intensity of the delayed fluorescence was considerably less (~10^{-5}) than the value expected on the basis of a $\gamma = 5\times10^{-11}$. It is likely that this delayed fluorescence is due to triplets generated via bulk reabsorption of the fluorescence rather than triplet-triplet annihilation on the surface. This is consistent with Kepler's results[6] on laser-induced delayed fluorescence, which showed delayed fluorescence in crystals but not in powders where the surface-to-volume ratio is very large.

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OBLATE NUCLEAR DEFORMATIONS*

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It has been pointed out[1] that there should exist a region of nuclear deformation among neutron-deficient isotopes in the vicinity of Ba in the periodic table. We have performed calculations of nuclear shapes in this region. We find, as expected, that these nuclei have large intrinsic quadrupole moments but, in addition, these quadrupole moments are negative, i.e., the nuclei are oblate, or disc-shaped. This is an unexpected result since all strongly deformed nuclei known so far have prolate shapes, i.e., positive quadrupole moments.

Our calculations are based on the pairing-plus-quadrupole[2] model of nuclear forces. The total Hamiltonian consists of four parts: (1) A single-particle, spherical part

$$H_S = \sum_\alpha \epsilon_\alpha c_\alpha^\dagger c_\alpha,$$

in which $c_\alpha^\dagger$ creates a nucleon in state α, specified by quantum numbers $(nljm)$. The wave functions are those of an isotropic harmonic oscillator and the energies ϵ_α (independent of m) give the sequence of shell-model levels in spherical nuclei. (2) and (3) Two pairing parts with different strengths g_p and g_n, one for protons and one for neutrons, each of the form

$$H_P = -\tfrac{1}{4} g \sum_{\alpha\gamma} c_\alpha^\dagger c_{-\alpha}^\dagger c_{-\gamma} c_\gamma,$$

where $-\alpha$ is the time-reversed state of α. (4) A quadrupole part with a single strength,

$$H_Q = -\tfrac{1}{2}\chi \sum_{\alpha\beta\gamma\delta M} \langle \alpha | r^2 Y_{2M} | \gamma\rangle \langle \delta | r^2 Y_{2M} | \beta\rangle \times c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma.$$

This Hamiltonian is treated in the Hartree-Bogolyubov approximation with a number of additional approximations, the net result being as follows.[3] First, one diagonalizes the single-particle Hamiltonian:

$$H_D = H_S - \sum_{\alpha\gamma} \langle \alpha | \sum_M D_M^* r^2 Y_{2M} | \gamma\rangle c_\alpha^\dagger c_\gamma,$$

which is essentially the Nilsson Hamiltonian,[4] for all values of the deformation parameters D_M. Then, one uses the pairing force to construct a BCS wave function, for each deformation, out of these Nilsson-like single-particle functions. Finally, one takes the expectation value of $H_S + H_P + H_Q$ for this BCS function. This is the total energy of the nucleus, which really depends on only two deformation parameters, for instance, the familiar β and γ.[5] The lowest minimum of this function, if it is at least a few MeV deep, gives the stable shape of the nucleus.

In applying this method to the region $50 < Z < 82$, $50 < N < 82$, we use essentially the ϵ_α's of Mottelson and Nilsson[6] for the protons and those of Nilsson[4] for the neutrons (see Figs. 1 and 2), except for a lowering of the $g_{9/2}$ level. We only include in the calculation the states of the two

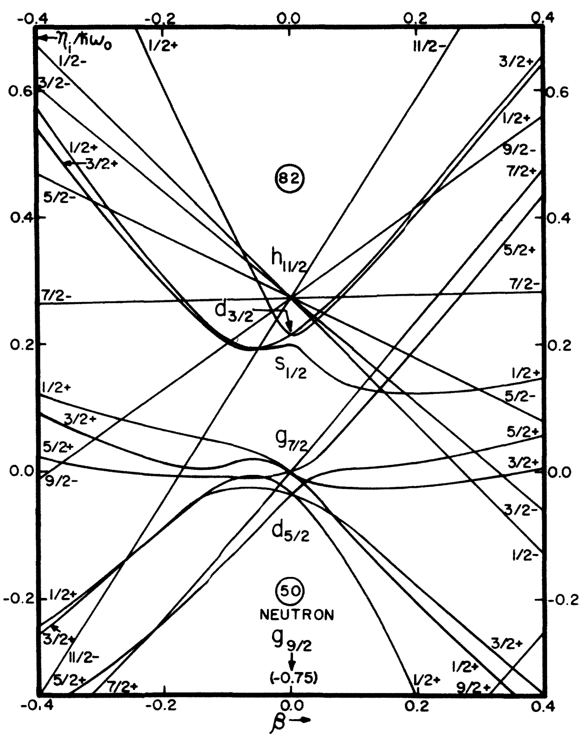

FIG. 1. The eigenvalues η_i of the deformed Hamiltonian H_D, in units of $\hbar\omega_0$, plotted against β for the neutron shell $50 < N < 82$. Oblate nuclei have negative β.

