches Feynman has investigated the problems which arise when one attempts to quantize a field possessing a non-Abelian infinite-dimensional invariance group (e.g., Yang-Mills field; gravitational field). In particular, he has posed the question: Is it possible, in perturbation theory, to calculate the radiative corrections for such a field while simultaneously (1) carrying out a consistent renormalization scheme, (2) maintaining manifest covariance, and (3) securing unitarity of the S matrix?

Feynman found that he could do these things in lowest order, i.e., for diagrams involving only single closed loops, by removing the cyclic products of retarded (or advanced) Green's functions from the loops, thereby breaking them open, and then rearranging the result into sets of associated tree diagrams having their external lines on the mass shell. He next restricted the summations involving external lines to the "transverse" or "physical" quantum states, and proved that the sum of all the thus restricted tree diagrams of given order is group invariant. This theorem is important, because for the fields in question "manifest covariance" means "group invariance" (or "gauge invariance") and not merely Lorentz invariance. Indeed, in the case of the gravitational field the theorem can be extended to arbitrary Riemannian manifolds and not merely to quasi-Minkowskian space-time.

The author has published an alternative version[1] of Feynman's results which removes some