to $\langle\varphi_2\rangle$. The first term is a massless particle state, as in the relativistic theories. The second term yields contributions from intermediate states whose energy becomes zero at zero three-momentum.[4] The third term arises from a "spurion" state; when this contributes no theorem holds.

This is the term discussed by Klein and Lee. The introduction of a disconnected vacuum state gives rise to a $\delta^4(k)$ term in the Fourier transform, a constant term in the commutator matrix element, which can be used to break the symmetry. In order to get a finite contribution, the coefficient of this state must have weight $1/V$, the total volume considered, as is the case for Klein and Lee.

An isolated eigenstate of energy-momentum with zero eigenvalues can only contribute <u>non</u>-relativistically. In a covariant theory, the $\delta^4(k)$ term introduced into the commutator by such a state must be multiplied by k_μ, the only vector available, and its contribution is identically zero.[5]

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[1]A. Klein and B. W. Lee, Phys. Rev. Letters <u>12</u>, 266 (1964).

[2]J. Goldstone, Nuovo Cimento <u>19</u>, 154 (1961).

[3]J. Goldstone, A. Salam, and S. Weinberg, Phys. Rev. <u>127</u>, 965 (1962).

[4]The second term only contributes if $\rho_5 = \bar{\rho}(k^2, nk)/|k|^2$. $\bar{\rho}$ is still further restricted, because in order that $\int d^3x\, j_0$ be conserved, $\int d^3x\, \partial_i j^i$ must vanish. This requires $\delta^3(k)k_0\bar{\rho}(k^2, nk)$ to vanish, and therefore $\bar{\rho}$ must require k_0 to be zero when $|k|$ is. $\bar{\rho}$ cannot contain a $\delta(nk)$ term alone because k_μ must be timelike.

[5]Another way of phrasing our argument is to observe that the expectation of a vector operator between any vacuum states is identically zero in a relativistic theory, since there exists no vector for it to depend on.

BROKEN SU(3) ⊗ SU(3) ⊗ SU(3) ⊗ SU(3) SYMMETRY OF STRONG INTERACTIONS*

Peter G. O. Freund and Yoichiro Nambu
The Enrico Fermi Institute for Nuclear Studies, The University of Chicago, Chicago, Illinois

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An ever increasing amount of experimental evidence is being accumulated in favor of the octet scheme[1] of SU(3) invariant strong interactions. We would like to suggest here, taking the SU(3) symmetry as a new starting point, that it is natural, or perhaps necessary, that we should consider a hierarchy of symmetries which are even larger and at the same time more dramatically broken than the SU(3). Specifically we shall enlarge the symmetry up to SU(3) ⊗ SU(3) ⊗ SU(3) ⊗ SU(3) from the following considerations:

(1) It is natural to combine the conserved vector currents in weak interactions with conserved axial vector currents, in which the pseudoscalar-meson octet will play a special role.[2]

(2) One way to understand the "reason" for the baryons to form a tensor (octet) representation is to dissociate the two indices and assign them to separate groups.[3] This would then make nine baryons necessary.

We divide the baryon spinor field into the left-handed and right-handed components $\psi_L = (1+\gamma_5)\psi/2$, $\psi_R = (1-\gamma_5)\psi/2$. Each is a 3×3 matrix $\psi_{\mu\nu}(\mu, \nu = 1, 2, 3)$ in the SU(3) space. Then consider the following distinct operations: $\psi_L \to S_L\psi_L$, $\psi_L \to \psi_L S_L'$, $\psi_R \to S_R\psi_R$, $\psi_R \to \psi_R S_R'$, where each S is in the form

$$S = \exp\left[i \sum_{j=1}^{8} \alpha_j \lambda_j\right]$$

using well known notations.[1] Calling the infinitesimal generators of these four operations G_L, H_L, G_R, H_R respectively, construct

$$F_i^V = G_{Li} + G_{Ri} - H_{Li} - H_{Ri},$$

$$D_i^V = G_{Li} + G_{Ri} + H_{Li} + H_{Ri},$$

$$F_i^A = G_{Li} - G_{Ri} - H_{Li} + H_{Ri} = F_i^V \gamma_5,$$

$$D_i^A = G_{Li} - G_{Ri} + H_{Li} - H_{Ri} = D_i^V \gamma_5, \quad i = 1, \cdots 8,$$

