Table I. Comparison of the predictions based on Coulomb energies to the experimentally determined excitation energies for the lowest lying $J=0$, $T=2$ levels.

Nucleus	Prediction from Coulomb energy differences (MeV)	Experimental value (MeV)
Mg^{24}	15.30 ± 0.05	15.4 ± 0.12
Ti^{44}	9.44 ± 0.1	9.35 ± 0.12
Fe^{54}	8.46 ± 0.1	8.48 ± 0.12

Further, the states appear sharp with much of the transition strength focused in a small energy interval. Thus it is seen that the (p,t) reaction does pick out states with $T = T_z + 2$ and these states lie near the energies expected from the Coulomb energy difference.

One of the authors (G. T. G.) gratefully axknowledges helpful and stimulating discussion with B. Bayman and R. Sherr, and the hospitality of the Lawrence Radiation Laboratory.

†This work was supported by the U. S. Atomic Energy Commission and the Higgins Scientific Trust Fund.

[1]J. D. Anderson, C. Wong, and J. W. McClure, Phys. Rev. 126, 2170 (1962); J. D. Anderson and C. Wong, Phys. Rev. Letters 7, 250 (1961).

[2]A. M. Lane and J. M. Soper, Phys. Rev. Letters 7, 420 (1961).

[3]J. D. Fox, C. F. Moore, and D. Robson, Phys. Rev. Letters 12, 198 (1964).

[4]R. Sherr, M. E. Rickey, and C. G. Hoot, Bull. Am. Phys. Soc. 9, 458 (1964); E. Rost, B. Bayman, and R. Sherr, Bull. Am. Phys. Soc. 9, 458 (1964).

[5]G. T. Garvey and B. F. Bayman, to be published; also Argonne National Laboratory Report 6848, pp. 125-129 (unpublished).

[6]J. D. McCullen, B. F. Bayman, and L. Zamick, Phys. Rev. 134, B515 (1964).

[7]J. Ginocchio and J. B. French, Phys. Letters 7, 137 (1963).

[8]F. S. Goulding, D. A. Landis, J. Cerny, and R. H. Pehl, to be published.

[9]G. Bassani, N. M. Hintz, and C. D. Kovaloski, Annual Progress Report, University of Minnesota Linear Accelerator Laboratory, 1963 (unpublished).

[10]L. A. König, J. H. E. Mattauch, and A. H. Wapstra, Nucl. Phys. 31, 18 (1962).

[11]R. Sherr, private communication.

GEOMETRIC TEST OF THE ISOSPIN MULTIPLET NATURE OF NUCLEAR ANALOG STATES

S. Barshay and G. M. Temmer*
Rutgers, The State University, New Brunswick, New Jersey
(Received 15 May 1964)

Recently, a number of experimental and theoretical studies have appeared bearing on the matter of nuclear states and their isobaric analogs. On the one hand, high-energy electron scattering experiments[1] and their analysis[2] have provided a new probe of the extent to which, say, the fundamental three-body nuclear systems, He^3 and H^3, constitute an isobaric doublet. On the other hand, apparent isobaric analog states have been discovered in fairly heavy nuclei.[3] In this note we propose a possible geometric test of the isospin multiplet nature of a wide variety of nuclear analog states, based upon the following theorem:

Theorem.—Consider the nuclear reaction

$$A + B \rightarrow C + C',$$

where C and C' are members of the same isospin multiplet (isobaric analogs) and where the isobaric spin T of either A or B is zero. If T is strictly conserved, and if C and C' are exactly connected by a rotation in isospace, then the differential cross section of the reaction products will exhibit symmetry about 90 degrees in the center-of-mass system, independently of reaction mechanism. The reaction can be taken as proceeding in either direction.

Proof.—If we think of the system in the isospin formalism, C and C' are identical particles obeying Bose or Fermi statistics, depending upon whether they contain an even or odd number of nucleons. This imposes an overall symmetry condition upon the wave function describing the two particles. Because of the way we have set up the reaction, only one value of T can contribute, so that the product of the space and spin portions of the wave function has definite symmetry. Hence even angular momenta and odd angular momenta are associated with orthogonal spin wave functions which cannot introduce interference terms; the angular distribution will therefore contain only even powers of $\cos\theta$, leading to the conclusion stated above.