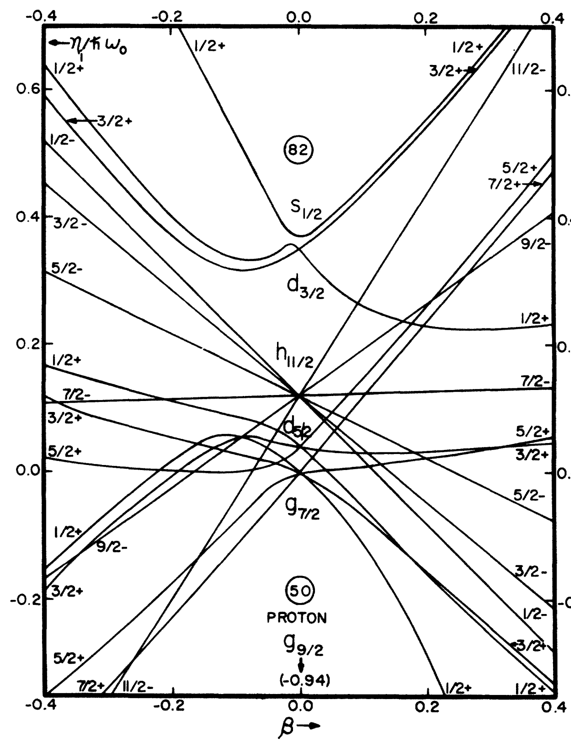

FIG. 2. Same as Fig. 1 for the proton shell $50 < Z < 82$.

partially filled proton and neutron major shells. Besides the argument of simplicity, one reason for doing this is that the quadrupole force is unreasonable, and gives unreasonable results, when extended over too many configurations. It is well known[2] that this restriction in the number of states is equivalent to a renormalization of χ. The values of g and χ are obtained mostly by fitting the known properties of deformed nuclei in the rare-earth region (where quadrupole moments are positive). They are $g_p = 6.5A^{-2/3}$ MeV, $g_n = 4.8A^{-2/3}$ MeV, and $\chi = 310A^{-5/3}(m\omega_0/\hbar)^2$ MeV, which agree fairly well with the values obtained by Kisslinger and Sorensen[7] in fitting spherical nuclei in adjoining regions. When these parameters are used to compute the energy of Ba^{124}, Ba^{126}, and Ba^{128}, it turns out that negative β (and $\gamma = 0°$) is favored over positive β by 3.0, 2.3, and 1.3 MeV, respectively. Other nuclei in the vicinity are similar.

Now, we shall try to understand how these negative quadrupole moments come about. First, we note that, while the magnitude of the nuclear deformation is a rather sensitive function of χ, g_p, and g_n, the energy difference between an oblate and a prolate deformation of the same magnitude is not nearly so sensitive. Hence, a simplified argument disregarding pairing has considerable validity. We can also disregard asymmetric shapes since it is known that pairing effects enforce axial symmetry. Then, the total energy of the nucleus is simply

$$E = \langle H_S \rangle - \tfrac{1}{2}\chi q^2,$$

where q is the mass quadrupole moment computed with a Slater determinant of Nilsson wave functions. Both terms in E depend on D, whose single nonvanishing component is D_0. When E is minimum, the self-consistency condition[2,3]

$$D = \chi q$$

is satisfied. It is convenient to add to E the quantity $\tfrac{1}{2}(D - \chi q)^2/\chi$, which does not change the position and the value of the minimum. The resulting energy E' can be written

$$E' = \langle H_D \rangle + \tfrac{1}{2}D^2/\chi$$

and is very close to the expression minimized by Mottelson and Nilsson[6] in their investigation of nuclear shapes; $\langle H_D \rangle$ is simply the sum of the filled Nilsson levels, but without the deformationdependent scale which they use and whose role is played in our formulation by $\frac{1}{2}D^2/\chi$. E' is convenient because it does not contain the unknown function $q(D)$ and can be estimated by looking at a Nilsson diagram. Figure 1 shows such a diagram for $50 < N < 82$. It is seen from Fig. 1 and calculation confirms that, for a given value of $|\beta|$, it is more advantageous to introduce neutron holes into the filled shell at negative β than at positive β. The effect should be particularly marked for $N = 66$, 68, and 70. Nothing of the sort happens for the proton levels shown in Fig. 2. If anything, they display a slight preference for positive β around $Z = 68$. Close examination reveals that this difference between neutrons and protons is due mostly to the shift of the $h_{11/2}$ level. We repeated the calculations with the levels of Kisslinger and Sorensen,[7] which show a similar shift, and the results were the same. We recall[8] that this difference in location of the $h_{11/2}$ level in the two diagrams is thought to be due to the fact that Fig. 2 is used for nuclei in which the $h_{11/2}$ and $h_{9/2}$ neutron levels are filled or filling, while Fig. 1 is used for nuclei in which the $h_{11/2}$ and $h_{9/2}$ proton levels are practically empty. We should also mention that, for both the rare-earth region and the region under investigation here, the neutrons rather than the protons seem to be responsible for determing the sign of β.

In summary, the sign of the equilibrium deformation is very sensitive to the single-particle spherical levels. If those shown in Fig. 1 are approximately correct, the nuclei in question are oblate. One must conclude that the simplified arguments[9] advanced to explain the general occurrence of prolate nuclei in nature are of limited validity. Though they account correctly for the preponderence of prolate shapes, the present work shows that oblate shapes cannot be ruled out. It would obviously be desirable to measure directly the signs of the quadrupole moments involved. Unfortunately, these nuclei are radioactive and the usual methods of determining the sign might prove difficult. It may be significant that the known quadrupole moments of nearby odd spherical nuclei (Sb^{121}, Sb^{123}, Te^{125*}, I^{125}, I^{127}, I^{129}, I^{131}) are negative.[10]

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