so that we have

$$[F_i^V, F_j^{V,A}] = i f_{ijk} F_k^{V,A},$$

$$[F_i^{\,V}, D_j^{\,V,A}] = if_{ijk} D_k^{\,V,A},$$

$$[F_i^{\,A}, D_j^{\,V,A}] = if_{ijk} D_k^{\,A,V},$$

$$[D_i^{\,V}, D_j^{\,V,A}] = if_{ijk} F_k^{\,V,A},$$

$$[D_i^{\,A}, D_j^{\,V,A}] = if_{ijk} F_k^{\,A,V}.$$

These will generate conserved vector (V) and axial vector (A) currents of type F and D corresponding to the antisymmetric and symmetric octet parts of 8×8.[1] Out of these 32 generators the following Lie algebras can be constructed:

$$A_{10} = \{F_i^{\,V}\},$$

$$A_{11}{}^{D} = \{F_i^{\,V}, D_j^{\,A}\},$$

$$A_{11}{}^{F} = \{F_i^{\,V}, F_j^{\,A}\},$$

$$A_{20} = \{F_i^{\,V}, D_j^{\,V}\},$$

$$A_{21}{}^{\pm} = \{F_i^{\,V}, D_j^{\,V}, F_k^{\,A} \pm D_k^{\,A}\},$$

$$A_{22} = \{F_i^{\,V}, D_j^{\,V}, F_k^{\,A}, D_l^{\,A}\}.$$

The structure of the algebras is $A_{nm} = [\mathrm{SU}(3)]^{m+n}$ as can readily be seen from the definitions of $F_i^{\,V}$, $F_i^{\,A}$, $D_i^{\,V}$, $D_i^{\,A}$.[4] A_{10} is the usual SU(3). $A_{11}{}^{D}$ contains conserved F-type vector currents and D-type axial vector currents, etc.

In this hierarchy of possible symmetries, presumably some are better manifested in nature than the others. Now indications are that (1) the axial vector weak currents are predominantly D;[5] (2) the baryon-pseudoscalar meson coupling is also predominantly D,[6] which is quite consistent with the Goldberger-Treiman relation (or the conserved axial vector currents). Since the D symmetry seems significant, one might also ask whether the D-type vector current is meaningful. We are thus led to $A_{11}{}^{D}$, A_{20}, $A_{21}{}^{\pm}$, and A_{22} as interesting algebras, which we will examine below.

(1) A_{20}.—We assign the 8 baryons $\psi_8 = (\Sigma, \Lambda, N, \Xi)$ together with $\psi_1 = Y_0{}^*(1405\text{ MeV})$ to a 9-dimensional representation of A_{20}. We have four quantum numbers: Y, I_3, Y', and I_3' where the first two are the ordinary (F-type) hyper-

Table I. Y' and I_3' assignments for baryons.

Baryon	p	n	Ξ^-	Ξ^0	Σ^+	Σ^-	Σ^0	Λ^0	$Y_0'{}^*$
Y' [a]	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$		
I_3' [b]	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	0	0			

[a]An empty space means that the respective baryon is not an eigenstate of Y'.
[b]Same as reference a for I_3'.

charge and third component of isotopic spin. The assignment of Y' and I_3' to the baryons is given in Table I. Due to the fact that the neutral $Y=0$ baryons (Σ^0, Λ, $Y_0{}^*$) are not eigenstates of Y' and I_3', the selection rules derived from the latter will be weak for these baryons. Excluding them, we thus find, for example, the selection rules

$$N + \Xi \not\to \Sigma + \Sigma,$$

$$\bar{N} + N \not\to \bar{\Xi} + \Xi \text{ in } I = 1 \text{ channel}.$$

The second relation, together with relations implied by F-type SU(3) only,[7] leads to

$$\sigma(\bar{p}p \to \bar{\Xi}^+\Xi^-) = \sigma(pp \to \bar{\Sigma}^+\Sigma^-)$$

which is compatible with the existing data.[8]

As for the mesons, there should be two octets of vector mesons, V_F and V_D.[9] Such a double octet scheme of vector mesons may have some support in the high-momentum behavior of nucleon form factors.[10] As for the pseudoscalar mesons P, there should be also two octets,[11] with π, K, $\bar{K}$, η belonging to the D type.