As soon as we relax either the condition of strict isospin conservation, or of the strict analog relationship between C and C', symmetry about 90 degrees is no longer required since

Table I. Examples of reactions producing isospin multiplets.

Case	Reaction	Q value (MeV)	T of reaction	T_z	t of multiplet
I	$d + He^4 \rightleftarrows H^3 + He^3$	-14.320	0	0	$\frac{1}{2}$
II	$C^{12} + d \rightarrow Li^7 + Be^7$	-17.542	0	0	$\frac{1}{2}$
	$B^{10} + He^4 \rightarrow Li^7 + Be^7$	-16.200			
III(a)	$Mg^{26} + d \rightarrow C^{14} + N^{14*}$ (2.311)	-11.274	1	+1	1
	$Ne^{22} + Li^6 \rightarrow C^{14} + N^{14*}$ (2.311)	- 2.130			
	$O^{18} + B^{10} \rightarrow C^{14} + N^{14*}$ (2.311)	+ 3.075			
	$C^{14} + N^{14} \rightarrow C^{14} + N^{14*}$ (2.311)	- 2.311			
III(b)	$Mg^{24} + He^4 \rightarrow C^{14} + O^{14}$	-22.533	0	0	1
	$O^{16} + C^{12} \rightarrow C^{14} + O^{14}$	-15.764			

we cannot invoke the symmetry condition which governs identical particles in a pure T state.

Clearly, the departures from fore-and-aft symmetry will reflect the combined effects of (a) isospin impurities brought in by the nonidentical reacting particles A and B; (b) the impurity introduced by the reaction itself in the intermediate stage; and (c) the breakdown of isobaric analogy between C and C'. We believe that effects (a) and (b) can, at least approximately, be separated from effect (c), by experiment alone. Before discussing this separation further, it is worthwhile to consider a few illustrative examples of nuclear reactions to which the theorem is applicable.

Table I lists two examples of fermion isobaric doublets, and one of a boson triplet, which can be studied by producing two members at a time. Other examples involving any desired pair of isobaric analogs can easily be constructed. The listed Q values refer to the lowest isobaric multiplets (involving ground states); of course, these considerations apply equally well to excited multiplets. Note the variety of ingoing channels which can be used to produce a given multiplet. The availability of variable-energy heavy-ion beams with small energy spreads, coupled with solid-state detection and particle-identification schemes of high energy resolution, renders such experiments feasible for the first time. In all these cases one must resolve particles of equal mass, differing by one, or at most two, units of charge. Moreover, the $T_z = +1$ cases usually require one partner to be left in an excited state; this can be insured either by coincident detection of the appropriate de-excitation gamma radiation, or more simply by the kinematic characteristics, detecting both outgoing particles in coincidence.

We must, of course, insure that there be no a fortiori reasons for fore-and-aft symmetry. Such reasons are the following: (1) A and B are identical particles (e.g., if in reaction III(b) of Table I above, we use $N^{14} + N^{14}$ for $A + B$); (2) only one dominant orbital angular momentum contributes to the reaction (e.g., in pure s-wave interactions); (3) the reaction proceeds mainly through a compound state $(A + B)$ of sharp angular momentum and parity. (1) is trivial and is easily avoided, although it may be used to advantage to eliminate instrumental asymmetries. Any possible influence of effects (2) and (3) can, in principle, be ascertained empirically by examining some other outgoing channels $D + E$ where D and E are not isobaric analogs. For example, using Case II of Table I, one would select bombarding energies for which the reaction

$$C^{12} + d \rightleftarrows B^{10} + He^4$$

shows marked asymmetry. Alternatively, reaction II leading to $Li^7 + Be^7$, where one of the latter nuclei is in an excited state, would serve the same purpose.