The D-type symmetry is presumably badly broken. First there will be mass splittings between ψ_1 and ψ_8, V_F and V_D, etc., which still keeps F-type symmetry intact. Then the Gell-Mann–Okubo splitting will break F, causing also mixing of $Y_0{}^*$ and Λ. If the ψ_1-ψ_8 splitting is such that $m_1 \approx 1400$ MeV, $m_8 \approx 1150$ MeV (before turning on the Gell-Mann–Okubo splitting), the $Y_0{}^*$ should have spin-parity $\frac{1}{2}^+$. If the splitting is so large that $m_0 \approx -1400$ MeV, $m_8 \approx 1150$ MeV, $Y_0{}^*$ should be $\frac{1}{2}^-$.[12] At the same time a Goldberger-Treiman-type argument[13] would suggest the existence of an octet $\{S_i{}^D\}$ of scalar mesons that strongly couples to the divergence of the D currents. The strongest coupling should result from the terms $\sim(|m_1| - |m_8|)\bar{\psi}_{8i}\psi_1 S_i + \text{H.c.}$ (even relative parity) or $\sim i(|m_1| + |m_8|)\bar{\psi}_{8i}\gamma_5\psi_1 S_i + \text{H.c.}$ (odd relative parity), and hence copious production of S mesons is expected in processes of the type $K^- + p \to \text{virtual } Y_0{}^* \to \text{baryon} + S$. In addition, there would be an "incomplete" octet $\{S_i^F\}$ arising from the partial breaking of F symmetry.[14]

(2) A_{11}^D.—This symmetry does not produce the above extra quantum numbers Y', I_3', but leads to conserved D-type axial vector currents. This can be realized by assuming the baryon mass to be of purely dynamical origin arising from spontaneous breakdown of chiral symmetry.[15] The pseudoscalar octet should have D-type coupling to the baryons and should be massless in the limit of exact D symmetry. The axial vector current conservation is broken only by the masses of the P octet, and not by baryon masses.

(3) $A_{21}^{\pm}$.—This symmetry incorporates both the new quantum numbers Y', I_3', and the conserved axial vector currents of type $F+D$ or $F-D$. Neither has definite "R parity," which may be a desirable feature in understanding why the $\{10\}$ baryon resonance multiplet seems favored by nature over $\{\overline{10}\}$ and $\{27\}$.[16] On this basis A_{21}^+ is the preferable one, which means that the coupling $g_{\Xi\Xi\pi} \approx 0$, $g_{\Sigma NK} \approx 0$. The $\Delta S = 1$ leptonic decays of baryons will have the feature that $\Sigma^- \to n$ is pure vector. We have two vector octets V_F, V_D, one axial vector octet A, one pseudoscalar octet P, and two scalar octets S^D and S^F (incomplete). A and P in this case have also definite quantum numbers $Y' = Y$, $I_3' = I_3$. That would imply selection rules such as

$$\bar{K} + N \not\to \pi + \Sigma$$

$$\pi + N \not\to K + \Sigma$$

$$\bar{K} + N \not\to K + \Xi.$$

Of course A_{21} is broken and these rules are not absolute.[17]

In this scheme, the $\frac{3}{2}^+$ decuplet $\{10\}$ becomes a member of a supermultiplet $\{45\}$ which decomposes according to $\{10\}+\{8\}+\{27\}$ under ordinary SU(3). One might tentatively identify $\{8\}$ with the $\frac{3}{2}^-$ π-N second resonance multiplet. As was pointed out above, the parity of $\{8\}$ and $\{27\}$ can be made arbitrary and a mass splitting between the $\{10\}$ and $\{27\}$ of ~3000 MeV would not be surprising.

(4) A_{22}.—This is the largest of our algebras. It does not fix the coupling of pseudoscalar mesons to baryons and leads to the existence of 16 (or less) scalar,[12] 16 pseudoscalar, 16 vector, and 16 axial vector mesons.

We expect the symmetries described above to come to their own at very high energies where the baryon masses themselves become insignificant. Their main predictions, which could be tested in the presently available energy ranges, are the existence of the $0^{\pm}$ and $1^{\pm}$ meson states described above. Should such meson states be discovered in the future the algebras A_{mn} will be a good place to welcome them. The details of this work will be published elsewhere.

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[1]M. Gell-Mann, California Institute of Technology Report CTSL-20 (1961); Phys. Rev. 125, 1067 (1962). The algebras A_{11}^F and A_{11}^D, in our classification below, have previously been discussed by Gell-Mann in this second reference. Y. Ne'eman, Nucl. Phys. 26, 222 (1961).

[2]Y. Nambu, Phys. Rev. Letters 4, 380 (1960); J. Bernstein, S. Fubini, M. Gell-Mann, and W. Thirring, Nuovo Cimento 17, 757 (1960).

[3]In this respect our scheme bears a certain degree of similarity to the W_3-invariance recently proposed by J. Schwinger, Phys. Rev. Letters 12, 237 (1964).

[4]The semisimple algebras A_{mn} can be further embedded in simple algebras A_{mn}^S. One possible choice is $A_{mn}^S = \mathrm{SU}[3(m+n)]$.

[5]N. Cabibbo, Phys. Rev. Letters 10, 531 (1963).

[6]See, e.g., R. Cutkosky, Ann. Phys. (N.Y.) 23, 415 (1963); A. W. Martin and K. C. Wali, Phys. Rev. 130, 2455 (1963); J. J. de Swart and C. K. Iddings, Phys. Rev. 130, 319 (1963).

[7]P. G. O. Freund, Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 924.

[8]R. Armenteros et al., Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 236.

[9]V_D couples to ψ_1 and ψ_8 as opposed to V_F which couples only to ψ_8. We imply here that V_D, V_F are the "gauge fields." However, the existence of another singlet $V_0 \sim \mathrm{Tr}\bar{\psi}\psi$ need not be excluded. As for the new quantum numbers, the combinations $V_F \pm V_D$ have $Y' = \pm Y$, $I_3' = \pm I_3$.

[10]A. P. Balachandran, P. G. O. Freund, and C. R. Schumacher, Phys. Rev. Letters 12, 209 (1964).

[11]The Y' and I_3' assignments for the 16 pseudoscalar mesons are the same as those for the vector mesons mentioned in reference 9.

[12]In the Dirac equation, a γ_5 transformation $\psi \to i\gamma_5\psi$ reverses the sign of mass term and the relative parity.

[13]Y. Nambu and J. J. Sakurai, Phys. Rev. Letters 11, 42 (1963).

[14]The $\kappa(725)$ might be a member of $\{S^D\}+\{S^F\}$. Cf. reference 13.

[15]Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961); 124, 246 (1961). Here we can avoid the

doubling of baryon states [W. Thirring, Nucl. Phys. 10, 97 (1959); F. Gürsey, Ann. Phys. (N.Y.) 12, 91 (1961)] since the vacuum becomes infinitely degenerate. The physical world is built on one of the vacua, and is not invariant under $D_i{}^A$.

[16]The D:F mixing ratio derived from this consideration (for $\{P_i\}$, reference 6) as well as from the weak currents (for $\{A_i\}$, reference 5) is somewhere in the neighborhood of 3:1.

[17]E.g., the first of these processes may well proceed via a $Y_0{}^*(\bar{K}N \rightarrow Y_0{}^*$ is allowed) as long as $Y_0{}^*$ is unstable against decay into $\Sigma\pi$.

ERRATA

NUCLEAR POLARIZATION IN InSb BY A dc CURRENT. W. G. Clark and G. Feher [Phys. Rev. Letters 10, 134 (1963)].

The value of the electron concentration (n) appearing in Figs. 2 and 3 is a transcription error. The value given elsewhere in the Letter, $n = 4\times10^{15}/\mathrm{cm}^3$, is correct. We thank R. L. Mieher for bringing this discrepancy to our attention.

COMPILATION OF RESULTS ON THE TWO-PION DECAY OF THE ω. G. Lütjens and J. Steinberger [Phys. Rev. Letters 12, 517 (1964)].

Reference f in Table III is erroneous. It should read "J. Vander Velde (private communication)."