In an attempt to separate off the asymmetry introduced by the combined effects (a) and (b) above, consider studying the following reaction in conjunction with example III(b) of Table I:

$$Mg^{24} + He^4 \ (T = 0) \rightarrow \begin{cases} N^{14} + N^{14*}(2.31\ \text{MeV}, T = 1) & (1) \\ N^{14} + N^{14}(T = 0). & (2) \end{cases}$$

The square root of the ratio of the intensities of the "forbidden" reaction (1) to the "allowed" reaction (2) will provide an estimate of the "hard core" asymmetry to be expected from reaction III(b). Any additional asymmetry in that reaction

can then be ascribed to the breakdown of the mirror relationship between the outgoing pair of nuclei.

Considering this breakdown further, one can distinguish theoretically between two effects which combine to produce the fore-and-aft asymmetry. One might be called the "static" departure from the situation where the two nuclei are related to each other by a rotation in isospace. Thus, for instance, the two like nucleons in He^3 and H^3 may be spatially correlated in a different manner,[1,2] leading to the breakdown of an exact mirror relationship. Note that this effect varies linearly with the amplitude parameter expressing the deviation from exact isobaric analogy between C and C'. The second effect is the "analyzing power" of a particular nuclear reaction, i.e., its ability to distinguish such differences in the angular distribution. It is conceivable that even though two states, C and C', are not exact analogs in isospace, a particular reaction might be relatively insensitive to the difference, treating the states, in effect, as components of a multiplet, and hence leading to fore-and-aft symmetry. A trivial example of this situation is given in point (3) above. A measured asymmetry attributed to a breakdown of isobaric analogy between C and C' must be analyzed as a result of these two factors, the former depending linearly upon static differences between C and C', the latter varying from reaction to reaction.

To sum up, then, we point out the possibility of some incisive tests of isospin analogy, based on simple geometric properties of reaction cross sections, and applying equally well to ground states and excited states of nuclei, from the lightest mirror pairs to the heaviest isobaric analogs. Recent technological advances place the necessary reactions well within the realm of present-day capabilities.

We are grateful to Professor B. F. Bayman for enlightening discussions.

Note added in proof.—It has just come to our attention that recent experimental data exist on Case I of Table I [B. Kuehn and B. Schlenk, Nucl. Phys. 48, 353 (1963)]. The angular distributions exhibit striking symmetry about 90° at all four measured energies, with a slight but significant degree of fore-and-aft asymmetry (~10%), whereas other reaction channels at the same energies show no symmetry at all.

*Work supported in part by the National Science Foundation.

[1]H. Collard et al., Phys. Rev. Letters 11, 132 (1963).

[2]L. I. Schiff, Phys. Rev. 133, B802 (1964); L. I. Schiff, H. Collard, R. Hofstadter, A. Johansson, and M. R. Yearian, Phys. Rev. Letters 11, 387 (1963).

[3]J. D. Fox, C. F. Moore, and D. Robson, Phys. Rev. Letters 12, 198 (1964), and private communication.

OBSERVATION OF THE JAHN-TELLER SPLITTING OF THREE-VALENT d^7 IONS VIA ORBACH RELAXATION*

Ulrich Höchli
Physik Institut der Universität Zürich, Zurich, Switzerland

and

K. A. Müller
IBM Zurich Research Laboratory, Rüschlikon, Zurich, Switzerland

(Received 30 April 1964)

In this Letter an investigation on the line broadening of octahedrally coordinated d^7 ions with t^6e configuration is reported. This broadening is due to an Orbach resonance relaxation. For Pt^{3+} in Al_2O_3 and for Ni^{3+} in $SrTiO_3$ the obtained energy separation Δ between the lowest two levels is shown to be the Jahn-Teller (J-T) splitting of the ^2E ground state.

Relaxation between the two states of a Kramers doublet by the Orbach process takes place when there is a third energy level lying within the phonon spectrum.[1] The relaxation time T_1 for this process obeys the relation

$$T_1 = A \exp(\Delta/kT),$$

where Δ is the separation of the first excited level from the ground doublet and A is the lifetime of the excited state.

In transition metal ions this type of relaxation was first observed by Zverev and Petelina[2] for